ON PREAMBLE-BASED CHANNEL ESTIMATION IN OFDM/OQAM SYSTEMS

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ABSTRACT

In this paper, the problem of preamble-based channel estimation in OFDM offset QAM (OFDM/OQAM) systems under a deterministic channel assumption is studied. Recent results indicate that full (all tones carrying pilot symbols) preambles with equal pilot symbols are locally optimal, in the sense that they yield a local minimum of the mean square error (MSE) performance metric of the estimated channel frequency response (CFR), subject to a total training energy constraint. We study the problem of finding the globally optimal full preamble. We show that under a channel constancy assumption, the global optimum coincides with the full preamble of equal symbols. The same result holds even if the channel constancy assumption is removed. Numerical simulations are presented to support the derived results.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is currently enjoying great popularity both in wireless and wired communications [2]. Its basic feature is that it allows to counteract multipath fading and the severe transmission rate degradation due to this fading. The key mechanism of an OFDM system's operation is the use of a cyclic prefix (CP) as a guard interval. With its aid, OFDM can transform a frequency selective channel into a set of parallel flat channels with independent noise disturbances. This greatly simplifies the tasks of channel estimation and data detection at the receiver. The price paid for these attributes is an increased sensitivity to frequency offset and Doppler spread. This is due to Heisenberg's principle: although the subcarrier functions are perfectly localized in time[1], they suffer from spectral leakage in the frequency domain, since they have infinite bandwidth. Moreover, the inclusion of the CP entails a loss in spectral efficiency, which, in practical systems, can become as high as 25% [2].

An alternative to CP-OFDM is a filter bank-based variant employing offset quadrature modulation (OQAM), known as OFDM/OQAM [9]. In this system, good pulse localization both in time and frequency domains is possible at the expense of sacrificing the complex field orthogonality. Nevertheless, the good localization of the pulses increases the robustness of the system to frequency offsets and Doppler effects [8]. Moreover, the spectral containment is enhanced [1].

As mentioned previously, the price paid for the good localization of the pulses both in time and frequency domains is the sacrifice of their complex field orthogonality. The pulses are now orthogonal only over the real field. This implies that there is always an *intrinsic* imaginary interference among (neighboring) subcarriers [4]. As expected, the loss of complex field orthogonality of the pulses affects both the tasks of channel estimation and signal detection. OFDM/OQAM channel estimation has been recently studied for both preamble-based [10, 12] and scattered pilots-based [4, 11] training schemes. Furthermore, in [7], many possible preambles were analyzed and compared. It was shown that the optimal sparse (containing as many pilot tones as the channel impulse response (CIR) taps) preamble for the OFDM/OQAM system contains equipowered and equidistant pilot tones, while a locally optimal full preamble contains equal pilot tones. The notion of optimality in the aforementioned cases is with respect to minimizing the mean square error (MSE) subject to a transmit training energy constraint.

The focus of this paper is on proving the global optimality of the full preamble with equal pilot tones, when the training energy constraint is placed at the output of the transmitter[2] and the rest of the assumptions in [7] are preserved. The key contribution of this paper is the proof of the latter global optimality. The problem of finding the globally optimal full preamble is initially studied under the *channel constancy (CC)* assumption, which is very common in the OFDM/OQAM literature [5, 10, 11, 12]. According to the CC assumption, the CFR coefficients do not vary significantly over neighboring subcarriers. Under this assumption, also used in [7], it is shown that the full preamble containing equal pilot tones is the global optimum. It is also shown that the same full preamble is globally optimum, even when the CC assumption is dropped.

The rest of the paper is organized as follows: Section 2 presents the system model and some necessary mathematical definitions, useful in the following. In Section 3, the proof of the global optimality of the full preamble with equal symbols under the CC assumption is given. Section 4 proves that the same full preamble is globally optimum even when the channel constancy assumption is dropped. Numerical simulations are provided in Section 5, while Section 6 concludes the paper.

