HOS BASED ONLINE CALIBRATION

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ABSTRACT

Antenna array calibration is an important task. In this paper, an online calibration method is proposed for antenna arrays where the antennas have unknown gain/phase mismatch and mutual coupling. The mutual coupling matrix is unstructured and the array is randomly deployed on a plane. Two reference sensors are assumed to be perfectly calibrated. The proposed technique uses higher order statistics (HOS) and the reference sensors to estimate the direction-of-arrivals (DOA) of multiple signals. The gain/phase mismatch and mutual coupling parameters are also estimated with a direct approach. Several simulations are done to show the effectiveness of the proposed method.

1. INTRODUCTION

Practical antenna arrays usually require gain/phase and mutual coupling calibration. This task can be done as offline or online. In online calibration, the unknown parameters are found using the information collected by the array for some unknown incoming sources with unknown DOAs.

Online calibration problem is investigated in detail previously in [2] - [5]. In [2] an iterative procedure to compensate the mutual coupling and perturbation of gain and phase parameters is proposed. This method is applicable for only Uniform Linear Array (ULA) and Uniform Circular Array (UCA). Also the performance of the algorithm highly depends on the initial conditions. A similar iterative approach is proposed for randomly deployed sensors in [3]. In this method, it is assumed that there are no gain/phase errors and the mutual coupling matrix is symmetric. As an alternative for the iterative approaches, direct solutions are proposed in [4] and [5]. However, the applicability of these methods is limited for either special arrays or for partly calibrated arrays. In [4], the gain/phase errors are not considered. DOA estimation in partly calibrated arrays is proposed in [5]. In [5], antenna mutual coupling is not considered and each subarray is perfectly calibrated while there are imperfections between subarrays.

In this paper, a new method is proposed to estimate the DOA angles of multiple sources, gain/phase and mutual coupling parameters jointly by using a noniterative or a direct approach. The proposed method is applicable for any arbitrary sensor geometry. It only requires two reference sensors that are perfectly calibrated, which is much less than the required calibrated sensors in [5]. Therefore, considering the previous literature, it can be seen that the proposed method gives a solution for a more general problem. In the proposed method, higher order statistics, more specifically cumulants, are used since it is possible to find the DOA angles and actual array steering matrix directly without being affected by the unknown gain/phase and mutual coupling parameters. Previously, it has been shown that HOS approach is effective for the joint estimation of DOA and sensor positions [1]. Once the actual array steering matrix and DOA angles are found, nominal array steering matrix and unknown parameters are estimated in least squares sense. Simulation results demonstrate the effectiveness of the proposed method.

2. PROBLEM STATEMENT

It is assumed that the planar array is composed of two reference sensors and $K-2$ randomly deployed sensors. The reference sensors are perfectly calibrated and there is no interaction between the reference sensors and the remaining sensors. Therefore, the mutual coupling coefficients between the reference sensors and $K-2$ sensors are zero. Note that the magnitude of the mutual coupling between sensors are inversely proportional with the distance between sensors and may become negligible if the distance exceeds a few wavelength [2]. Hence, the zero assumptions for the mutual coupling between the reference sensors and the remaining sensors can be realized by placing $K-2$ sensors a few wavelength away from the reference sensors. The reference sensors are placed at $(0,0)$ and $(\Delta,0)$ without loss of generality where $\Delta \leq \lambda/2$ and λ is the wavelength. It is assumed that there are $L \leq K$ far field sources that are on the same plane with the sensors and source signals have a non-Gaussian distribution since fourth order cumulants are used. The array output vector, $\mathbf{x}(t) = [x_1(t),...,x_K(t)]^T$, for the narrowband case can be written as,

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{\Gamma}\mathbf{A}(\Theta)\mathbf{s}(t) + \mathbf{v}(t), \qquad t = 1,2,\ldots,N \qquad (1) \\ &= \overline{\mathbf{A}}(\Theta)\mathbf{s}(t) + \mathbf{v}(t) \end{aligned}$$

where, N is the number of snapshots, $\mathbf{s}(t) = [s_1(t),...,s_L(t)]^T$ is the $L \times 1$ vector of L sources, $\mathbf{v}(t) = [v_1(t),...,v_K(t)]^T$ is the $K \times 1$ vector of Gaussian noise. $\mathbf{A}(\Theta)$ and $\overline{\mathbf{A}}(\Theta)$ are the nominal and actual array steering matrices, respectively. $\Theta = [\theta_1,...,\theta_L]$ is the DOA angles of L sources. $\mathbf{\Gamma}$ is the array distortion matrix which is the product of $K \times K$ complex mutual coupling matrix, $\mathbf{M}$, and $K \times K$ diagonal gain/phase mismatch matrix, $\mathbf{T}$, i.e.,

