Energy-Efficient Link Adaptation on Parallel Channels

Christian Isheden and Gerhard P. Fettweis
Technische Universität Dresden, Vodafone Chair Mobile Communications Systems,
D-01069 Dresden, Germany
Email: (see http://www.vodafone-chair.com/)

***Abstract*—Energy-efficient link adaptation is studied for transmission on a parallel channel. The total power dissipation model includes circuit power and a power amplifier inefficiency parameter. Earlier results are derived in various ways based on convex minimization problems and concave maximization problems, respectively. It is shown that the fixed-point algorithm proposed earlier by the authors is equivalent to the Dinkelbach method for solving nonlinear fractional programs.**

I. Introduction

Energy efficiency in mobile communication devices is becoming more and more important because of the increasing gap between power consumption of signal processing circuits and battery capacity. Improved energy efficiency involves minimizing the energy consumption per bit [1] or equivalently maximizing a "throughput per Joule" metric [2]. Previous work on optimization of energy efficiency during transmission has modeled the power dissipated in a mobile terminal during transmission as the sum of a constant power dissipation in the processing circuit and the transmission power divided by the power amplifier drain efficiency.

Energy-efficient link adaptation for parallel channels is an important problem, since both frequency-selective, block fading channels [3] and MIMO channels (via singular value decomposition [4]) can be described with this model.

In this paper, we develop our previous results on energy-efficient link adaptation for parallel channels in [5]. The main results from that paper are derived with the aid of transformed problems that are concave in the case of maximization and convex in the case of minimization, respectively. Thus, the optimum is guaranteed to be global. Furthermore, it is shown that the algorithm proposed in [5] is equivalent to the Dinkelbach method for solving nonlinear fractional programs.

The paper is organized as follows. In Section II, the energy efficiency is maximized over power based on the transformation to a concave program. In Section III, the inverse problem of minimizing energy consumption per bit over rate is considered. In Section IV, the equivalent results of these two optimization problems form the basis of a fixed-point algorithm that is shown to be equivalent to the Dinkelbach method in Section V. Section VI concludes the paper.

II. Maximizing the energy efficiency

Consider a parallel AWGN channel consisting of a set of K non-interfering subcarriers, where the noise is independent across subcarriers. Assuming that perfect channel state information is available at both transmitter and receiver, the maximum rate (in bits/s) that can be reliably transmitted over each subcarrier is

$$r_i = W \log_2(1 + \gamma_i p_i), \quad i = 1, \ldots, K, \tag{1}$$

where W denotes the subcarrier spacing, p_i is the transmit power spectral density of subcarrier i, and γ_i is the channel to noise ratio (CNR) of subcarrier i, given by

$$\gamma_i = \frac{|h_i|^2}{N_0},$$

where $|h_i|^2$ denotes the subcarrier power gain and N_0 is the noise power spectral density.

Efficiency in general is utility divided by cost. In the case of energy efficiency, the utility is the amount of data transferred and the cost is the amount of energy needed for the transmission. Since the energy efficiency (EE) can be calculated as sum rate over total power dissipation, we have the optimization problem

$$\underset{\boldsymbol{p} \in \mathbb{R}_+^K}{\text{maximize}} \quad EE(\boldsymbol{p}) = \frac{\sum_{i=1}^K W \cdot r_i(p_i)}{P_C + \varepsilon \sum_{i=1}^K W \cdot p_i}, \tag{2}$$

where P_C is the circuit power dissipation, and ε is a parameter that expresses power amplifier inefficiency. For simplicity, P_C is assumed to be constant. Since the numerator is concave and the denominator affine (hence convex) in $\boldsymbol{p}$, this is a strictly quasiconcave maximization problem.