Notation. Vectors and matrices are denoted by bold lowercase and uppercase letters, respectively. Superscripts ^{T} and ^{H} stand for transposition and conjugate transposition. The complex conjugate of a complex number z is denoted by z^*. Also, $\imath = \sqrt{-1}$. $\|\cdot\|$ is the Euclidean norm and $|\cdot|$ the absolute value of a real number. For a matrix $\mathbf{A}$, $\mathbf{A}_{i,j}$ denotes its (i, j) entry. The expectation and matrix trace operators are

[1]Therefore forming an orthogonal basis over the complex field

[2]The global optimality of the full preamble with equal pilot tones when the training energy constraint is placed in front of the Synthesis Filter Bank (SFB) has been shown in [7].

denoted by $E(\cdot)$ and $\mathrm{tr}(\cdot)$, respectively. $\mathbf{I}_m$ denotes the mth-order identity matrix, $\mathbf{0}_{m\times n}, \mathbf{1}_{m\times n}$ the $m\times n$ all zeros and all ones matrices, respectively. Finally, $\mathbb{Z}$ denotes the set of integer numbers.

2. SYSTEM MODEL

The baseband discrete-time signal at time instant l, at the output of an OFDM/OQAM Synthesis Filter Bank (SFB) is given by [13]:

$$s(l) = \sum_{m=0}^{M-1} \sum_{n\in\mathbb{Z}} a_{m,n} g_{m,n}(l), \tag{1}$$

where $a_{m,n}$ are real (OQAM) symbols, and $g_{m,n}$ is commonly defined as

$$g_{m,n}(l) = g\left(l - n\frac{M}{2}\right) e^{\imath\frac{2\pi}{M}m\left(l-\frac{L_g-1}{2}\right)} e^{\imath\varphi_{m,n}}, \tag{2}$$

with g being the *real symmetric* prototype filter impulse response (assumed here of unit energy) of length L_g, M being the *even* number of subcarriers, and $\varphi_{m,n} = \varphi_0 + \frac{\pi}{2}(m+n) \bmod \pi$, where φ_0 can be arbitrarily chosen[3] [13]. The filter g is usually designed to have length $L_g = KM$, where K, the overlapping factor, takes on integer values $1 \le K \le 5$ in practice. The double subscript $(\cdot)_{m,n}$ denotes the (m,n)-th frequency-time (FT) point. Thus, m is the subcarrier index and n the OFDM/OQAM symbol time index.

The pulse g is designed so that the associated subcarrier (basis) functions $g_{m,n}$ are orthogonal in the real field, that is

$$\Re\left\{\sum_l g_{m,n}(l) g^*_{p,q}(l)\right\} = \delta_{m,p}\delta_{n,q}$$

where $\delta_{i,j}$ is the Kronecker delta (i.e., $\delta_{i,j} = 1$ if $i = j$ and 0 otherwise). This implies that even in the absence of channel distortion and noise, and with perfect time and frequency synchronization, there will be some intercarrier (and/or intersymbol) interference at the output of the Analysis Filter Bank (AFB), which is purely imaginary, i.e.,

$$\sum_l g_{m,n}(l) g^*_{p,q}(l) = \imath u^{p,q}_{m,n}$$

and it is known as *intrinsic* interference [10]. Here $u^{p,q}_{m,n}$ is a real number.

Adopting the commonly used assumption that the channel is (approximately) frequency flat at each subcarrier and constant over the duration of the prototype filter [10], which is true for practical values of the channel length L_h and L_g and for well time-localized g's, one can express the AFB output at the pth subcarrier and qth OFDM/OQAM symbol as:

$$y_{p,q} = H_{p,q}a_{p,q} + \underbrace{\imath \underbrace{\sum_{m=0}^{M-1}\sum_{n}}_{(m,n)\neq(p,q)} H_{m,n}a_{m,n}u^{p,q}_{m,n}}_{I_{p,q}} + \eta_{p,q}, \tag{3}$$

[3]For example, in [13], $\varphi_{m,n}$ is defined as $(m+n)\frac{\pi}{2} - mn\pi$. An alternative definition would be $\varphi_{m,n} = (m+n)\frac{\pi}{2} - m(n+1)\pi$.

where $H_{p,q}$ is the CFR at that FT point and $I_{p,q}$ the associated interference component. $\eta_{p,q}$ is the additive noise component at the (p,q)-th FT point, which is considered to be the result of a zero mean, white, complex gaussian noise process of variance σ^2 filtered by the AFB at the receiver.