$$\mathbf{\Gamma} = \mathbf{M}\mathbf{T} \qquad (2)$$

The matrices $\mathbf{A}(\Theta)$, $\mathbf{M}$ and $\mathbf{T}$ are defined as

$$\mathbf{A}(\Theta) = \begin{bmatrix} 1 & \dots & 1 \\ e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_1)} & \dots & e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_L)} \\ e^{j\frac{2\pi}{\lambda}\mathbf{p}_3\mathbf{u}(\theta_1)} & \dots & e^{j\frac{2\pi}{\lambda}\mathbf{p}_3\mathbf{u}(\theta_L)} \\ \vdots & & \\ e^{j\frac{2\pi}{\lambda}\mathbf{p}_K\mathbf{u}(\theta_1)} & \dots & e^{j\frac{2\pi}{\lambda}\mathbf{p}_K\mathbf{u}(\theta_L)} \end{bmatrix} \quad (3)$$

$$\mathbf{M} = \begin{bmatrix} 1 & m_{12} & 0 & 0 & 0 & \dots & 0 \\ m_{21} & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & m_{34} & m_{35} & \dots & m_{3K} \\ 0 & 0 & m_{43} & 1 & c_{45} & \dots & m_{4K} \\ 0 & 0 & m_{53} & m_{54} & 1 & \dots & m_{5K} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & m_{K3} & m_{K4} & m_{K5} & \dots & 1 \end{bmatrix} \quad (4)$$

$$\mathbf{T} = diag\begin{pmatrix} 1 & 1 & \alpha_3 e^{j\beta_3} & \dots & \alpha_K e^{j\beta_K} \end{pmatrix} \quad (5)$$

where, θ_i is the direction-of-arrival of i^{th} source in azimuth, $\mathbf{u}(\theta_i) = [cos(\theta_i), sin(\theta_i)]^T$ is the unit direction vector of the i^{th} source, $\mathbf{p}_m = [p_{m,x}, p_{m,y}]$ is the 2D position of the m^{th} sensor. Note that the position of the first and second sensors are $[0,0]$ and $[\Delta, 0]$, respectively. $m_{i,j}$ is the complex mutual coupling between sensor i and j. α_i and β_i are the gain and phase errors of the i^{th} sensor, respectively. Since the reference sensors are assumed to be perfectly calibrated, m_{12} and m_{21} are known and the mutual coupling between the reference sensors and the other sensors are zero, as in (4). Note that due to the normalization in (3) and (4) with respect to the first and diagonal elements, the unknown parameters are found up to a complex scale factor.

The goal in this paper is to estimate both the DOAs of L sources, $\{\theta_i\}_{i=1}^{L}$, and the unknown parameters in $\mathbf{M}$ and $\mathbf{T}$ matrices, $\{m_{i,j}, \alpha_i, \beta_i\}_{i,j=3}^{K}$. There are $2(K-2)^2$ number of real unknown parameters in general.

3. HOS BASED ONLINE CALIBRATION

In this section, HOS based online calibration algorithm is introduced for a solution to the problem described in Section 2. First the cumulant matrix structure is presented and then, the proposed method is discussed.

3.1 Cumulant Matrix

When the sensor i and sensor j are selected as the reference sensors, the cumulant matrix is written as

$$\mathbf{C}_{ij}(k,l) = Cum\left(x_i(t), x_j^*(t), x_k(t), x_l^*(t)\right) \quad (6)$$

where $x_i(t)$ is the output signal of the i^{th} sensor, defined as

$$x_i(t) = \overline{\mathbf{a}}_{ri}\mathbf{s}(t) + v_i(t) \quad (7)$$

where $\overline{\mathbf{a}}_{ri}$ is the i^{th} row of actual array steering matrix, $\overline{\mathbf{A}}$, in (1). By substituting (7) into (6), the fourth order cumulant can be rewritten as,

$$\mathbf{C}_{ij}(k,l) = Cum\left(\overline{\mathbf{a}}_{ri}\mathbf{s}(t) + v_i(t), \overline{\mathbf{a}}_{rj}^*\mathbf{s}^*(t) + v_j^*(t), \overline{\mathbf{a}}_{rk}\mathbf{s}(t) + v_k(t), \overline{\mathbf{a}}_{rl}^*\mathbf{s}^*(t) + v_l^*(t)\right) \quad (8)$$