To simplify the mathematical analysis, we express the sum rate in nats/s and introduce the parameter

$$\mu = \frac{P_C}{\varepsilon W}$$

that expresses the relative weight of the terms in the cost function. Thus, the problem to be solved is

$$\underset{\boldsymbol{p} \in \mathbb{R}_+^K}{\text{maximize}} \quad q(\boldsymbol{p}) = \frac{\boldsymbol{1}^T \boldsymbol{\rho}(\boldsymbol{p})}{\mu + \boldsymbol{1}^T \boldsymbol{p}}, \tag{3}$$

where

$$\rho_i = \ln(1 + \gamma_i p_i), \quad i = 1, \ldots, K.$$

Note that EE in (2) is related to q by

$$EE = \frac{q}{\varepsilon \ln 2}$$

and that

$$r_i = \frac{W}{\ln 2} \cdot \rho_i.$$

A. Transformation to a concave problem

By the transformation

$$t = 1/(\mu + \mathbf{1}^T \boldsymbol{p}), \quad \boldsymbol{y} = \boldsymbol{p}/(\mu + \mathbf{1}^T \boldsymbol{p}), \quad \boldsymbol{y} \in \mathbb{R}_+^K, \quad t > 0,$$

we obtain the concave maximization problem [6]

$$\begin{array}{ll} \underset{\boldsymbol{y}\in\mathbb{R}_+^K, t>0}{\text{maximize}} & q(\boldsymbol{y}/t) = t\mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}/t) \\ \text{subject to} & \mu t + \mathbf{1}^T\boldsymbol{y} = 1, \end{array} \tag{4}$$

where $\rho_i = \ln\left(1 + \gamma_i \cdot \frac{y_i}{t}\right)$ and the parameter t corresponds to the inverse of the total power dissipation. The problem above is concave since the perspective function in the objective preserves concavity and the equality constraint is affine. By fixing the value of t, we see that the problem of maximizing the energy efficiency in a multi-carrier system is closely related to that of maximizing the sum rate for a given total transmission power allocated to a set of communication channels. It is well known that this problem has an analytical solution given by water-filling of transmission power, see Appendix for details.

B. Mathematical analysis

Since the objective function in (4) is concave and continuously differentiable and the equality constraint function is affine, the KKT conditions are both necessary and sufficient for optimality. After the introduction of a Lagrange multiplier $\nu \in \mathbb{R}$ for the equality constraint, the Lagrangian is

$$\mathcal{L}(\boldsymbol{y}, t, \nu) = t\mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}/t) - \nu(\mu t + \mathbf{1}^T\boldsymbol{y} - 1),$$

hence the KKT conditions are

$$\mu t^* + \mathbf{1}^T\boldsymbol{y}^* = 1,$$

$$t^* \cdot \frac{\partial \rho_i(\boldsymbol{y}^*/t^*)}{\partial y_i} - \nu^* = 0, \quad i = 1, \ldots, K,$$

$$\mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}^*/t^*) + t^* \sum_{i=1}^{K} \frac{\partial \rho_i(\boldsymbol{y}^*/t^*)}{\partial t} - \nu^* \cdot \mu = 0,$$

with

$$\frac{\partial \rho_i}{\partial y_i} = \frac{\frac{1}{t}}{\frac{1}{\gamma_i} + \frac{y_i}{t}}$$

$$\frac{\partial \rho_i}{\partial t} = -\frac{y_i}{t} \cdot \frac{\partial \rho_i}{\partial y_i}.$$

From the second KKT condition,

$$\nu^* = \frac{1}{\frac{1}{\gamma_i} + \frac{y_i^*}{t^*}}, i = 1, \ldots, K.$$

Solving for y_i^* yields

$$y_i^* = t^*\left(\frac{1}{\nu^*} - \frac{1}{\gamma_i}\right), i = 1, \ldots, K.$$

If $\frac{1}{\gamma_i} > \frac{1}{\nu^* \varepsilon \ln 2}$, the resulting y_i^* would be negative, so in this case $y_i^* = 0$. Thus, we have the optimal power allocation

$$p_i^* = \frac{y_i^*}{t^*} = \left[\frac{1}{\nu^*} - \frac{1}{\gamma_i}\right]^+,$$

which is water-filling with a cutoff CNR equal to ν. The corresponding optimal rate allocation can be calculated from

$$\rho_i^* = \left[\ln\frac{1}{\nu^*} - \ln\frac{1}{\gamma_i}\right]^+, i = 1, \ldots, K.$$

Since the unknowns can be calculated as functions of ν, the problem reduces to finding ν^* from the last KKT condition,

$$\nu^* \cdot \mu = \mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}^*/t^*) - t^* \sum_{i=1}^{K} \frac{y_i^*}{t^*} \frac{\partial \rho_i}{\partial y_i^*}.$$