The good localization of the pulses in the time and frequency domains usually leads to concentration of the intrinsic interference in a neighborhood $\Omega_{p,q}$ around the FT point (p,q). With this assumption, (3) is written as:

$$y_{p,q} = H_{p,q}a_{p,q} + \imath \underbrace{\sum_m\sum_n}_{(m,n)\in\Omega_{p,q}} H_{m,n}a_{m,n}u^{p,q}_{m,n} + \eta_{p,q}. \tag{4}$$

In practice, we consider $\Omega_{p,q}$ to be the *first*-order neighborhood of (p,q), denoting it as $\Omega^1_{p,q}$, i.e., $\Omega^1_{p,q} = \{(p\pm1, q\pm1), (p, q\pm1), (p\pm1, q)\}$.

In the rest of the paper, the set $\{0,1,\ldots,M-1\}$ will be viewed as *circular*, in the sense that, e.g., when $m=0$, then $m-1 = M-1$.

3. GLOBALLY OPTIMAL FULL PREAMBLE

As in [7], we will consider a preamble structure consisting of two vector symbols, namely a nonzero training vector which is transmitted at $n=0$, followed by a zero vector. The time index $n=0$ will be henceforth omitted for simplicity. Then, any frame or subframe will have the form:

$$\begin{bmatrix} a_0 & 0 & d_{0,2} & \cdots & d_{0,T_F-3} \\ a_1 & 0 & d_{1,2} & \cdots & d_{1,T_F-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{M-1} & 0 & d_{M-1,2} & \cdots & d_{M-1,T_F-3} \end{bmatrix}$$

where $d_{m,n}$ is a data symbol at the (m,n)-th FT point. T_F is the frame length, while $[a_0, a_1, \ldots, a_{M-1}]^T$ is the initial training vector. The zero vector in the preamble aims at protecting the nonzero training vector from intrinsic interference due to the data section of the frame. We will call the nonzero training vector, a *preamble vector* [10]. In wireless communications standards (e.g., WiMAX [2]), there are sufficiently long guard periods between the uplink and downlink subframes and between frames. Thus, there is no need to worry about intrinsic interference on the preamble vector from previous frames. The preamble vector is called *full*, if it contains pilots in all subcarriers. It is called *sparse*, if it contains L_h isolated pilots and zeros at the rest of its entries.

For the sake of facilitating the analysis, and with a harmless abuse of the OQAM modulation, we will henceforth include the phase factor $e^{\imath\varphi_{m,n}}$ in the preamble symbols, i.e., $x_m = a_m e^{\imath\varphi_m}$, where $\varphi_m \in \{0, \pm\pi, \pm\frac{\pi}{2}\}$. This of course also requires that we accordingly modify the previous definition of $g_{m,0}(l)$ for this part of the frame/subframe. Then the intrinsic interference will be purely real, equal to $u^{p,0}_{m,0}$. Since $g(l)$ is a real and (evenly) symmetric prototype function, $g^2(l)$ is also real and (evenly) symmetric. With this in mind, one can readily see that the interference concerning the entries of the nonzero preamble vector is such that

$$\sum_l g_{m-1}(l) g^*_m(l) = \sum_l g_{m+1}(l) g^*_m(l) = \beta,$$

where β is a real number[4]. For commonly used pulses, it is $|\beta| \leq 1/2$. We use the same assumption as in [7] that $\beta > 0$. This can always be achieved, for if $\beta < 0$, we can incorporate the sign factor $e^{\imath\varphi'_{m,0}} = e^{\imath m\pi}$ to the g_m's above, yielding a positive β.

For the received sample corresponding to the training symbol x_m, we can then write:

$$y_m = H_m x_m + H_{m-1} x_{m-1}\beta + H_{m+1} x_{m+1}\beta + \eta_m. \quad (5)$$

Using the CC assumption, we can consider that $H_m \approx H_{m-1} \approx H_{m+1}$, and the last equation is written as:

$$y_m \approx H_m \left(x_m + x_{m-1}\beta + x_{m+1}\beta\right) + \eta_m. \quad (6)$$

