ARRAY CALIBRATION WITH MODIFIED ITERATIVE HOS-SOS (MIHOSS) ALGORITHM

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ABSTRACT

Joint direction-of-arrival (DOA) and sensor position estimation for randomly deployed sensors is introduced in Iterative HOS-SOS (IHOSS) algorithm [1]. IHOSS algorithm exploits the advantages of both higher-order-statistics (HOS) and second-order-statistics (SOS) with an iterative algorithm using two reference sensors. The iterative algorithm is guaranteed to converge. IHOSS algorithm solves the position ambiguity by using source signals observed at multiple frequencies and hence it is applicable for wideband signals. In this paper, we propose Modified-IHOSS (MIHOSS) algorithm to solve the same problem for narrowband signals. In MIHOSS, it is assumed that the nominal sensor positions are known. It is shown that ambiguity problem is solved effectively without any assumption on the position perturbations. The upper bound of perturbations for unambiguous sensor position estimation is presented. The performance of MIHOSS approaches to the Cramér-Rao bound (CRB) for both DOA and position estimation.

1. INTRODUCTION

The deviation of array parameters from the assumed model generates errors for the array processing applications including DOA estimation and beamforming. Gain/phase mismatch of antennas, mutual coupling and antenna position errors are some examples of array modeling errors. Array calibration is the task of estimating the errors in array model as well as the DOA and source parameters. In this paper, we focus on array calibration for the sensor position errors. Sensor position errors are important in practical applications where the sensors are distributed in a wide area or there are sensor displacements due to the platform as in the case of sensors on the wing tips of a plane.

In this paper, joint DOA and sensor position estimation is done in a setting where the nominal sensor positions are known. In this case, it is assumed that there are two reference sensors whose positions are known perfectly. The rest of the sensors are distributed randomly in a large area. While the nominal positions of the distributed sensors are known, there is no assumption on the perturbation for the sensor positions. Note that the problem defined above is different than the partially calibrated arrays (PCA) [3] since the number of sources is not restricted to be less than the number of reference sensors. Furthermore, the sensor positions are estimated in our case as opposed to [3].

In the literature, array calibration problem for the sensor position errors is investigated in two settings, namely small error [4] and large error approximations [5]. In small error approximation, the perturbations are assumed to be small and array calibration is performed by using a first order approximation. The first order approximation is not applicable as the perturbations are increased. Large error approximation [5] is proposed to circumvent the limitations of the small error approximation. However the DOA estimation problem is considered for a uniform circular array and for some fixed source DOAs. One of the main problems in sensor position estimation from the knowledge of source observed data is the ambiguity in sensor positions. Ambiguity arises due to the wrap around in array steering matrix phase terms.

Previously IHOSS algorithm [1], [6] is presented, which jointly uses HOS and SOS approaches iteratively for the estimation of both source DOAs and sensor positions. The iterative process is guaranteed to converge. In IHOSS algorithm, except the two reference sensors there is no a priori information about the sensor positions. The positions of the two reference sensors are assumed to be known. IHOSS algorithm considers the ambiguity problem in sensor position estimation and solves the problem by using the source signals observed at multiple frequencies. Hence it is applicable for wideband signals. In this paper, IHOSS algorithm is modified for the narrowband signals and the new algorithm is called as MIHOSS. Since for the narrowband case, source signals can only be observed at single frequency, MIHOSS requires to know the nominal sensor positions to solve the ambiguity problem. It is proved that the ambiguity problem can be solved if the perturbations are bounded. The upper bound for the perturbations is also presented.

Both IHOSS and MIHOSS algorithms can effectively be used in the array calibration problem for the sensor position errors for different applications. Since different assumptions are used for IHOSS and MIHOSS algorithms, the comparison between them is not fair.