Combining this with the second KKT condition, we have

$$\nu^* \cdot \mu = \mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}^*/t^*) - \sum_{i=1}^{K} \frac{y_i^*}{t^*} \cdot \nu^*,$$

or

$$\nu^* = \frac{\mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}^*/t^*)}{\mu + \mathbf{1}^T\boldsymbol{y}^*/t^*} = t^*\mathbf{1}^T\boldsymbol{\rho}(\boldsymbol{y}^*/t^*) = q(\boldsymbol{y}^*/t^*),$$

since $\mu + \mathbf{1}^T\boldsymbol{y}^*/t^* = 1/t^*$. Thus, at the optimum point the Lagrange multiplier ν is equal to q.

III. Minimizing the energy consumption per bit

Since the numerator and denominator in problem (3) are both positive, equivalently the inverse can be minimized.

Solving (1) for p_i, we get

$$p_i = \frac{2^{r_i/W} - 1}{\gamma_i}, \quad i = 1, \ldots, K, \tag{5}$$

which is convex and continuously differentiable in r_i. For energy-efficient communication, we wish to minimize the cost function energy consumption per bit (denoted E_a), corresponding to the dissipated power divided by the throughput, thus the problem to be solved can be stated as

$$\underset{\boldsymbol{r}\in\mathbb{R}_+^K}{\text{minimize}} \quad E_a(\boldsymbol{r}) = \frac{P_C + \varepsilon\mathbf{1}^T\boldsymbol{p}(\boldsymbol{r})}{\mathbf{1}^T\boldsymbol{r}}, \tag{6}$$

where $p_i(r_i)$ is given by (5).

With the same simplifications as in the maximization case, we get

$$\underset{\boldsymbol{\rho}\in\mathbb{R}_+^K}{\text{minimize}} \quad \frac{1}{q(\boldsymbol{\rho})} = \frac{\mu + \mathbf{1}^T\boldsymbol{p}(\boldsymbol{\rho})}{\mathbf{1}^T\boldsymbol{\rho}}, \tag{7}$$

where

$$p_i = \frac{e^{\rho_i} - 1}{\gamma_i}, \quad i = 1, \ldots, K.$$

Note that the energy consumption per bit E_a can be calculated from

$$E_a = \frac{1}{EE} = \frac{\varepsilon \ln 2}{q}.$$

A. Perspective transformation

The optimization problem (7) is a convex-concave fractional problem with an affine denominator [7],

$$\begin{array}{lll} \underset{x}{\text{minimize}} & f_0(\boldsymbol{x})/(\boldsymbol{c}^T\boldsymbol{x}+d) & \\ \text{subject to} & f_i(\boldsymbol{x}) \le 0, & i=1,\dots,m \\ & A\boldsymbol{x}=\boldsymbol{b}, & \end{array}$$

where $f_0, f_1, \dots, f_m$ are convex, and the domain of the objective function is defined as $\{\boldsymbol{x} \in \mathbf{dom} f_0 | \boldsymbol{c}^T\boldsymbol{x}+d > 0\}$. It can be shown that this problem is strictly quasiconvex [8]. By the one-to-one variable transformation

$$\boldsymbol{y}=\boldsymbol{x}/(\boldsymbol{c}^T\boldsymbol{x}+d), \quad t=1/(\boldsymbol{c}^T\boldsymbol{x}+d), \quad \boldsymbol{y} \in \mathbb{R}_+^K, \quad t>0$$

the problem above becomes

$$\begin{array}{lll} \underset{\boldsymbol{y},t}{\text{minimize}} & t f_0(\boldsymbol{y}/t) & \\ \text{subject to} & t f_i(\boldsymbol{y}/t) \le 0, & i=1,\dots,m \\ & A\boldsymbol{y}=\boldsymbol{b}t & \\ & \boldsymbol{c}^T\boldsymbol{y}+dt=1, & \end{array}$$

where $t f_0(\boldsymbol{y}/t)$ is called the perspective of f_0. This problem is convex since the perspective operation conserves convexity and the equality constraint is affine.