Placing all the received observations in a vector, we can write:

$$\mathbf{y} = \mathbf{T}\mathbf{H} + \eta$$

where $\mathbf{y} = [y_0, y_1, \ldots, y_{M-1}]^T$, $\mathbf{H} = [H_0, H_1, \ldots, H_{M-1}]^T$, $\eta = [\eta_0, \eta_1, \ldots, \eta_{M-1}]^T$ and $\mathbf{T} = \mathrm{diag}\left(T_0, T_1, \ldots, T_{M-1}\right)$. Here, $T_0 = x_0 + x_1\beta \pm x_{M-1}\beta$, $T_{M-1} = x_{M-1} + x_{M-2}\beta \pm x_0\beta$, $T_j = x_j + x_{j-1}\beta + x_{j+1}\beta$, $j = 1, 2, \ldots, M-2$. In the following, for homogeneity reasons, we will assume that $T_0 = x_0 + x_1\beta + x_{M-1}\beta$, $T_{M-1} = x_{M-1} + x_{M-2}\beta + x_0\beta$, or we can consider that the border subcarriers carry null symbols[5].

If there is no statistical assumption about the channel, the least-squares (LS) criterion is meaningful. Then our channel estimate will be given by:[6]

$$\hat{\mathbf{H}} = \mathbf{T}^{-1}\mathbf{y}. \quad (7)$$

The error covariance matrix for (7) is $\mathbf{C}_{\hat{\mathbf{H}}} = \mathbf{T}^{-1}\mathbf{C}_\eta\mathbf{T}^{-H}$, where $\mathbf{C}_\eta = E[\eta\eta^H]$ is given by [7]: $\mathbf{C}_\eta = \sigma^2\mathbf{B}$. $\mathbf{B}$ is the circulant matrix with first row $[1, \beta, 0, \ldots, 0, \beta]$.

Lemma 1 $\mathbf{C}_\eta$ *is invertible if and only if* $\cos(2\pi m/M) \neq -1/(2\beta), m = 0, 1, \ldots, M-1$.

Proof

To obtain the condition of the lemma, simply observe that $\mathbf{C}_\eta = \sigma^2\mathbf{B}$ and $\mathbf{B}$ is circulant. Thus, $\mathbf{C}_\eta$ is diagonalizable by the DFT matrix, with eigenvalues equal to $\{\sigma^2(1 + 2\beta\cos(2\pi m/M))\}_{m=0}^{M-1}$. $\mathbf{C}_\eta$ is invertible iff all these eigenvalues are $\neq 0$, which yields the desired condition. ■

According to [7], the performance of the LS estimator given by (7) will be enhanced if $\hat{\mathbf{H}}$, produced by a full preamble, is further processed by $\mathbf{K} = \mathbf{F}_{M\times L_h}\left(\mathbf{F}^H_{M\times L_h}\mathbf{F}_{M\times L_h}\right)^{-1}\mathbf{F}^H_{M\times L_h}$, where $\mathbf{F}_{M\times L_h}$ is the left $M \times L_h$ submatrix of the normalized $M \times M$ DFT matrix $\mathbf{W}$. In essence, the LS performance is improved based on our a priori knowledge about the channel length, L_h. The filter $\mathbf{K}$ projects our LS CFR estimate onto the space of the CFR's generated by CIR's of length L_h. The optimal training design problem is thus formulated as follows:

$$\min_{x_m, m=0,1,\ldots,M-1} E\left[\left\|\mathbf{K}\hat{\mathbf{H}} - \mathbf{H}\right\|^2\right] \quad (8)$$

$$\text{s.t.} \quad \sum_{m=0}^{M-1}\left(|x_m|^2 + \beta x_m x^*_{m-1} + \beta x_m x^*_{m+1}\right) \leq \mathscr{E}, \quad (9)$$

where the constraint (9) is given by eq. (48) in [7]. The constraint can be seen to incorporate the energy of the input, namely $\sum_{m=0}^{M-1}|x_m|^2$, and two extra terms due to the cross-correlation of adjacent pulses in $\Omega^1_{m,0}$.

For immediate mathematical tractability of the selection of the T_i's and consequently the x_i's, the analysis in [7] is performed for the channel estimator (7), before the application of the filter $\mathbf{K}$. For the same reasons and to be in accordance with [7], we will deal with the optimization problem of the training symbols, before the filtering of $\hat{\mathbf{H}}$ by $\mathbf{K}$ in the rest of the paper[7].

