AN EXTENSION OF THE MUSIC ALGORITHM TO BROADBAND SCENARIOS USING A POLYNOMIAL EIGENVALUE DECOMPOSITION

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ABSTRACT

The multiple signal classification (MUSIC) algorithm for direction of arrival estimation is defined for narrowband scenarios. In this paper, a generalisation to the broadband case is presented, based on a description of broadband systems by polynomial space-time covariance matrices. A polynomial eigenvalue decomposition is used to determine the noise-only subspace of the this matrix, which can be scanned by appropriately defined broadband steering vectors. Two broadband MUSIC algorithm versions are presented, which resolve either angle of arrival alone or in combination with the frequency range over which sources are active. Initial results for these approaches are presented and demonstrate a significant benefit over independent frequency bin processing using narrowband MUSIC.

1. INTRODUCTION

The multiple signal classification (MUSIC) algorithm [10] is a powerful and accurate technique for determining angles of arrival in narrowband array processing applications. Since its publication, the algorithm has found numerous applications and inspired solutions ranging from spectrum estimation to source localisation problems, see e.g. [5, 8].

While the original MUSIC algorithm has been defined for the narrowband scenario, recently a number of efforts to derive broadband algorithms for direction of arrival estimation have been pursued. In [4], a discrete frequency domain approach leads to a decoupling of source parameters such as frequency and angle of arrival, over which estimation is performed. A wideband time domain approach reported in [9] is based on a Markov chain Monte-Carlo method. While [4, 9] are broadband approaches but unrelated to the MUSIC algorithm, [2, 3] introduce a parameterised spatial covariance matrix, which depends on the angle of arrival ϑ, and subsequently the relative time delay between array elements, as a continuous variable. Based on this description, a number of algebraic techniques including a broadband MUSIC algorithm have been derived [1, 11, 12].

In this paper, we utilise a polynomial form of the space-time covariance matrix in combination with a polynomial eigenvalue decomposition (PEVD) derived in [7] to determine its signal-plus-noise and noise-only subspaces. Different from [2], the covariance matrix and its decomposition are discrete and finite, and the angle of arrival is a continuous variable only in the definition of a broadband steering vector, which is used to scan the noise-only subspace in MUSIC-style.

In the following, we briefly review the narrowband MUSIC algorithm in Sec. 2. Sec. 3 reviews the polynomial space-time covariance matrix, and then introduces a polynomial steering vector which enable the proposed broadband generalisation of the MUSIC algorithm. Two versions of the proposed broadband MUSIC algorithm, resolving sources either spatially or spatio-spectrally, are demonstrated in Sec. 4 and compared to an independent frequency bin (IFB) approach, whereby a narrowband MUSIC algorithm is independently applied in every frequency bin. Conclusions are drawn in Sec. 5.

Notation. Vector and matrix quantities are represented by lower and uppercase bold variables, respectively, such as $\mathbf{a}$ and $\mathbf{A}$. The Hermitian transpose of $\mathbf{A}$ is denoted as $\mathbf{A}^{\mathrm{H}}$. Polynomial vectors and matrices are written as $\mathbf{a}(z)$ and $\mathbf{A}(z)$, with the parahermitian $\tilde{\mathbf{A}}(z) = \mathbf{A}^{\mathrm{H}}(z^{-1})$. A transform pair $a[n]$ and $A(z) = \sum_{n=-\infty}^{\infty} a[n]z^{-n}$ is abbreviated as $a[n] \circ\!\!-\!\!\bullet\, A(z)$.

2. NARROWBAND MUSIC

We first review the narrowband MUSIC algorithm, based on a description of array data by steering vectors and the narrowband spatial covariance matrix.