Since the noise is assumed to be Gaussian, using the cumulant properties [CP1], [CP2] and [CP4] stated in [6], the cumulants in (8) is simplified as,

$$\begin{aligned} \mathbf{C}_{ij}(k,l) &= Cum\left(\overline{\mathbf{a}}_{ri}\mathbf{s}(t), \overline{\mathbf{a}}_{rj}^*\mathbf{s}^*(t), \overline{\mathbf{a}}_{rk}\mathbf{s}(t), \overline{\mathbf{a}}_{rl}^*\mathbf{s}^*(t)\right) \\ &= \left(\overline{\mathbf{a}}_{rk} \otimes \overline{\mathbf{a}}_{rj}^*\right)\mathbf{C}_s\left(\overline{\mathbf{a}}_{rl} \otimes \overline{\mathbf{a}}_{ri}^*\right)^H \end{aligned} \quad (9)$$

where $\mathbf{C}_s$ is the $L^2 \times L^2$ source cumulant matrix in the form of [1],

$$\mathbf{C}_s(i,j) = Cum\left(s_f(t), s_g^*(t), s_m(t), s_n^*(t)\right) \quad (10)$$
$$i = L(m-1) + g, \quad 1 \le m, g \le L$$
$$j = L(n-1) + f, \quad 1 \le n, f \le L$$

Due to the cumulant property [CP5] stated in [6], when the source signals are statistically independent, there are only L nonzero elements in source cumulant matrix, $\mathbf{C}_s$, i.e.,

$$\mathbf{C}_s = diag(\gamma_1, 0, \dots, 0, \gamma_2, 0, \dots, 0, \gamma_L) \quad (11)$$

and

$$\begin{aligned} \gamma_i &= Cum(s_i(t), s_i^*(t), s_i(t), s_i^*(t)) \quad (12) \\ &= \mathbf{C}_s\left(L(i-1)+i, L(i-1)+i\right), \quad 1 \le i \le L \end{aligned}$$

As shown in (12), the non-zero diagonal elements, γ_i, are located with the indices $L(i-1)+i$ for $1 \le i \le L$. Substituting (11) into (9) simplifies the relation in matrix form as

$$\mathbf{C}_{ij} = \overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{rj}}^H \mathbf{R}_s^{HOS} \mathbf{D}_{\overline{\mathbf{a}}_{ri}} \overline{\mathbf{A}}^H \quad (13)$$

where $L \times L$ diagonal matrices $\mathbf{R}_s^{HOS}$ and $\mathbf{D}_{\overline{\mathbf{a}}_{ri}}$ are defined as

$$\mathbf{R}_s^{HOS} = diag(\gamma_1, \gamma_2, \dots, \gamma_L) \quad (14)$$
$$\mathbf{D}_{\overline{\mathbf{a}}_{ri}} = diag(\overline{\mathbf{a}}_{ri}(1), \overline{\mathbf{a}}_{ri}(2), \dots, \overline{\mathbf{a}}_{ri}(L)) \quad (15)$$

3.2 Steering Matrix Estimation

In Section 2, sensor 1 and sensor 2 are selected as the reference sensors in order to find the DOA angles unambiguously. However, it is possible to use any two sensor pair as reference in order to find actual array steering matrix. This is required in order to cope with the number of unknown parameters.

The actual steering matrix is estimated by using the relations between the cumulant matrices for different reference sensors, $\mathbf{C}_{ij}$, $1 \le i, j \le K$. Let $\mathbf{B}_{ij}$ is the $2K \times 2K$ matrix defined as

$$\begin{aligned} \mathbf{B}_{ij} &= \begin{bmatrix} \mathbf{C}_{ii} & \mathbf{C}_{ji} \\ \mathbf{C}_{ij} & \mathbf{C}_{jj} \end{bmatrix} \\ &= \begin{bmatrix} \overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{ri}}^H \mathbf{R}_s^{HOS} \mathbf{D}_{\overline{\mathbf{a}}_{ri}} \overline{\mathbf{A}}^H & \overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{ri}}^H \mathbf{R}_s^{HOS} \mathbf{D}_{\overline{\mathbf{a}}_{rj}} \overline{\mathbf{A}}^H \\ \overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{rj}}^H \mathbf{R}_s^{HOS} \mathbf{D}_{\overline{\mathbf{a}}_{ri}} \overline{\mathbf{A}}^H & \overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{rj}}^H \mathbf{R}_s^{HOS} \mathbf{D}_{\overline{\mathbf{a}}_{rj}} \overline{\mathbf{A}}^H \end{bmatrix} \end{aligned} \quad (16)$$