The optimal training design problem, subject to a total training energy constraint at the *output* of the SFB, is formulated as follows:

$$\min_{x_m, m=0,1,\ldots,M-1} \mathrm{tr}\left\{\mathbf{T}^{-1}\mathbf{C}_\eta\mathbf{T}^{-H}\right\} \quad (10)$$

$$\text{s.t.} \quad \sum_{m=0}^{M-1}\left(|x_m|^2 + \beta x_m x^*_{m-1} + \beta x_m x^*_{m+1}\right) \leq \mathscr{E}. \quad (11)$$

We now prove that the full preamble with equal symbols is also a *globally* optimal solution with this energy constraint.

Theorem 1 *Assuming that* $\mathbf{C}_\eta$ *is nonsingular, the global minimizer for the optimization problem (10)-(11) is the full preamble with equal symbols.*

Proof

The objective function (10) can be written as:

$$\mathrm{tr}\left\{\mathbf{T}^{-1}\mathbf{C}_\eta\mathbf{T}^{-H}\right\} = \sum_{i=0}^{M-1}\frac{\sigma^2}{|T_i|^2}. \quad (12)$$

In essence, due to the fact that $\mathbf{T}$ is diagonal, the problem is equivalent to considering $\mathbf{C}_\eta = \sigma^2\mathbf{I}_M$ instead of its circulant form.

Due to the stability of the SFB, i.e., the stability of all the subchannel filters, the constraint (11) implies that there is an upper bound to the energy at the input of the SFB, namely, $\sum_{m=0}^{M-1}|x_m|^2 \leq E$.[8] Using this natural bound, in Proposition 4 of [7], it is shown that $\sum_{i=0}^{M-1}|T_i|^2 \leq E(1+2\beta)^2$ for any choice of $\{T_i\}_{i=0}^{M-1}$. To derive this bound, (11) has been used. This bound is actually tight, in the sense that there are choices $\{x_m\}_{m=0}^{M-1}$ achieving it, e.g., $x_0 = x_1 = \cdots = x_{M-1} = x = \sqrt{\frac{E}{M}}e^{\imath\theta}$, $\theta \in \{0, \pm\pi, \pm\pi/2\}$. This constraint can replace constraint (11), when seeking for the optimal training vector, as long as a possible minimizer for this new problem yields our original constraint (11) tight.The latter is trivial in our case, as the following analysis demonstrates.

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[4]There might be a peculiarity with respect to the boundary subcarriers. The interference introduced by the 0th subcarrier to the $(M-1)$th and vice versa can be $\pm\beta$ depending on the used pulse.

[5]The placement of nulls in the boundary subcarriers is a common practice for spectral masking purposes, as well.

[6]It is straightforward to check that the last estimator delivers an unbiased estimate with the same error covariance as $\left(\mathbf{T}^H\mathbf{C}_\eta^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{C}_\eta^{-1}\mathbf{y}$.

[7]Although the performances presented in the simulations section take $\mathbf{K}$ into account (see also [7] for clarifications).

[8]Following [7], it can be readily seen that $E = \mathscr{E}/(1+2\beta)$.

Using the Arithmetic-Harmonic mean (AH) inequality, we can write:

$$\sum_{i=0}^{M-1} \frac{\sigma^2}{|T_i|^2} \geq \frac{\sigma^2 M^2}{\sum_{i=0}^{M-1} |T_i|^2} \geq \frac{\sigma^2 M^2}{E(1+2\beta)^2}$$

with equality iff $|T_i|^2 = E(1+2\beta)^2/M$ for all i. An obvious choice for this condition to be achieved is that $x_0 = x_1 = \cdots = x_{M-1} = x = \sqrt{\frac{E}{M}} e^{\imath\theta}$. It is trivial to see that the constraints $\sum_{m=0}^{M-1} |x_m|^2 \leq E$, $\sum_{i=0}^{M-1} |T_i|^2 \leq E(1+2\beta)^2$ and (11) are tight at the optimal solution $x_0 = x_1 = \cdots = x_{M-1} = x = \sqrt{\frac{E}{M}} e^{\imath\theta}$. This completes our proof. ■

4. DROPPING THE CHANNEL CONSTANCY ASSUMPTION

It is intuitive to expect that due to "symmetry" of the system in the channel and noise directions, the equal pilot symbols will constitute a good preamble. By "symmetry" of the system, we mean that the additive noises in all subcarriers have the same variance, while the crosscorrelations of adjacent pulses are the same. Also, the symmetry of the system is associated with an assumption of lack of side channel state information. Side channel state information could be, for example, the covariances of the individual CFR coefficients that would impose a greater power loading on certain subcarriers as opposed to others, so that the total variance of the CFR estimators is minimized. It is intuitive to expect that the described symmetry would not allow some of the CFR coefficients to be favorably treated as opposed to others.