2.1 Array Signals and Steering Vector

Consider a vector of samples $\mathbf{x}[n] \in \mathbb{C}^M$

$$\mathbf{x}[n] = \begin{bmatrix} x_0[n] \\ x_1[n] \\ \vdots \\ x_{M-1}[n] \end{bmatrix} , \quad (1)$$

acquired by an M element sensor array at discrete time n. A signal emerging from a single source arrives at these sensors with time shifts due to the propagation delays as the wavefront travels across the array. For simplicity, we here assume the farfield case, with no amplitude difference between the sensors due to attenuation in the medium. Taking the first sensor signal, $x_0[n]$, as reference, the relative delays of the remaining sensor signals can be characterised as

$$\mathbf{x}[n] = \begin{bmatrix} x_0[n] \\ x_0[n-\Delta\tau_1] \\ \vdots \\ x_0[n-\Delta\tau_{M-1}] \end{bmatrix} . \quad (2)$$

For a narrowband source with normalised angular frequency Ω, with a reference signal $x_0[n] = e^{j\Omega n}$, the time delays $\Delta\tau_m$

collapse to simple phase shifts

$$\mathbf{x}[n] = \begin{bmatrix} 1 \\ e^{-j\Omega\Delta\tau_1} \\ \vdots \\ e^{-j\Omega\Delta\tau_{M-1}} \end{bmatrix} e^{j\Omega} = \mathbf{a}_{\Omega,\vartheta} e^{j\Omega} \quad , \tag{3}$$

where $\mathbf{a}_{\Omega,\vartheta}$ is termed the narrowband steering vector.
Example. To derive the steering vector of a linear equidistant array with critical sensor spacing $d = \frac{\lambda_{\min}}{2} = \frac{c}{2f_{\max}} = \frac{c}{f_s}$, based on the propagation speed c in the medium and critical sampling $f_s = 2f_{\max}$. A source illuminates the array with a complex exponential $e^{j\omega t}$ from an angle ϑ measured against broadside. Once sampled at $t = nT_s = \frac{n}{f_s}$, the reference signal $e^{j\Omega n}$ with normalised angular sampling rate $\Omega = \omega/f_s$ arrives with delays of $\Delta\tau = \frac{\lambda \sin\vartheta}{c} = \Omega \sin\vartheta$, such that

$$\mathbf{a}_{\Omega,\vartheta} = \begin{bmatrix} 1 \\ e^{-j\Omega\sin\vartheta} \\ \vdots \\ e^{-j(M-1)\Omega\sin\vartheta} \end{bmatrix} \quad . \tag{4}$$

In the presence of L narrowband sources $s_l[n]$ characterised by pairs $\{\Omega_l, \vartheta_l\}$, the array vector is given by

$$\mathbf{x}[n] = \sum_{l=1}^{L} \mathbf{a}_{\Omega_l,\vartheta_l} s_l[n] + \mathbf{v}[n] \quad , \tag{5}$$

with independent and identically distributed white noise $\mathbf{v}[n]$, such that $\mathscr{E}\{\mathbf{v}[n]\mathbf{v}^{\mathrm{H}}[n-\tau]\} = \delta[\tau]\sigma_v^2\mathbf{I}$.

2.2 Narrowband Covariance Matrix

If the signals can be characterised as narrowband with frequency Ω, only correlations for lag zero need to be considered on the covariance matrix $\mathbf{R} \in \mathbb{C}^{M\times M}$,

$$\mathbf{R} = \mathscr{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n]\} \quad , \tag{6}$$

where $\mathscr{E}\{\cdot\}$ is the expectation operator. This covariance matrix entirely describes the data as modelled in (5), since for the case of L independent source signals of power σ_l^2, $l \in (1, L)$,

$$\mathbf{R} = \sum_{l=1}^{L} \sigma_l^2 \mathbf{a}_{\Omega,\vartheta_l} \mathbf{a}_{\Omega,\vartheta_l}^{\mathrm{H}} + \sigma_v^2 \mathbf{I} \quad . \tag{7}$$

The maximum rank of $\mathbf{R}$ is M, which is achieved in the case of linear independence of all steering vectors.