Array steering matrix estimate for the j^{th} sensor is found from the eigenvalue decomposition of $\mathbf{B}_{ij}$, i.e.,

$$\begin{bmatrix} \mathbf{C}_{ii} & \mathbf{C}_{ji} \\ \mathbf{C}_{ij} & \mathbf{C}_{jj} \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \end{bmatrix} \mathbf{\Lambda}_s \quad (17)$$

where $\mathbf{\Lambda}_s$ is the diagonal matrix composed of the L largest eigenvalues of the matrix $\mathbf{B}_{ij}$ and $2K \times L$ matrix $\mathbf{S} =$

$\left[\mathbf{S}_1^T \ \ \mathbf{S}_2^T\right]^T$ is obtained from the eigenvectors corresponding to these eigenvalues. $\mathbf{S}_1$ and $\mathbf{S}_2$ are $K \times L$ matrices. Substituting (16) into (17) simplifies the relation, i.e.,

$$\overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{ri}}^H \boldsymbol{\Phi} = \mathbf{S}_1 \quad (18)$$

$$\overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{rj}}^H \boldsymbol{\Phi} = \mathbf{S}_2 \quad (19)$$

where $L \times L$ matrix $\boldsymbol{\Phi}$ is defined as

$$\boldsymbol{\Phi} = \left(\mathbf{R}_s^{HOS}\mathbf{D}_{\overline{\mathbf{a}}_{ri}}\overline{\mathbf{A}}^H\mathbf{S}_1 + \mathbf{R}_s^{HOS}\mathbf{D}_{\overline{\mathbf{a}}_{rj}}\overline{\mathbf{A}}^H\mathbf{S}_2\right)\boldsymbol{\Lambda}_s^{-1} \quad (20)$$

Using (18) and (19), the following relation is obtained, i.e.,

$$\begin{aligned} \mathbf{S}_2 &= \mathbf{S}_1\boldsymbol{\Phi}^{-1}\left(\mathbf{D}_{\overline{\mathbf{a}}_{ri}}^H\right)^{-1}\mathbf{D}_{\overline{\mathbf{a}}_{rj}}^H\boldsymbol{\Phi} \\ \mathbf{S}_1^\dagger\mathbf{S}_2\boldsymbol{\Phi}^{-1} &= \boldsymbol{\Phi}^{-1}\mathbf{D}_{ij}^H \end{aligned} \quad (21)$$

where $\mathbf{S}_1^\dagger = \left(\mathbf{S}_1^H\mathbf{S}_1\right)^{-1}\mathbf{S}_1^H$ is the Moore-Penrose pseudoinverse operator and $L \times L$ diagonal matrix $\mathbf{D}_{ij}$ is defined as

$$\begin{aligned} \mathbf{D}_{ij} &= \mathbf{D}_{\overline{\mathbf{a}}_{ri}}^{-1}\mathbf{D}_{\overline{\mathbf{a}}_{rj}} \\ &= diag\left(\overline{\mathbf{a}}_{rj} \oslash \overline{\mathbf{a}}_{ri}\right) \\ &= diag\left(\frac{\overline{\mathbf{a}}_{rj}(1)}{\overline{\mathbf{a}}_{ri}(1)}, \frac{\overline{\mathbf{a}}_{rj}(2)}{\overline{\mathbf{a}}_{ri}(2)}, \ldots, \frac{\overline{\mathbf{a}}_{rj}(L)}{\overline{\mathbf{a}}_{ri}(L)}\right) \end{aligned} \quad (22)$$

where $\oslash$ is the element by element division.

As it is seen from (21), $L \times L$ diagonal matrix $\mathbf{D}_{ij}^H$ can be found from the eigenvalue decomposition of matrix $\mathbf{S}_1^\dagger\mathbf{S}_2$. The matrix $\mathbf{D}_{ij}^H$ is composed of the eigenvalues and $\boldsymbol{\Phi}^{-1}$ is the matrix composed of the corresponding eigenvectors of $\mathbf{S}_1^\dagger\mathbf{S}_2$, respectively..

It is important to note that the solution in (21) has a permutation ambiguity. This is due to the fact that there is no a priori information to guarantee that the p^{th} eigenvalue corresponds to the p^{th} source. Let $\mathbf{P}_{ij}$ be a $L \times L$ permutation matrix in case of the $\mathbf{D}_{ij}$ in (21). Then, (21) can be written in more general form as

$$\mathbf{S}_1^\dagger\mathbf{S}_2\boldsymbol{\Phi}^{-1}\mathbf{P}_{ij} = \boldsymbol{\Phi}^{-1}\mathbf{D}_{ij}^H\mathbf{P}_{ij} \quad (23)$$