2. PROBLEM STATEMENT

It is assumed that the array is composed of randomly deployed M sensors and there are L far-field sources. Two sensors are selected as the reference sensors. The sensor positions are randomly perturbed from their nominal positions except the reference sensors. The positions of the reference sensors are assumed to be known and the distance between them is less than or equal to $\lambda/2$, where λ is the wavelength of the incoming source signals. Under these assumptions, the received signal vector for the sensor array can be written as,

$$\mathbf{x}(t) = \mathbf{A}(\Theta, \mathbf{P^0} + \tilde{\mathbf{P}})\mathbf{s}(t) + \mathbf{v}(t), \qquad t = 1, 2, \ldots, N \quad (1)$$

where, N is the number of snapshots, $\mathbf{s}(t) = [s_1(t), ..., s_L(t)]^T$ is the $L \times 1$ vector of L sources, $\mathbf{v}(t)$ is the $M \times 1$ vector of Gaussian noise. Source signals are assumed to be non-Gaussian and they can be correlated but not coherent. Noise is assumed to be statistically independent with the source signals. $\mathbf{\Theta} = [\theta_1, ..., \theta_L]$ is the source DOA vector, $\mathbf{P}^0 = [\mathbf{p}_1^{0^T}, ..., \mathbf{p}_M^{0^T}]^T$ and $\tilde{\mathbf{P}} = [\tilde{\mathbf{p}}_1^T, ..., \tilde{\mathbf{p}}_M^T]^T$ are the nominal sensor positions and the perturbations in positions, respectively. $\mathbf{A}(\mathbf{\Theta}, \mathbf{P})$ is the $M \times L$ array steering matrix, composed of,

$$\left[\mathbf{A}(\mathbf{\Theta}, \mathbf{P^0} + \tilde{\mathbf{P}})\right]_{mi} = exp\left\{ j\frac{2\pi}{\lambda} \left[\left(p_{m,x}^0 + \tilde{p}_{m,x}\right) cos\theta_i + \left(p_{m,y}^0 + \tilde{p}_{m,y}\right) sin\theta_i \right] \right\} \quad (2)$$

where, θ_i is the direction-of-arrival of i^{th} source in azimuth, $\mathbf{p}_m^0 = [p_{m,x}^0, p_{m,y}^0]$ and $\tilde{\mathbf{p}}_m = [\tilde{p}_{m,x}, \tilde{p}_{m,y}]$ are the 2D nominal position of the m^{th} sensor and the 2D perturbation of the m^{th} sensor position, respectively. Since the positions of the two reference sensors are known, their perturbations are zero, i.e., $\tilde{\mathbf{p}}_m = \mathbf{0}, \ \ m = 1, 2$. $(.)^T$ is the transpose operator.

The goal in this paper is to estimate both DOAs of L sources and the perturbation parameters of $M-2$ sensors.

3. MIHOSS ALGORITHM

In this section, MIHOSS algorithm is introduced for a solution to the problem described in Section 2. MIHOSS algorithm is based on the IHOSS algorithm [1], which uses the HOS and SOS approaches jointly. The basic difference between the IHOSS and MIHOSS is their solution of the ambiguity in sensor positions. IHOSS algorithm requires observations at multiple frequencies. On the other hand, MIHOSS uses the nominal sensor positions to solve the ambiguity problem and can be applied for narrowband signals.

3.1 HOS Based Blind DOA Estimation

In [7], it is shown that HOS approach can be used to find the DOA and array steering matrix estimates for random sensor geometries without knowing the sensor positions except the two reference sensors. In this respect cumulant matrix composed of fourth-order cumulants are used together with the ESPRIT algorithm. On the other hand, as explained in [1], this approach can be employed for DOA estimation as long as the source signals are independent. IHOSS algorithm [1] overcomes this limitation by proposing a new cumulant matrix estimation technique, which is more robust to the dependency between source signals, i.e.,

$$\mathbf{C} = \sum_{i=1}^{L} \begin{bmatrix} \overline{\mathbf{A}}_1^{(i)} \mathbf{C}_s \left(\overline{\mathbf{A}}_1^{(i)}\right)^H & \overline{\mathbf{A}}_1^{(i)} \mathbf{C}_s \left(\overline{\mathbf{A}}_2^{(i)}\right)^H \\ \overline{\mathbf{A}}_2^{(i)} \mathbf{C}_s \left(\overline{\mathbf{A}}_1^{(i)}\right)^H & \overline{\mathbf{A}}_1^{(i)} \mathbf{C}_s \left(\overline{\mathbf{A}}_1^{(i)}\right)^H \end{bmatrix} \quad (3)$$