If data is acquired over a data window of N samples, then the data matrix

$$\mathbf{X}_n = [\mathbf{x}[n-N+1] \ldots \mathbf{x}[n-1]\; \mathbf{x}[n]] \tag{8}$$

can be used to estimate the covariance matrix as

$$\hat{\mathbf{R}}_n = \frac{1}{N}\mathbf{X}_n\mathbf{X}_n^{\mathrm{H}} \quad . \tag{9}$$

While below the analysis is continued with $\mathbf{R}$, an appropriate estimation of this covariance matrix according to (9) is assumed.

2.3 Narrowband MUSIC Algorithm

To retrieve the angles of arrival of sources from $\mathbf{R}$, the extraction of eigenvectors from $\mathbf{R}$ as steering vector estimates can only be successful if all steering vectors in (7) are orthogonal. Otherwise, the eigenvalue decomposition (EVD)

$$\mathbf{R} = [\mathbf{Q}_s \mathbf{Q}_n] \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \Lambda_n \end{bmatrix} \begin{bmatrix} \mathbf{Q}_s^{\mathrm{H}} \\ \mathbf{Q}_n^{\mathrm{H}} \end{bmatrix} \tag{10}$$

may extract the steering vector of the strongest source correctly, but otherwise contain orthonormalised basis vectors of the signal subspace in $\mathbf{Q}_s$.

The idea of the MUSIC algorithm is to scan the noise-only subspace $\mathbf{Q}_n$, characterised by eigenvalues close to the noise floor, $\Lambda_n \approx \sigma_v^2\mathbf{I}$. Steering vectors of sources contributing to $\mathbf{R}$ will define the signal-plus-noise subspace $\mathbf{Q}_s$ and therefore lie in the nullspace of its complement $\mathbf{Q}_n$. Thus, the vector $\mathbf{Q}_n^{\mathrm{H}}\mathbf{a}_{\Omega,\vartheta}$ has to be close to zero for $\mathbf{a}_{\Omega,\vartheta}$ to be a steering vector of a contributing source. The MUSIC algorithm therefore calculates the inverse of the squared Euclidean norm of this vector as the MUSIC spectrum $P_{\mathrm{MU}}(\vartheta)$,

$$P_{\mathrm{MU}}(\vartheta) = \frac{1}{\mathbf{a}_{\Omega,\vartheta}^{\mathrm{H}} \mathbf{Q}_n \mathbf{Q}_n^{\mathrm{H}} \mathbf{a}_{\Omega,\vartheta}} \quad , \tag{11}$$

as proposed by [10].

3. BROADBAND MUSIC

To generalise (11) to the broadband case, we first define a polynomial space-time covariance matrix, which can be decomposed by McWhirter's PEVD [7], followed by an appropriate selection of a broadband steering vector.

3.1 Space-Time Covariance Matrix

Unlike the narrowband case, time delays arising from signal wavefronts travelling across the array at finite speed cannot merely be represented by phase shift, but require to be addressed as lags. This motivates a polynomial space-time covariance matrix $\mathbf{R}(z)$ •—○ $\mathbf{R}[\tau]$,

$$\mathbf{R}[\tau] = \mathscr{E}\{\mathbf{x}[n]\mathbf{x}^{\mathrm{H}}[n-\tau]\} \, .$$

This power spectral matrix can be decomposed by an iterative algorithm [7] to yield a polynomial EVD

$$\mathbf{R}(z) = \mathbf{Q}(z)\Lambda(z)\tilde{\mathbf{Q}}(z) = \sum_{m=0}^{M-1} \lambda_m(z)\mathbf{q}_m(z)\tilde{\mathbf{q}}_m(z)$$

with paraunitary $\mathbf{Q}(z)$, i.e. $\mathbf{Q}(z)\tilde{\mathbf{Q}}(z) = \mathbf{I}$. The diagonal matrix $\Lambda(z)$ contains the polynomial eigenvalues $\lambda_m(z)$. Thresholding the latter reveals the number of independent broadband sources contributing to $\mathbf{R}(z)$, and permits a distinction between signal-plus-noise and noise only subspaces,