Since matrix $\mathbf{D}_{ij}$ is a diagonal matrix, the right hand side of (23) can be written as

$$\boldsymbol{\Phi}^{-1}\mathbf{D}_{ij}^H\mathbf{P}_{ij} = \boldsymbol{\Phi}^{-1}\mathbf{P}_{ij}\tilde{\mathbf{D}}_{ij}^H \quad (24)$$

where

$$\tilde{\mathbf{D}}_{ij} = diag\left(\left(\overline{\mathbf{a}}_{rj} \oslash \overline{\mathbf{a}}_{ri}\right)\mathbf{P}_{ij}\right) \quad (25)$$

Substituting (24) into (23) results

$$\mathbf{S}_1^\dagger\mathbf{S}_2\left(\boldsymbol{\Phi}^{-1}\mathbf{P}_{ij}\right) = \left(\boldsymbol{\Phi}^{-1}\mathbf{P}_{ij}\right)\tilde{\mathbf{D}}_{ij}^H \quad (26)$$

It is shown in (25) and (26) that the j^{th} row of actual array steering matrix, $\overline{\mathbf{A}}$, is estimated up to a scale factor and column permutation from the eigenvalue decomposition of matrix $\mathbf{S}_1^\dagger\mathbf{S}_2$, i.e., $diag(\tilde{\mathbf{D}}_{ij}) = \left(\overline{\mathbf{a}}_{rj} \oslash \overline{\mathbf{a}}_{ri}\right)\mathbf{P}_{ij}$.

The permutation ambiguity is solved by aligning each estimated row of actual array steering matrix according two reference sensors, specifically sensor 1 and sensor 2. This is done by considering $\mathbf{P}_{12}$ as it is and by changing the order of the columns of $\mathbf{P}_{1j}$ and $\mathbf{P}_{2j}$ permutation matrices. While $\mathbf{B}_{12}$ and $\tilde{\mathbf{D}}_{12}$ are found once, as the value of j changes from 3 to K, $\mathbf{B}_{1j}$, $\mathbf{B}_{2j}$, $\tilde{\mathbf{D}}_{1j}$ and $\tilde{\mathbf{D}}_{2j}$ are found by using (16) - (26). The cost function used in this process is selected as

$$\left\{m_j^{(n)}, k_j^{(n)}\right\} = arg \min_{1 \le m_j, k_j \le L} \left\|\overline{\mathbf{a}}_{r2/r1}(n)\overline{\mathbf{a}}_{rj/r2}(k_j) - \overline{\mathbf{a}}_{rj/r1}(m_j)\right\|^2 \quad (27)$$

where $\overline{\mathbf{a}}_{rj/ri}(k)$ is the k^{th} element of $1 \times L$ vector $\left(\overline{\mathbf{a}}_{rj} \oslash \overline{\mathbf{a}}_{ri}\right)\mathbf{P}_{ij}$. $m_j^{(n)}$ and $k_j^{(n)}$ are the indices for alignment, i.e.,

$$\begin{aligned} \mathbf{P}_{12} &= \left[\begin{array}{cccc} \mathbf{p}_{12}(1) & \mathbf{p}_{12}(2) & \ldots & \mathbf{p}_{12}(L) \end{array}\right] \\ &= \left[\begin{array}{cccc} \mathbf{p}_{1j}(m_j^{(1)}) & \mathbf{p}_{1j}(m_j^{(2)}) & \ldots & \mathbf{p}_{1j}(m_j^{(L)}) \end{array}\right] \\ &= \left[\begin{array}{cccc} \mathbf{p}_{2j}(k_j^{(1)}) & \mathbf{p}_{2j}(k_j^{(2)}) & \ldots & \mathbf{p}_{2j}(k_j^{(L)}) \end{array}\right] \end{aligned} \quad (28)$$

where $\mathbf{p}_{ij}(k)$ is the k^{th} column of matrix $\mathbf{P}_{ij}$. Then, using the relation in (28) the array steering matrix is found up to scale factor as

$$\begin{aligned} \tilde{\overline{\mathbf{A}}} &= \left[\begin{array}{c} \mathbf{1}_L \\ \left(\overline{\mathbf{a}}_{r2} \oslash \overline{\mathbf{a}}_{r1}\right)\mathbf{P}_{12} \\ \left(\overline{\mathbf{a}}_{r3} \oslash \overline{\mathbf{a}}_{r1}\right)\mathbf{P}_{12} \\ \vdots \\ \left(\overline{\mathbf{a}}_{rK} \oslash \overline{\mathbf{a}}_{r1}\right)\mathbf{P}_{12} \end{array}\right] = \left\{\left[\begin{array}{c} \overline{\mathbf{a}}_{r1} \\ \overline{\mathbf{a}}_{r2} \\ \overline{\mathbf{a}}_{r3} \\ \vdots \\ \overline{\mathbf{a}}_{rK} \end{array}\right]\mathbf{D}_{\overline{\mathbf{a}}_{r1}}^{-1}\right\}\mathbf{P}_{12} \\ &= \left(\overline{\mathbf{A}}\mathbf{D}_{\overline{\mathbf{a}}_{r1}}^{-1}\right)\mathbf{P}_{12} \end{aligned} \quad (29)$$