where $\hat{\mathbf{A}}$ is the estimate of the array steering matrix, $\mathbf{A}$, and $\overline{\mathbf{A}}_j^{(i)} = \mathbf{Q}_i \mathbf{A} \otimes \mathbf{q}_j^{(i)^*} \mathbf{A}^*, \ \ j \in \{1, 2\}$. $\mathbf{q}_j^{(i)^*}$ is the complex conjugate of the j^{th} row of the matrix $\mathbf{Q}_i = \mathbf{I} - \hat{\mathbf{A}} \mathbf{Z}_i \hat{\mathbf{A}}^\dagger$. $(.)^\dagger$ is the Moore-Penrose pseudoinverse operator. $\mathbf{Z}_i$ is the $L \times L$ diagonal matrix whose diagonal elements are one except the i^{th} element. The i^{th} element is set to zero. $\mathbf{C}_s$ is the $L^2 \times L^2$ source cumulant matrix in the form of,

$$\mathbf{C}_s(i, j) = Cum\left(s_k(t), s_l^*(t), s_m(t), s_n^*(t)\right) \quad (4)$$
$$i = L(m-1) + l, \qquad 1 \le m, l \le L$$
$$j = L(n-1) + k, \qquad 1 \le n, k \le L$$

Note that the cumulant matrix estimate in (3) is a generalized cumulant matrix estimate which improves the parameter estimates depending on the accuracy of the array steering matrix estimation. In [1], it is shown that if the actual array steering matrix is known, i.e., $\hat{\mathbf{A}} = \mathbf{A}$, the cumulant matrix estimate in (3) simplifies to,

$$\mathbf{C} = \begin{bmatrix} \mathbf{A}\mathbf{R}_s^{HOS}\mathbf{A}^H & \mathbf{A}\mathbf{R}_s^{HOS}\mathbf{D}\mathbf{A}^H \\ \mathbf{A}\mathbf{D}^H\mathbf{R}_s^{HOS}\mathbf{A}^H & \mathbf{A}\mathbf{R}_s^{HOS}\mathbf{A}^H \end{bmatrix} \quad (5)$$

$L \times L$ diagonal matrices $\mathbf{R}_s^{HOS}$ and $\mathbf{D}$ are defined as,

$$\mathbf{R}_s^{HOS} = diag(\gamma_1, \gamma_2, \ldots, \gamma_L) \quad (6)$$
$$\mathbf{D} = diag\left(e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_1)}, \ldots, e^{j\frac{2\pi}{\lambda}\Delta cos(\theta_L)}\right) \quad (7)$$

where $\gamma_i = Cum(s_i(t), s_i^*(t), s_i(t), s_i^*(t))$ and $\Delta \le \lambda/2$ is the distance between the two reference sensors. The reference sensors are assumed to be located at $(0,0)$ and $(\Delta, 0)$ on the coordinate system for simplicity. Note that, the cumulant matrix in (5) is in the same form of the correlation matrix in the ESPRIT algorithm. The only difference is the source correlation matrix defined for SOS is replaced by $\mathbf{R}_s^{HOS}$. Since the cumulant matrix in (5) is not available in practice, its estimate in (3) is used for the parameter estimation.

The DOA and array steering matrix estimates are found from the eigenvalue decomposition of $\mathbf{C}$, i.e., $\mathbf{CS} = \mathbf{S}\mathbf{\Lambda}_s$ as in the ESPRIT algorithm. $\mathbf{\Lambda}_s$ is the diagonal matrix composed of the L largest eigenvalues of the matrix $\mathbf{C}$ and $2M \times L$ matrix $\mathbf{S} = [\ \mathbf{S}_1^T \ \ \mathbf{S}_2^T\ ]^T$ is obtained from the eigenvectors corresponding to these eigenvalues. $\mathbf{S}_1$ and $\mathbf{S}_2$ are $M \times L$ matrices. The DOA and the array steering matrix estimates are found by applying the ESPRIT algorithm, i.e.,