We now prove that the full preamble with equal symbols is still the optimal preamble in the case that the CC condition is dropped. In that case,[9] the matrix $\mathbf{T}$ becomes $\mathbf{BD}$, where $\mathbf{D} = \text{diag}(x_0, x_1, \ldots, x_{M-1})$.

Theorem 2 *Assuming that $\mathbf{C}_\eta$ is invertible, the full preamble with equal symbols is the global minimizer of (10)-(11) for $\mathbf{T} = \mathbf{BD}$.*

Proof

Consider the full preamble with equal pilot symbols. Then $\mathbf{T} = x\mathbf{B}$, where $x = x_0 = x_1 = \cdots = x_{M-1}$. It is straightforward to check that such a symbol selection is efficient in the sense that it can satisfy the constraint (11) or the constraint $\sum_{i=0}^{M-1} |T_i|^2 \leq E(1+2\beta)^2$ with equality for appropriate x. With this $\mathbf{T}$, $\mathbf{C}_{\hat{\mathbf{H}}} = (\sigma^2/|x|^2)\mathbf{B}^{-1}$, with eigenvalues $\left\{\lambda_m = (\sigma^2/|x|^2)/\left(1+2\beta\cos\left(\frac{2\pi}{M}m\right)\right)\right\}_{m=0}^{M-1}$ by Lemma 1. If we denote $w = e^{-\imath\frac{2\pi}{M}}$, then $(1/\sqrt{M})w^{ij}$, $i,j = 0,1,\ldots,M-1$ is the (i,j)-th element of the DFT matrix $\mathbf{W}$. Then, the j-th diagonal entry of $\mathbf{C}_{\hat{\mathbf{H}}}$ is written as:

$$(\mathbf{C}_{\hat{\mathbf{H}}})_{jj} = \frac{1}{M}\sum_{m=0}^{M-1} \lambda_m |w^{mj}|^2 = \frac{1}{M}\sum_{m=0}^{M-1} \frac{\sigma^2/|x|^2}{1+2\beta\cos\left(\frac{2\pi}{M}m\right)}$$

for all j. This shows that the corresponding LS CFR estimator delivers CFR estimates with the same error variance (aforementioned symmetry of the system).

[9] Generally speaking, the CC assumption becomes inaccurate for time domain channel lengths approximately equal to or greater than $M/4$.

Moreover, it is very easy to see that the corresponding objective function $\text{tr}\{\mathbf{C}_{\hat{\mathbf{H}}}\}$ is written as: $\sum_{m=0}^{M-1} \alpha/|x_m|^2$, where $\alpha = (1/M)\sum_{m=0}^{M-1}[\sigma^2/(1+2\beta\cos(2\pi m/M))]$. The global optimality of the full preamble with equal symbols is ensured by using the same argument as in the proof of Theorem 1, with the constraint $\sum_{i=0}^{M-1} |T_i|^2 \leq E(1+2\beta)^2$ replaced in this case by $\sum_{i=0}^{M-1} |x_i|^2 \leq E$ due to SFB stability. Again the optimal equal pilot solution is tight for both the last two constraints and (11). ■