$$\mathbf{R}(z) = [\mathbf{Q}_s(z)\mathbf{Q}_n(z)] \begin{bmatrix} \Lambda_s(z) & \mathbf{0} \\ \mathbf{0} & \Lambda_n(z) \end{bmatrix} \begin{bmatrix} \mathbf{Q}_s^{\mathrm{H}}(z) \\ \mathbf{Q}_n^{\mathrm{H}}(z) \end{bmatrix} \tag{12}$$

similar to the narrowband EVD in (10). To probe the nullspace of $\tilde{\mathbf{Q}}_n(z)$,

$$\tilde{\mathbf{Q}}_n(z) = \begin{bmatrix} \tilde{\mathbf{q}}_L(z) \\ \vdots \\ \tilde{\mathbf{q}}_{M-1}(z) \end{bmatrix} \tag{13}$$

a broadband steering vector is required instead of the narrowband one in (3).

3.2 Broadband Steering Vector

To accurately reflect the time delays required to describe (2), a polynomial vector containing fractional delay transfer functions is proposed here. One possibility to implement these fractional delays is by means of an appropriately sampled sinc function, such that

$$a_l[n] = \mathrm{sinc}(nT_s - \Delta\tau_l) \quad . \tag{14}$$

With $A_l(z) \bullet\!\!-\!\!\circ\, a_l[n]$, a broadband steering vector can be defined as

$$\mathbf{a}_\vartheta(z) = \begin{bmatrix} A_0(z) \\ \vdots \\ A_{M-1}(z) \end{bmatrix} \quad . \tag{15}$$

The parameter ϑ on the l.h.s. of (15) indicates the dependency of $\Delta\tau_l$ on the angle of arrival.

3.3 Polynomial MUSIC Algorithm

Following the rationale of the narrowband MUSIC algorithm, the generalised quantity

$$\Gamma_\vartheta(z) = \tilde{\mathbf{a}}_\vartheta(z)\mathbf{Q}_n(z)\tilde{\mathbf{Q}}_n(z)\mathbf{a}_\vartheta(z)$$

is no longer a norm measuring the vicinity of $\mathbf{a}_\vartheta(z)$ to the nullspace of $\tilde{\mathbf{Q}}_n(z)$, but a power spectral density. This motivates two versions of the a polynomial MUSIC (P-MUSIC) algorithm, which are detailed next.

Spatial P-MUSIC. To measure the energy contained in the signal vector $\tilde{\mathbf{Q}}_n(z)\mathbf{a}_\vartheta(z)$, the auto-correlation-type sequence $\gamma_\vartheta[\tau] \circ\!\!-\!\!\bullet\, \Gamma_\vartheta(z)$ provides a suitable quantity with the zero lag term $\gamma_\vartheta[0]$. This measure only depends on the angle of arrival ϑ, and collects all energy across the spectrum. Instead search for the steering vectors providing minimum energy, the reciprocal

$$P_{SP-MU}(\vartheta) = \frac{1}{\gamma_\vartheta[0]} \quad . \tag{16}$$

is maximised by the angle of arrival ϑ of sources.

Spatio-Spectral P-MUSIC. Since (3.3) represent a power spectral density term, spectral information can be exploited in addition to the spatial clues extracted by (16). Therefore,

$$P_{SSP-MU}(\vartheta,\Omega) = \left(\sum_{\tau=-\infty}^{\infty} \gamma_\vartheta[\tau] e^{-j\Omega\tau} \right)^{-1} \tag{17}$$

not only localise sources with respect to ϑ, but can also determine over which frequency range sources in the direction defined by the steering vector $\mathbf{a}_\vartheta(z)$ are active.

4. SIMULATIONS AND RESULTS

To assess the two P-MUSIC versions defined in (16) and (17), first some controlled scenarios are presented to highlight the ability for spatial discrimination of the proposed method. Thereafter, a more complex scenario is tested for (17) and benchmarked against an IFB MUSIC algorithm.