$$\hat{\theta}_i = cos^{-1}\left(-\frac{\angle \mathbf{\Phi}(i,i)}{2\pi\Delta}\lambda\right) \quad (8)$$

$$\overline{\mathbf{A}} = \mathbf{S}_1 \mathbf{\Psi} \quad (9)$$

where $\angle \mathbf{\Phi}(i,i)$ is the phase term of the i^{th} diagonal element of the matrix $\mathbf{\Phi}$. $L \times L$ diagonal matrix, $\mathbf{\Phi}$, and $L \times L$ matrix, $\mathbf{\Psi}$, are related as,

$$\mathbf{S}_1^\dagger \mathbf{S}_2 \mathbf{\Psi} = \mathbf{\Psi}\mathbf{\Phi} \quad (10)$$

Note that knowing the distance and the direction between the two reference sensors are sufficient for the DOA estimation as in (8). However, it is not the case for the array steering matrix estimation. In the ESPRIT algorithm the array steering matrix estimation is found up to an unknown scale factor as in (9). To find the scale factor, in addition to the distance and the direction between the two reference sensors, it is required to know one of the reference sensor position. Since it is assumed that the first reference sensor is located at (0,0), the first row of the array steering matrix has to be consist of all ones. Then, the actual array steering matrix can be found from (9), i.e.,

$$\hat{\mathbf{A}} = \overline{\mathbf{A}}\mathbf{H}^{-1} \quad (11)$$

where $\mathbf{H} = diag(\overline{a}_{11}, \overline{a}_{12}, \ldots, \overline{a}_{1L})$ and $\overline{a}_{ij}$ is the i^{th} row and j^{th} column of matrix $\overline{\mathbf{A}}$.

3.2 Unambiguous Sensor Localization

Once the DOA and array steering matrix estimations are found, sensor locations can be estimated using (2). Due to 2π ambiguity, the elements of the array steering matrix in (2), corresponding to m^{th} sensor and i^{th} source can be rewritten in the following form,

$$a_{m,i} = e^{j\frac{2\pi}{\lambda}\left[\left(\mathbf{p}_m^0+\tilde{\mathbf{p}}_m\right)\mathbf{u}(\theta_i)-\lambda k_{m,i}\right]} \tag{12}$$

where, $k_{m,i}$ is an integer specified for the m^{th} sensor and the i^{th} source. $\mathbf{u}(\theta_i) = [-cos(\theta_i), -sin(\theta_i)]^T$ is the unit direction vector of the i^{th} incoming source. When all the incoming sources are considered, the following relation can be written,

$$\left(\mathbf{p}_m^0+\tilde{\mathbf{p}}_m\right)\mathbf{U}(\hat{\Theta}) = \frac{\lambda}{2\pi}\hat{\Xi}_m + \lambda\mathbf{k}_m, \qquad 1 \le m \le M \tag{13}$$

where

$$\begin{aligned}
\hat{\Xi}_m &= \left[\ \angle(\hat{a}_{m,1}) \quad \angle(\hat{a}_{m,2}) \quad \dots \quad \angle(\hat{a}_{m,L})\ \right] && (14)\\
\mathbf{k}_m &= \left[\ k_{m,1} \quad k_{m,2} \quad \dots \quad k_{m,L}\ \right] && (15)\\
\mathbf{U}(\hat{\Theta}) &= \left[\ \mathbf{u}(\hat{\theta}_1) \quad \dots \quad \mathbf{u}(\hat{\theta}_L)\ \right] && (16)
\end{aligned}$$

$\hat{x}$ stands for the estimation of x and $\angle(\hat{a}_{m,i})$ is the phase term of the array steering matrix element estimate in (12).

The position perturbation of the m^{th} sensor can easily be found from (13) in the least squares sense as,

$$\hat{\tilde{\mathbf{p}}}_m(\mathbf{k}_m) = \left(\frac{\lambda}{2\pi}\hat{\Xi}_m + \lambda\mathbf{k}_m\right)\mathbf{U}^\dagger(\hat{\Theta}) - \mathbf{p}_m^0, \qquad 1 \le m \le M \tag{17}$$

Note that the position perturbation estimate in (17) takes different values for different $\mathbf{k}_m$ values. Therefore, $\hat{\tilde{\mathbf{p}}}_m(\mathbf{k}_m)$ values are considered as the ambiguous position perturbation estimates of the m^{th} sensor.