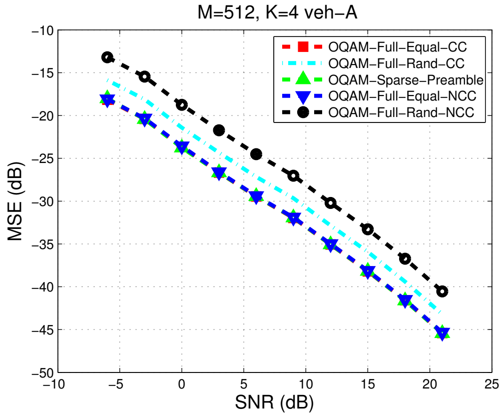
Remark: Using the fact that $1+2\beta\cos(2\pi m/M) \leq 1+2\beta$, it can be easily checked that with the same available training energy, the optimal LS estimator assuming channel constancy is better than the last one, even when the CC assumption becomes almost invalid. This is a direct consequence of comparing the optimal values of (10) for the optimal $\mathbf{T}$'s in the CC and the non CC cases. Intuitively, considering the CC assumption, the optimal full-preamble LS estimator uses pilots $x(1+2\beta)$. Thus, it essentially trades accuracy in the frequency direction for greater noise reduction. Within our framework, this tradeoff turns out to be in favor of the LS estimator using the CC assumption, even when this assumption is approximately invalid.

5. NUMERICAL EXAMPLES

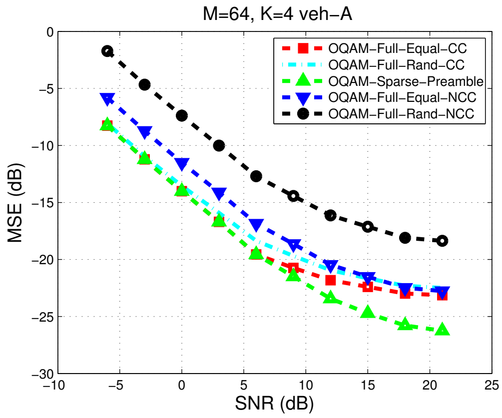
In this section, we present simulation results to verify our analysis. The channel follows the veh-A model [2]. The CIR is initially generated with 29 taps and then zero padded to the closest power of two, that is, $L_h = 32$ taps. For the OFDM/OQAM system, we have used filter banks as given in [3]. We plot the normalized MSE (NMSE), i.e., $E(\|\mathbf{H}-\hat{\mathbf{H}}\|^2/\|\mathbf{H}\|^2)$, versus the transmit SNR.

The optimal full (OQAM-Full-Equal-CC) and sparse preambles (OQAM-Sparse-Preamble) for the OFDM/OQAM system under the CC assumption are compared in Fig. 1 for different M,K. According to [6, 7], the optimal sparse preamble consists of L_h equispaced and equipowered pilot symbols. Note that the optimal sparse preamble is not affected by the validity or not of the CC assumption, due to its sparsity. The OQAM-Full-Rand-CC scheme uses the CC assumption to estimate the CFR coefficients, while it employs randomly selected but otherwise co-phased training symbols. The OQAM-Full-Equal-NCC and OQAM-Full-Rand-NCC schemes ("NCC" stands for "Not CC") do not use the CC assumption, while they employ the same symbols as OQAM-Full-Equal-CC and OQAM-Full-Rand-CC, respectively. All schemes have the same energy at the output of the SFB.

The global optimality of the OQAM-Full-Equal-CC preamble is demonstrated in both cases, while its relative performance with respect to the validity or not of the CC assumption against the optimal sparse preamble is also shown. In Fig. 1(a), the CC assumption holds due to the large ratio M/L_h. In this case, the curves of the optimal sparse, full-equal-cc and full-equal-ncc preambles coincide, while these preambles outperform the preambles with different power loadings. In Fig. 1(b), however, the CC assumption is almost invalidated because of the small ratio M/L_h, i.e., the neighboring channel coefficients may differ significantly. All the CC and the optimal sparse schemes outperform the NCC schemes, verifying the remark at the end of the last section. Observe the error floor exhibited at high SNR values



(a)



(b)

Figure 1: NMSE performance with $K = 4$ and (a) $M = 512$, (b) $M = 64$.

in Fig. 1(b). This is due to the fact that the CC assumption is not valid in this case, and hence residual interference results, which prevails over background noise in the high SNR regime. Notice that this effect is less significant in the sparse preamble, due to its relative robustness to interference.

6. CONCLUSION

The problem of optimal preamble design for LS channel estimation in OFDM/OQAM systems was addressed in this paper, in the case of full preambles. The full preamble with equal symbols was proved to be a globally optimal preamble under the CC assumption or even in the case that the CC assumption is removed, when the assumed analysis framework is the same as in [7]. Furthermore, it is generally better to assume CC, even when this assumption is approximately invalid. Our analytical results were supported by numerical simulations.

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