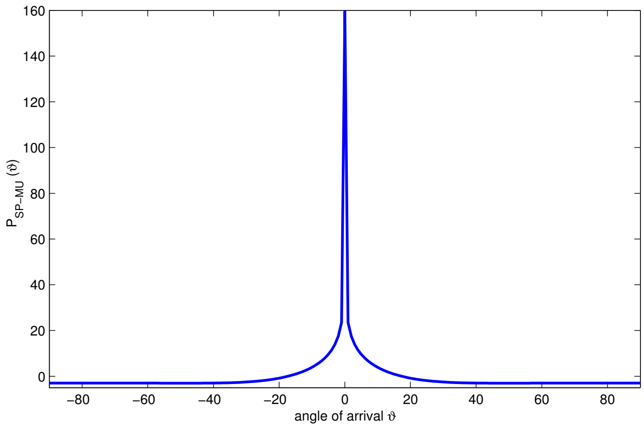

Figure 1: Spatial P-MUSIC $P_{\mathrm{SP-MU}}(\vartheta)$ for a source located at broadside ($\vartheta = 0°$).

4.1 Simple Angle-of-Arrival Scenarios

The broadband steering vector for a linear uniform array with $M = 4$ sensors separated by distances $d = \frac{c}{f_s}$ takes on simple forms for the broadside and end fire directions, which are evaluated below.

Example 1. Assuming a source at broadside, the steering vector for an $M = 4$ element linear uniform array is given by

$$\mathbf{a}_{\vartheta=0°}(z) = [1\ 1\ 1\ 1]^{\mathrm{T}} \quad . \tag{18}$$

If the source emits an uncorrelated random signal with zero mean and unit variance, the resulting covariance matrix is

$$\mathbf{R}_1(z) = \mathbf{a}_{\vartheta=0°}(z)\tilde{\mathbf{a}}_{\vartheta=0°}(z) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad . \tag{19}$$

This matrix has rank one, and the spatial P-MUSIC approach is identical to the MUSIC except for the use of broadband steering vectors in evaluating (16). The results for the latter are given in Fig. 1.

Example 2. If instead of the above situation, the source is located at end fire position $\vartheta = -90°$, the wavefront arrives at each sensor delayed by exactly one sampling period, i.e.

$$\mathbf{a}_{\vartheta=-90°}(z) = [1\ z^{-1}\ z^{-2}\ z^{-3}]^{\mathrm{T}} \quad . \tag{20}$$

As a result, the spatio-temporal covariance matrix is

$$\mathbf{R}_2(z) = \begin{bmatrix} 1 & z^1 & z^2 & z^3 \\ z^{-1} & 1 & z^1 & z^2 \\ z^{-2} & z^{-1} & 1 & z^1 \\ z^{-3} & z^{-2} & z^{-1} & 1 \end{bmatrix} \quad . \tag{21}$$

The result for the spatial P-MUSIC algorithm is shown in Fig. 2, correctly identifying the angle of arrival of the source.

Example 3. Assuming two independent sources, one at broadside and the other at end-fire position, the space-time covariance matrix is given by $\mathbf{R}_3(z) = \mathbf{R}_1(z) + \mathbf{R}_2(z)$. The Spatial P-MUSIC algorithm identifies two large polynomial eigenvalues, and from the noise-only subspace, the spatial P-MUSIC algorithm derives the result in Fig. 3.