where $\mathbf{1}_L$ is the $1 \times L$ vector with all ones. Note that $\mathbf{P}_{12}$ is taken as the reference during the permutation ambiguity resolution. The effect of this on the final steering matrix estimate in (29) is a possible change in the order of the columns of the actual array steering matrix. This does not pose any problem since sources and the corresponding DOAs can be ordered arbitrarily. Therefore, $\mathbf{P}_{12}$ in (29) is replaced by identity without loss of generality in the following parts.

3.3 DOA Estimation

The DOA angles are estimated using the relation between the first and the second rows of the actual array steering matrix, i.e., $\overline{\mathbf{a}}_{r2/r1}(k)$, given in (25) for $i = 1$ and $j = 2$. After substituting (2) - (5) into (1), $\overline{\mathbf{a}}_{r2/r1}(k)$ can be written as,

$$\overline{\mathbf{a}}_{r2/r1}(k) = \frac{m_{21} + e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_k)}}{1 + m_{12}e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_k)}} \quad (30)$$

Then, since m_{12} and m_{21} are assumed to be known, DOA angle of the k^{th} source is found as

$$\theta_k = cos^{-1}\left(\frac{\lambda}{2\pi\Delta}arg\left(\frac{\overline{\mathbf{a}}_{r2/r1}(k) - m_{21}}{1 - \overline{\mathbf{a}}_{r2/r1}(k)m_{12}}\right)\right) \quad (31)$$

where $arg(x)$ is the argument of complex variable x.

3.4 Calibration Parameter Estimation

After finding the DOA angles, $\{\theta_i\}_{i=1}^L$, as in (31), the diagonal matrix, $\mathbf{D}_{\overline{\mathbf{a}}_{r1}}$, is found by substituting (31) into (1), i.e.,

$$\mathbf{D}_{\overline{\mathbf{a}}_{r1}} = diag\left(1 + m_{12}e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_1)}, \ldots, 1 + m_{12}e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_L)}\right) \quad (32)$$

Then, the actual array steering matrix, $\overline{\mathbf{A}}$, is estimated by substituting (32) into (29), i.e.,

$$\overline{\mathbf{A}} = \tilde{\overline{\mathbf{A}}}\mathbf{D}_{\overline{\mathbf{a}}_{r1}} \tag{33}$$

Nominal array steering matrix, $\mathbf{A}$, is found by substituting (31) into (3). After that, the calibration parameters are found in least square sense by minimizing the cost function

$$\begin{aligned} Q &= \|\mathbf{\Gamma}\mathbf{A} - \overline{\mathbf{A}}\|^2 \\ &= \|\left(\mathbf{I}_{K\times K} \otimes \mathbf{A}^T\right)\mathbf{z} - \overline{\mathbf{a}}_r\|^2 \end{aligned} \tag{34}$$

where $\mathbf{z} = vec(\mathbf{\Gamma})$ is the $K^2 \times 1$ vector composed of the entries of matrix $\mathbf{\Gamma}$ in row-wise order and $\overline{\mathbf{a}}_r = vec(\overline{\mathbf{A}})$ is the $KL \times 1$ vector composed of matrix $\overline{\mathbf{A}}$ in row-wise order. However, if some elements of the vector $\mathbf{z}$ are known, (34) can be solved more effectively. Note that this is the case for us since some elements of array distortion matrix are known as seen from (4) and (5). Let $\mathbf{z}_k$ and $\mathbf{z}_u$ are the column vectors composed of the known and unknown elements of vector $\mathbf{z}$, respectively. Then, $\mathbf{z}_u$ can be found as

$$\widehat{\mathbf{z}}_u = \mathbf{F}_u^{\dagger}(\overline{\mathbf{a}}_r - \mathbf{F}_k\mathbf{z}_k) \tag{35}$$