Applying the variable transformation above to problem (7), we obtain

$$\begin{array}{ll} \underset{\boldsymbol{y}\in\mathbb{R}_+^K, t>0}{\text{minimize}} & \mu t + t\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}/t) \\ \text{subject to} & \mathbf{1}^T\boldsymbol{y}=1, \end{array} \tag{8}$$

where

$$p_i(y_i/t) = \frac{e^{y_i/t}-1}{\gamma_i}, \quad i=1,\dots,K.$$

For this particular problem, t corresponds to the inverse sum rate (which can be interpreted as the average nat transmission time), and $\boldsymbol{y}$ corresponds to a normalized rate vector with the fractional distribution of the rates.

B. Mathematical Analysis

Since the objective function in problem (8) is a continuously differentiable convex function and the equality constraint is an affine function, the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient for optimality. After introduction of a Lagrange multiplier $\nu_r \in \mathbb{R}$ for the equality constraint, the Lagrangian is

$$\mathcal{L}(\boldsymbol{y}, t, \nu_r) = \mu t + t\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}/t) + \nu_r(1-\mathbf{1}^T\boldsymbol{y}),$$

hence the KKT conditions are

$$1-\mathbf{1}^T\boldsymbol{y}^*=0$$

$$t^*\frac{\partial p_i}{\partial y_i^*}-\nu_r^*=0, \qquad i=1,\dots,K,$$

$$\mu+\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}^*/t^*)+t^*\sum_{i=1}^K\frac{\partial p_i}{\partial t^*}=0,$$

with

$$\frac{\partial p_i}{\partial y_i} = \frac{e^{y_i/t}}{t\gamma_i}$$

$$\frac{\partial p_i}{\partial t} = -\frac{y_i}{t}\cdot\frac{\partial p_i}{\partial y_i}.$$

The second KKT condition yields

$$\nu_r^* = t^*\frac{\partial p_i}{\partial y_i^*} = \frac{e^{y_i^*/t^*}}{\gamma_i}, \quad i=1,\dots,K.$$

Solving for y_i^*, we get

$$y_i^* = t^*\left(\ln\nu_r^* - \ln\frac{1}{\gamma_i}\right), \quad i=1,\dots,K.$$

If $\frac{1}{\gamma_i} > \nu_r^*$, the resulting y_i^* would be negative, so in this case $y_i^*=0$. Thus, we have the optimal rate allocation

$$\rho_i^* = \frac{y_i^*}{t^*} = \left[\ln\nu_r^* - \ln\frac{1}{\gamma_i}\right]^+,$$

with the corresponding subcarrier power allocations

$$p_i^* = \left[\nu_r^* - \frac{1}{\gamma_i}\right]^+, i=1,\dots,K.$$

These expressions are identical to the ones obtained through maximizing the energy efficiency with $\nu^*=1/\nu_r^*$.

Since all unknowns have been expressed as explicit functions of ν_r^*, the problem reduces to finding ν_r^* from the last KKT condition,

$$\mu+\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}^*/t^*)+t^*\sum_{i=1}^K\left(-\frac{y_i^*}{t^*}\right)\cdot\frac{\partial p_i}{\partial y_i^*}=0.$$

Combining this with the second KKT condition, we have

$$\mu+\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}^*/t^*)-\nu_r^*\cdot\frac{\mathbf{1}^T\boldsymbol{y}^*}{t^*}=0,$$

which leads to the result

$$\nu_r^* = \frac{t^*(\mu+\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}^*/t^*))}{\mathbf{1}^T\boldsymbol{y}^*},$$

or, since $\mathbf{1}^T\boldsymbol{y}=1$,

$$\nu_r^* = t^*(\mu+\mathbf{1}^T\boldsymbol{p}(\boldsymbol{y}^*/t^*)) = E_a^*.$$

That is, at the optimum point we have $\nu_r = E_a$.