If the position perturbation is limited, the ambiguity problem can be solved by selecting the sensor position perturbation estimate with minimum norm, i.e.,

$$\hat{\tilde{\mathbf{p}}}_m = arg\min_{\mathbf{k}_m}\left\|\hat{\tilde{\mathbf{p}}}_m(\mathbf{k}_m)\right\| \tag{18}$$

The upper bound of perturbations for unambiguous sensor position estimations is given in Lemma-1.

Lemma-1: Let d_m be the minimum distance between actual and estimated position perturbation of the m^{th} sensor, i.e., $\left\|\hat{\tilde{\mathbf{p}}}_m(\mathbf{k}_m) - \tilde{\mathbf{p}}_m\right\| \le d_m$. Then, the ambiguity problem in sensor position estimation is solved if the following condition is satisfied for, $1 \le m \le M$,

$$\|\tilde{\mathbf{p}}_m\| < \frac{\lambda}{2}\min_{\mathbf{k}_m^{(1)}\neq\mathbf{k}_m^{(2)}\in\mathbf{k}_m}\left\|\left(\mathbf{k}_m^{(1)}-\mathbf{k}_m^{(2)}\right)\mathbf{U}^\dagger(\hat{\Theta})\right\| - \max_m d_m \tag{19}$$

The proof of Lemma-1 is not given due to space limitations.

3.3 SOS-Based MUSIC Algorithm

Sensor position matrix estimate, $\hat{\mathbf{P}} = \mathbf{P}^0 + \hat{\tilde{\mathbf{P}}}$, is constructed using (18) with the nominal sensor positions and used in the MUSIC algorithm to generate the MUSIC pseudospectrum, i.e., $\Gamma(\theta) = \left(\mathbf{a}^H(\theta,\hat{\mathbf{P}})\mathbf{G}\mathbf{G}^H\mathbf{a}(\theta,\hat{\mathbf{P}})\right)^{-1}$ where $\mathbf{G}$ is the $M \times (M-L)$ matrix whose columns are composed of the eigenvectors corresponding to $M-L$ smallest eigenvalues of the correlation matrix obtained in the SOS approach. The DOA and the array steering matrix estimates for the SOS approach are obtained by finding the L largest peaks of the MUSIC pseudospectrum, i.e.,

$$\begin{aligned}
\left\{\hat{\theta}_i\right\}_{i=1}^{L} &= arg\max_{\theta}\Gamma(\theta) && (20)\\
\hat{\mathbf{A}} &= \left[\mathbf{a}(\hat{\theta}_1,\hat{\mathbf{P}}),\mathbf{a}(\hat{\theta}_2,\hat{\mathbf{P}}),\dots,\mathbf{a}(\hat{\theta}_L,\hat{\mathbf{P}})\right] && (21)
\end{aligned}$$

3.4 The Cost Function and The Algorithmic Steps

MIHOSS algorithm iteratively updates the DOA and array steering matrix estimates using the HOS and SOS approaches sequentially as summarized in Table 1. The cost function used at each iteration to select the best array steering vector estimates for each source is defined by the MUSIC pseudospectrum, i.e.,

$$\Gamma(\hat{\mathbf{a}}_i) = \left(\hat{\mathbf{a}}_i^H\mathbf{G}\mathbf{G}^H\hat{\mathbf{a}}_i\right)^{-1} \tag{22}$$

where $\hat{\mathbf{a}}_i$ are the array steering vector estimate for the i^{th} source. Note that the cost function, $\Gamma(\hat{\mathbf{a}}_i)$ is non-negative. At each iteration, n, we have $\Gamma(\hat{\mathbf{a}}_i^{(n)}) \ge \Gamma(\hat{\mathbf{a}}_i^{(n-1)}) \ge 0$. Therefore, the proposed MIHOSS algorithm is guaranteed to converge to a certain value, $\overline{\Gamma}$, at the end of the iterations. However, the convergence to this value does not mean that the global optimum is reached as it is the general disadvantage of all iterative algorithms [8].