where $\mathbf{F}_k$ and $\mathbf{F}_u$ are the $KL \times N_k$ and $KL \times (K^2 - N_k)$ matrices composed of the columns of matrix $\left(\mathbf{I}_{K\times K} \otimes \mathbf{A}^T\right)$ corresponding to the indices of known and unknown elements of vector $\mathbf{z}$. N_k is the number of known array distortion parameters. Note that the array distortion parameters can be found from (35) if the matrix $\mathbf{F}_u$ is full-column rank which requires, $L \geq \frac{K^2-N_k}{K}$. At the same time, it is required that the number of sources has to be less than or equal to the number of sensors for solving the array steering matrix and DOA estimates using (23). Therefore, the required number of sources for array calibration is related with the number of sensor such as,

$$\frac{K^2 - N_k}{K} \leq L \leq K \tag{36}$$

Note that, under the assumptions given in Section 2, $N_k = 4K - 4$. In this case at least $K - 2$ sources are required for finding the array distortion parameters. The required number of sources can be reduced if further a priori information is available for the array distortion parameters.

After finding the unknown parameters in array distortion matrix, $\mathbf{\Gamma}$, the matrices $\mathbf{T}$ and $\mathbf{M}$ can be found as

$$\mathbf{T} = diag(\mathbf{\Gamma}) \tag{37}$$

$$\mathbf{M} = \mathbf{\Gamma}\mathbf{T}^{-1} \tag{38}$$

The relation in (37) and (38) is written since the matrix $\mathbf{T}$ is diagonal and the diagonal entries of matrix $\mathbf{M}$ is all one as given in (4) and (5).

4. PERFORMANCE EVALUATION

The performance of the proposed algorithm is evaluated for the DOA, gain/phase and mutual coupling parameter estimations. Mutual coupling parameters are the complex valued. Since to the best of our knowledge there is no online calibration for the random sensor deployment and unstructured calibration parameters, Cramér-Rao bound (CRB) is used to show the effectiveness of the proposed algorithm. The derivation of the CRB is not given due to space limitation.

It is assumed that there are $L = 4$ far-field sources and $K = 6$ sensors. Each sensor position except the two reference sensors is randomly selected from a uniform distribution in the deployment area of $2\lambda \times 2\lambda$. The reference sensors are placed at $(0,0)$ and $(\lambda/2,0)$. $N = 1000$ snapshots are collected. The performance results are the average of 100 trials. At each trial, source signals, noise, the sensor positions except the reference sensors, the gain/phase mismatch and mutual coupling parameters and the DOA angles of source signals are changed randomly. The DOA angles are selected randomly in the range of $(30,150)$ degrees. The gain terms in mutual coupling parameters are selected randomly in the range of $(0.1,0.3)$, while it is $(0.8,1.2)$ for the gain terms of the gain/phase mismatch parameters. The phase terms of both mutual coupling and gain/phase mismatch parameters are changed randomly in the range of $(-10,10)$ degrees. The source signals are statistically independent and a uniformly distributed. The noise is additive white Gaussian and uncorrelated with the source signals. There are $(K-2)^2 - (K-2) = 12$ complex unknown mutual coupling parameters that should be estimated. Note that these parameters are in an unstructured matrix given in (4). In addition, there are $K - 2 = 4$ complex unknown parameters for gain/phase mismatch as in (5). The proposed algorithm estimates L unknown DOA angles and $(K-2)^2 = 16$ unknown complex parameters jointly. The figures in this part show the average error for all parameters.

The DOA estimation performance of the proposed method is presented in Fig. 1. The algorithm accurately estimates the DOA angles as the signal-to-noise ratio (SNR) increases.

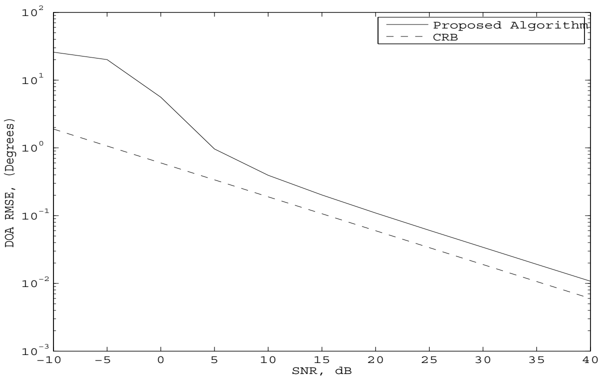

Figure 1: Performance of the proposed algorithm for DOA estimation.

The performance of the proposed algorithm for estimating the mutual coupling parameters is presented in Fig. 2 and Fig. 3. In Fig 2, the average error for the estimation of gain terms in mutual coupling matrix is presented while the error for the phase is given in Fig. 3. As it is seen from these figures, mutual coupling gain terms can be estimated accurately while there is a large margin of improvement for the phase terms. The main reason for the performance in Fig. 3 is due to the small gain values for the mutual coupling parameters.