IV. Numerical solution algorithm

Summarizing the results from the mathematical analysis of the two equivalent problem formulations, the necessary and sufficient KKT conditions result in the following set of equations at an optimal point (remember that $\nu = 1/\nu_r$):

$$\rho_i^* = \left[\ln\frac{1}{\nu^*} - \ln\frac{1}{\gamma_i}\right]^+, i=1,\dots,K \tag{9}$$

$$p_i^* = \left[\frac{1}{\nu^*} - \frac{1}{\gamma_i}\right]^+, i=1,\dots,K \tag{10}$$

$$\nu^* = \frac{\mathbf{1}^T\boldsymbol{\rho}^*}{\mu+\mathbf{1}^T\boldsymbol{p}^*} \tag{11}$$

Require: q_0 satisfying $F(q_0) \geq 0$, tolerance Δ
$n \leftarrow 0$
repeat
Solve problem (12) with $q = q_n$ to obtain $\boldsymbol{x}_n^*$
$q_{n+1} \leftarrow \frac{f(\boldsymbol{x}_n^*)}{g(\boldsymbol{x}_n^*)}$
$n \leftarrow n + 1$
until $|F(q_n)| \leq \Delta$

Fig. 1. The Dinkelbach method.

This set of nonlinear equations is difficult to solve explicitly. Therefore, in [5] we proposed the following fixed-point algorithm exhibiting superlinear convergence rate:

given initial value ν_0, tolerance Δ
repeat
1) Use ν_n to calculate new values for ρ_i and p_i, $i = 1, \ldots, K$
2) Use these values to compute ν_{n+1}
3) $n := n + 1$

until $|\nu_n - \nu_{n-1}| \leq \Delta$

V. Comparison with the Dinkelbach method

A concave-convex fractional program

$$\underset{\boldsymbol{x} \in \mathbf{S}}{\text{maximize}} \quad q(\boldsymbol{x}) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})},$$

can also be associated with a parametric concave program [10], [11],

$$\underset{\boldsymbol{x} \in \mathbf{S}}{\text{maximize}} \quad f(\boldsymbol{x}) - q g(\boldsymbol{x}), \tag{12}$$

where $q \in \mathbb{R}$ is treated as a parameter. The problem above might be mathematically more tractable than a concave-convex fractional program since it is concave.

Let $\boldsymbol{x}^*$ be an optimal point in problem (12). The optimal value of the objective function,

$$F(q) = f(\boldsymbol{x}^*) - q g(\boldsymbol{x}^*),$$

is a convex, continuous, and strictly decreasing function of q [10]. Moreover, let the optimal value of the objective function in the concave-convex fractional program be denoted by q^*. Then the following statements are equivalent:

$$F(q) > 0 \Leftrightarrow q < q^*$$
$$F(q) = 0 \Leftrightarrow q = q^*$$
$$F(q) < 0 \Leftrightarrow q > q^*$$

Thus, solving the concave-convex fractional program is equivalent to finding the root of the nonlinear equation $F(q) = 0$.

The algorithm described in Fig. 1, known as the Dinkelbach method [10], can be used to find this root. The algorithm is in fact the application of Newton's method to a nonlinear fractional program [11]. Therefore, the sequence converges to the optimal point with a superlinear convergence rate. A detailed convergence analysis can be found in [12]. The initial point can be any $q_0 = \frac{f(\tilde{\boldsymbol{x}})}{g(\tilde{\boldsymbol{x}})}$ with a feasible $\tilde{x}$ that satisfies $F(q_0) \geq 0$.

For problem (3), the corresponding parametric concave optimization problem is

$$\underset{\boldsymbol{p} \in \mathbb{R}_+^K}{\text{maximize}} \quad \mathbf{1}^T \boldsymbol{\rho}(\boldsymbol{p}) - q(\mu + \mathbf{1}^T \boldsymbol{p}), \tag{13}$$

where

$$\rho_i = \ln(1 + \gamma_i p_i), \quad i = 1, \ldots, K$$

and $q \in \mathbb{R}$ is a given parameter.

Problem (13) needs to be solved in each step of Dinkelbach's algorithm. Since it is obvious that a strictly feasible point exists for problem (13), strong duality holds according to Slater's condition [9] and the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. The stationarity condition (obtained by setting the derivative with respect to p_i equal to zero) is

$$\frac{\gamma_i}{1 + \gamma_i p_i^*} - q = 0, \quad i = 1, \ldots, K.$$

Solving this equation for p_i^*, we get

$$p_i^* = \frac{1}{q} - \frac{1}{\gamma_i}, \quad i = 1, \ldots, K.$$

Since the transmit power must be nonnegative, we have

$$p_i^* = \left[\frac{1}{q} - \frac{1}{\gamma_i}\right]^+, \quad i = 1, \ldots, K.$$

The corresponding optimal rate adaptation is given by

$$\rho_i = \left[\ln \frac{1}{q} - \ln \frac{1}{\gamma_i}\right]^+, \quad i = 1, \ldots, K.$$

Comparing these results with the algorithm proposed in [5], we see that the two numerical methods are equivalent with the parameter q corresponding to ν.