4. PERFORMANCE RESULTS

MIHOSS algorithm is compared with the MUSIC [2] and small error approximation [4], illustrated as SmallError in the figures, for DOA and sensor position estimations. CRB [1] is also evaluated for both DOA and sensor position estimation. While MIHOSS and SmallError algorithms are iterative methods, MUSIC algorithm is non iterative one. As stated in Table 1, MIHOSS starts with the SOS MUSIC algorithm and iterates HOS and SOS approaches to update both DOA and sensor position estimations. Also SmallError [4] algorithm starts with MUSIC algorithm and iteratively updates both DOA and sensor position estimations using SOS approach. Therefore, comparing MUSIC algorithm with MIHOSS and SmallError algorithms shows the effectiveness of the iteration processes. Note that for a fair comparison, the sensor position estimation algorithm described in Section 3.2 is also applied for the MUSIC algorithm.

It is assumed that there are two far-field sources and $M = 10$ sensors. Each sensor position except the two reference sensors is randomly selected from a uniform distribution in the deployment area of $2\lambda \times 2\lambda$. The reference sensors are placed at $(0,0)$ and $(\lambda/2,0)$. The positions of the sensors other than the reference sensors are arbitrarily perturbed. The perturbation values are randomly selected with a uniform distribution. For the parameter estimation, $N = 1000$ snapshots are collected. The performance results are the average of 100 trials. At each trial, source signals, noise, the sensor positions except the reference sensors, the perturbations and the DOA angles of source signals are changed randomly. The difference between the DOA angles of the source

Table 1: Pseudocode for MIHOSS algorithm.

1 $n = 0$. *Find the initial values of the array steering vector for each source,* $\hat{\mathbf{a}}_i^{(0)}$*, with SOS approach as in (21) using the nominal sensor positions.*;
2 *Termination* $= true$. *Estimate the proposed cumulant matrix from the array output and* $\hat{\mathbf{a}}_i^{(n)}$ *as in (3). Then, find the DOA estimates,* $\hat{\theta}_i^{HOS}$ *using (8) and the array steering matrix* $\hat{\mathbf{A}}^{HOS}$*, using (11), for* $1 \leq i \leq L$;
3 *Find the sensor position estimates,* $\hat{\mathbf{P}} = \mathbf{P}^0 + \hat{\tilde{\mathbf{P}}}$*, as in (18) using (16) and (14) with* $\hat{\theta}_i^{HOS}$ *and* $\hat{\mathbf{A}}^{HOS}$*, for* $1 \leq i \leq L$;
4 *Find* $\hat{\theta}_i^{(SOS)}$ *using* $\hat{\mathbf{P}}$ *as in (20). Then, find* $\hat{\mathbf{a}}_i^{(SOS)}$ *using* $\hat{\mathbf{P}}$ *and* $\hat{\theta}_i^{(SOS)}$ *as in (21)*;
5 **for** $i = 1$ **to** L **do**
6   **if** $\Gamma(\hat{\mathbf{a}}_i^{(SOS)}) \geq \Gamma(\hat{\mathbf{a}}_i^{(n)})$ **then**
7         $\hat{\mathbf{a}}_i^{(n+1)} = \hat{\mathbf{a}}_i^{(SOS)}$, $\hat{\theta}_i^{(n+1)} = \hat{\theta}_i^{(SOS)}$, $\Gamma(\hat{\mathbf{a}}_i^{(n+1)}) = \Gamma(\hat{\mathbf{a}}_i^{(SOS)})$, *Termination* $= false$;
8   **else**
9         $\hat{\mathbf{a}}_i^{(n+1)} = \hat{\mathbf{a}}_i^{(n)}$, $\hat{\theta}_i^{(n+1)} = \hat{\theta}_i^{(n)}$;
10   **end**
11 **end**
12 **if** *Termination* $= false$ **then**
13     $n = n + 1$, *Go to Step 2*;
14 **else**
15   *Find the final estimate of sensor positions using* $\hat{\theta}_i^{(n)}$ *and* $\hat{\mathbf{a}}_i^{(n)}$, $1 \leq i \leq L$;
16 **end**

signals is set to 40 degrees. The source signals have a uniform distribution and the noise is additive white Gaussian and uncorrelated with the source signals.