The estimation performances for gain/phase mismatch terms are presented in Fig. 4 and Fig. 5. In Fig. 4, the

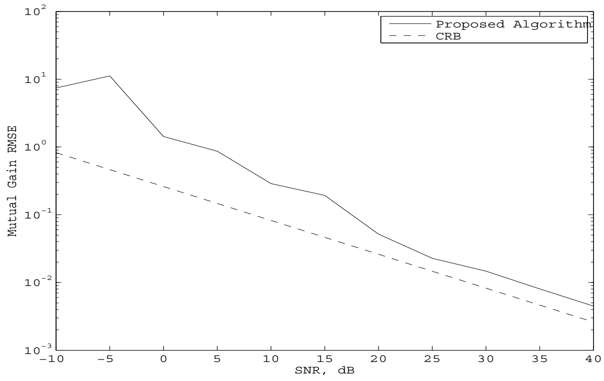

Figure 2: Performance of the proposed algorithm for the estimation of the gain terms of the mutual coupling matrix.

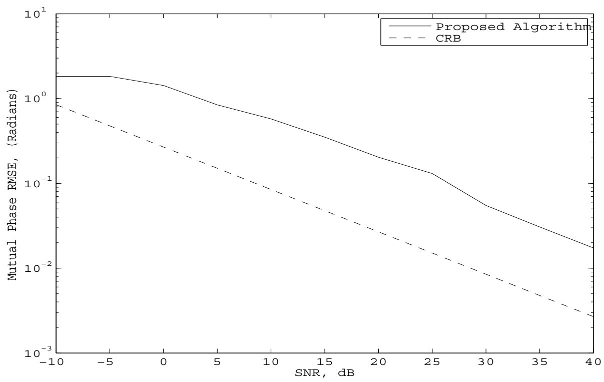

Figure 3: Performance of the proposed algorithm for the estimation of the phase terms of the mutual coupling matrix.

estimation performance of the gain terms is presented while Fig. 5 is for the phase terms. As it can be seen from these figures, both gain and phase terms can be estimated accurately as the SNR increases. Since the gain terms are sufficiently large, the estimation performance for the phase terms is also good.

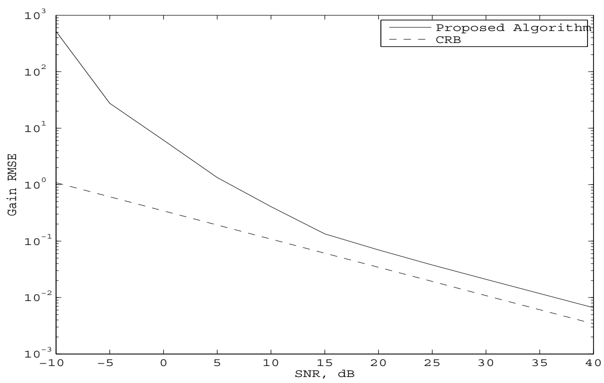

Figure 4: Performance of the proposed algorithm for the estimation of the gain terms of the gain/phase mismatch matrix.

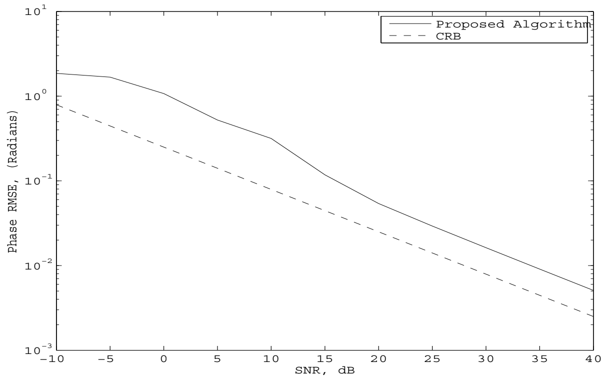

Figure 5: Performance of the proposed algorithm for the estimation of the phase terms of the gain/phase mismatch matrix.

5. CONCLUSION

In this paper, a new method is proposed for the joint estimation of DOA angles, mutual coupling and gain/phase mismatch for antenna arrays. The proposed online calibration method can be applied for sensor arrays randomly deployed on a plane with an unstructured mutual coupling matrix. The proposed technique uses Higher-Order-Statistics (HOS) for parameter estimation. This is specially convenient since DOA angles can be estimated when there are both mutual coupling and gain/phase mismatch errors. CRB expressions are derived in order to evaluate the performance of the proposed method. Several experiments are done and it is shown that parameters are estimated accurately by the proposed method.

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