VI. Conclusion

We have seen that the results in [5] can be derived in a number of ways based on concave maximization problems or convex minimization problems, respectively. The fixed-point algorithm turns out to be equivalent to the Dinkelbach method for solving concave-convex fractional programs. For detailed convergence properties of the algorithm, the reader is referred to [12].

Appendix A
Water-filling power allocation

Find the subcarrier power allocation p_i that maximizes the capacity under a total power constraint $\sum_{i=1}^K p_i \leq \hat{P}$:

$$\begin{aligned} \underset{\boldsymbol{p} \in \mathbb{R}_+^K}{\text{minimize}} \quad & -\textstyle\sum_{i=1}^K r_i(p_i) \\ \text{subject to} \quad & \textstyle\sum_{i=1}^K p_i \leq \hat{P} \end{aligned}$$

Since the objective and the inequality constraint functions are convex and continuously differentiable, the KKT conditions are both necessary and sufficient for optimality.

Introducing a Lagrange multiplier $\lambda \in \mathbb{R}$ for the inequality constraint, the Lagrangian is

$$\mathcal{L}(\boldsymbol{p}, \lambda) = -\sum_{i=1}^{K} r_i(p_i) + \lambda \left(\sum_{i=1}^{K} p_i - \hat{P} \right),$$

hence the KKT conditions (primal feasibility, dual feasibility, complementary slackness and stationarity) are

$$\begin{aligned} \sum_{i=1}^{K} p_i^* - \hat{P} &\leq 0, \\ \lambda^* &\geq 0, \\ \lambda^* \left(\sum_{i=1}^{K} p_i^* - \hat{P} \right) &= 0, \\ -\frac{dr_i}{dp_i^*} + \lambda^* &= 0, \qquad i = 1, \ldots, K, \end{aligned}$$

respectively. Since $r_i(p_i)$ is monotonically increasing, the last row has no solution if $\lambda^* = 0$, so λ^* has to be strictly greater than 0. Thus, the complementary slackness condition implies $\sum_{i=1}^{K} p_i^* - \hat{P} = 0$, so the total power constraint is always active.

Inserting the expression for $r_i(p_i)$ from (1) we obtain

$$-\frac{W}{\ln 2} \cdot \frac{\frac{\gamma_i}{W}}{1 + \frac{\gamma_i p_i^*}{W}} + \lambda^* = 0, \quad i = 1, \ldots, K.$$

Rearranging yields

$$p_i^* = W \left(\frac{1}{\lambda^* \ln 2} - \frac{1}{\gamma_i} \right), \quad i = 1, \ldots, K.$$

If $\frac{1}{\gamma_i} > \frac{1}{\lambda^* \ln 2}$, the resulting p_i^* would be negative, so in this case $p_i^* = 0$. Thus, we have the optimal power allocation

$$p_i^* = W \cdot \left[\frac{1}{\lambda^* \ln 2} - \frac{1}{\gamma_i} \right]^+, \quad i = 1, \ldots, K,$$

with λ chosen such that the total power constraint is met with equality,

$$W \sum_{i=1}^{K} \left[\frac{1}{\lambda^* \ln 2} - \frac{1}{\gamma_i} \right]^+ = \hat{P}.$$

The sum on the lefthand side is a piecewise-linear increasing function of $\frac{1}{\lambda^* \ln 2}$, with breakpoints at $\frac{1}{\gamma_i}$, so the equation has a unique solution.

The values $\frac{1}{\gamma_i}$ plotted as a function of the subcarrier index i can be thought of as tracing out the bottom of a vessel. If $\hat{P}$ units of water are filled into the vessel, the amount of water in subcarrier i is the power allocated to that subcarrier and $\frac{1}{\lambda^* \ln 2}$ is the water level.

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