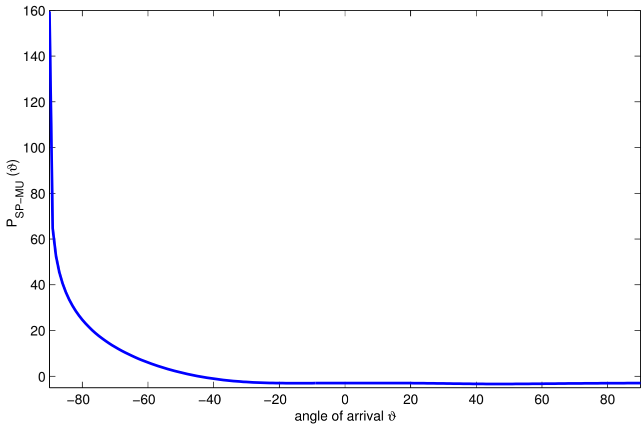

Figure 2: Spatial P-MUSIC $P_{\mathrm{SP-MU}}(\vartheta)$ for a single source located at end fire ($\vartheta = -90°$).

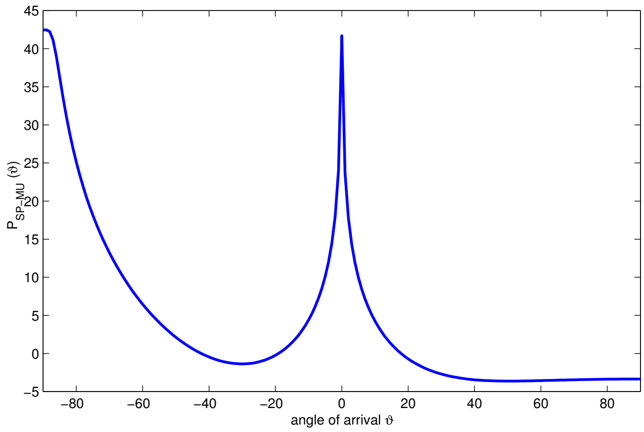

Figure 3: Spatial P-MUSIC $P_{\mathrm{SP-MU}}(\vartheta)$ for a scenario with two independent sources of equal strength located at broadside and end fire positions.

4.2 Spatio-Spectral Estimation

An $M = 8$ element array is illuminated by two broadband sources characterised as follows:

- source 1 — located at $\vartheta = 30°$, and active over a frequency range $\Omega \in [0.3125\pi,\, 0.7812\pi]$.
- source 2 — located at $\vartheta = -20°$, and active over a frequency range $\Omega \in [0.4688\pi, 0.9375\pi]$.
- the array signals are corrupted by independent and identically distributed complex Gaussian noise at 40dB SNR.

The source signals are simulated as a series of complex exponentials with randomised phases of 1000 samples length. The delay with which the waveform propagates across the array is implemented in the frequency domain by separately implementing the steering vector defined in (4) for every complex harmonic. The exact frequency locations of the source components can be varied to coincide or not coincide with the bin frequencies of an independent frequency bin processor. The latter provides the inputs to independent MUSIC algorithms operating in every frequency bin, which are used to benchmark the proposed SSP-MUSIC method.

The IFB approach with source frequencies coinciding with $N = 64$ bin frequencies is shown in Fig. 4. The result highlights the very accurate retrieval of both sources both in terms of angle of arrival as well as their spectral range.

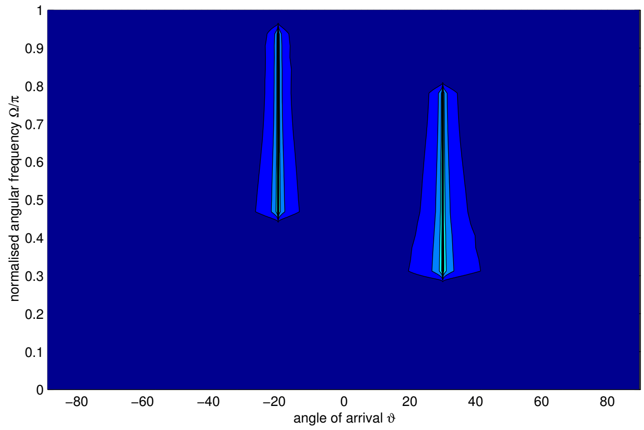

Figure 4: Narrowband MUSIC applied to independent frequency bins, for an $M = 8$ element array and bin frequencies coinciding with those of the sources.