The performance results for the DOA and sensor position estimations at different SNR values are illustrated in Fig. 1. The sensor position perturbation is limited to 0.1λ. It is seen that both MUSIC and small error approach algorithm (SmallError) have a flooring effect for both DOA and sensor position estimations. As it is seen in Fig. 1, SmallError algorithm slightly improves the MUSIC performance. It is also seen that after approximately SNR = 7 dB MIHOSS algorithm significantly outperforms and closely follows CRB for both DOA and sensor position estimations.

In Fig. 2, the performance of the algorithms is presented for different position perturbations. SNR is set to 30 dB. As it is seen in Fig. 2, the parameter estimation performance of MIHOSS algorithm is not affected from the value of perturbations and closely follows CRB. It is also observed in Fig. 2-(b) that, MIHOSS algorithm effectively solves the ambiguity problem up to a perturbation value of 0.42λ. The condition presented in Lemma-1 is not satisfied for further increase in perturbations and sensor positions can not be found unambiguously. Note that DOA estimation is accurate and is not affected by the sensor position ambiguity as shown in Fig. 2-(a). This is due to the fact that array steering matrix estimate is accurate while the positions are ambiguous. The performance of both MUSIC and SmallError algorithm degrade significantly for the large perturbation values. SmallError

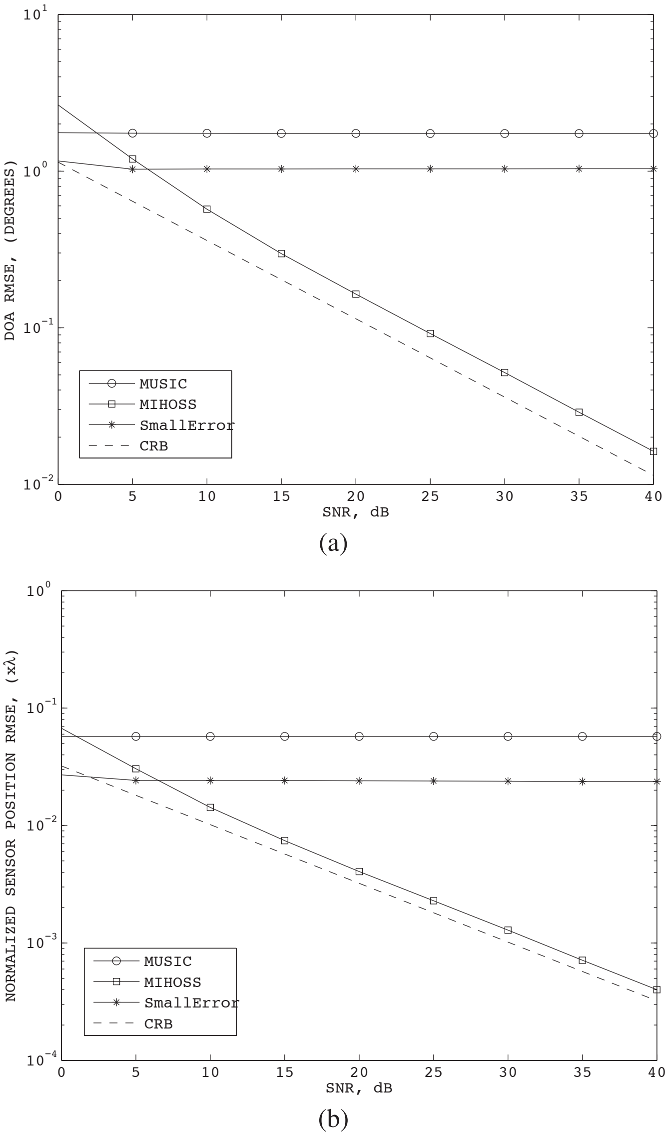
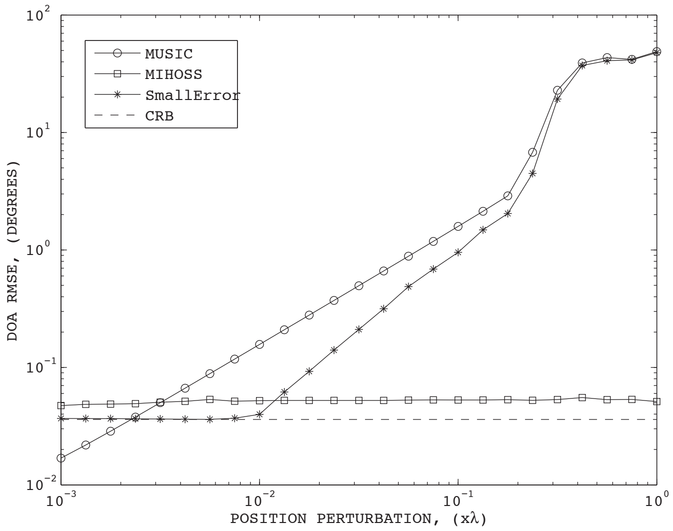


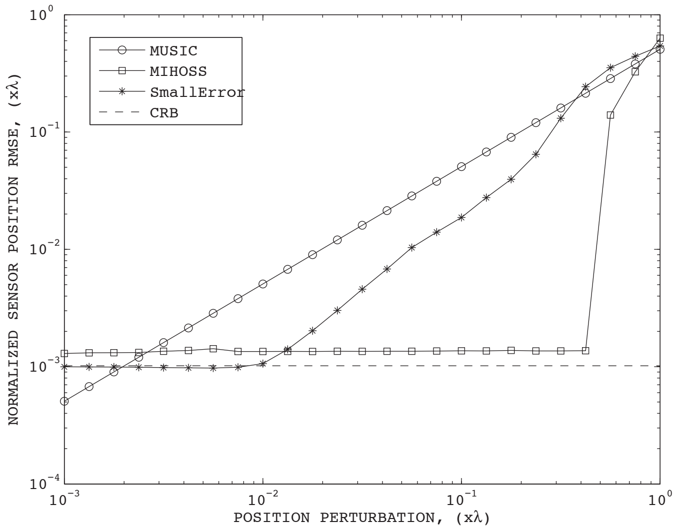
Figure 1: (a) DOA and (b) position estimation RMSE values for different SNR values and sensor position perturbation of 0.1λ.

algorithm slightly outperforms MIHOSS algorithm only for very small perturbations (less than 0.01λ). For the perturbations less than 0.0016λ MUSIC outperforms both MIHOSS and SmallEror algorithms as well as CRB. The reason for this fact is that iterative processes in MIHOSS and SmallError algorithms decrease the estimation performances for the extremely small perturbations. As shown in (17), pseudoinverse operator is used for sensor position estimation, which is not an exact solution. Iteratively updating sensor positions may result worse position estimation than the nominal sensor positions when the perturbation is extremely small. The similar explanation is also valid for the SmallError algorithm. While CRB does not specify any algorithm for sensor position estimation, it uses perturbations as unknown parameters and tries find the minimum variance for both DOA and sensor position estimations jointly. Hence, CRB assumes that there are always errors in sensor positions even if there is no.

On the other hand MUSIC algorithm finds the DOA and sensor position estimations in a single step. It does not assume that there are errors in sensor positions and does not update the estimations iteratively.



(a)



(b)

Figure 2: (a) DOA and (b) position estimation RMSE values for different sensor position perturbations and SNR = 30 dB.

5. CONCLUSION

A new method for joint DOA and sensor position estimation is presented when the sensors are randomly deployed and arbitrarily perturbed from their nominal positions. It is assumed that the distance and the direction between two reference sensors are known. HOS and SOS approaches are employed jointly in an iterative manner. The iterative method is guaranteed to converge. Several simulations are done and it is shown that the proposed method improves the performance of DOA and sensor position estimation significantly and approaches to the CRB.

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