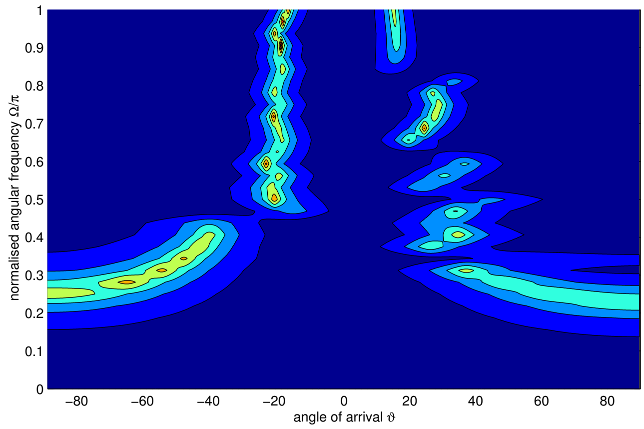

Figure 5: Narrowband MUSIC applied to independent frequency bins, for an $M = 8$ element array and bin frequencies not coinciding with those of the sources.

If the source frequencies do not coincide with bin frequencies, the IFB approach is known to yield a very poor worst-case performance [13], which is confirmed by the result in Fig. 5. The result exhibits a large number of spurious components in the lower frequency range, and is unable to extract the source at $\vartheta = 30°$ well.

The proposed spatio-spectral P-MUSIC approach (17) is applied to the same data as the IFB processor in Fig. 5. The result is shown in Fig. 6, and unlike Fig. 5 allows to accurately extract both angles of arrival and frequency range of the two contained broadband sources. The resolution is not as sharp as the IFB processor with coinciding frequency bins, which may be due to both the constructed nature of the IFB setup, as well as the fact that the fractional delay filters implementing the time delays associated with different angles ϑ are not accurate across the entire frequency range [6].

It is important to note that while the IFB processor is extremely sensitive to the location of the sinusoidal components, the proposed SSP-MUSIC algorithm has been found very robust.

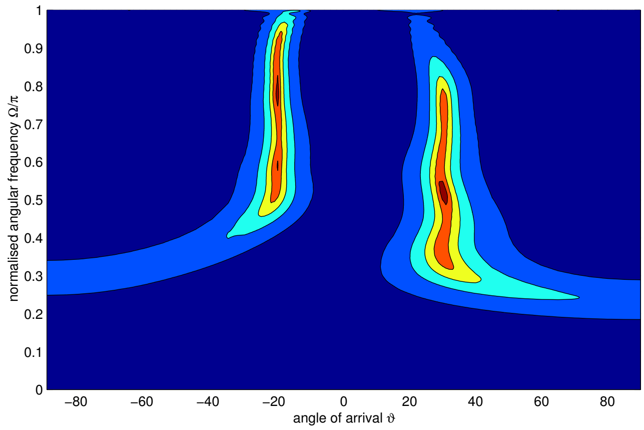

Figure 6: Spatio-Spectral P-MUSIC for the data analysed in Fig. 5.

5. CONCLUSIONS

We have proposed a broadband generalisation of the classical narrowband MUSIC algorithm, based on a polynomial covariance matrix, its polynomial EVD, and the definition of polynomial steering vectors. The derived polynomial MUSIC approach offers to variations, either analysing the angle of arrival only, or providing both spatial and spectral information on the sources contributing to the space-time covariance matrix.

In examples and simulations, we have highlighted some of the properties and benefits of the proposed method, in particular with respect to the independent frequency bin processor operating separate narrowband MUSIC algorithms in every frequency bin. This method was found to be very sensitive to the exact location of frequency components within the array data. In contrast, the proposed P-MUSIC algorithms have shown to be very robust in terms of signal properties.

Compared to IFB MUSIC with coinciding sources and frequency bins, the P-MUSIC algorithm is inferior. However, it is believed that further improvements in P-MUSIC are possible by carefully selecting well-behaved fractional delay filters [6] instead of the employed sinc functions.

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