

# DECENTRALIZED PHASE SYNCHRONIZATION SCHEME FOR COLLABORATIVE BEAMFORMING IN WIRELESS SENSOR NETWORKS

Lazar Berbakov, Javier Matamoros, Carles Antón-Haro

Centre Tecnològic de Telecomunicacions de Catalunya (CTTC)  
 Parc Mediterrani de la Tecnologia, Av. Carl Friedrich Gauss 7, 08860 Castelldefels, Spain  
 E-mail: {lazar.berbakov, javier.matamoros, carles.anton}@cttc.es

## ABSTRACT

In this paper, we propose a phase synchronization scheme suitable for collaborative beamforming with wireless sensor networks. The scheme does not require the BS to coordinate the allocation of sensors to the training timeslots or poll them individually (which can be burdensome for large networks): sensors randomly choose their respective training timeslots. In that sense, the synchronization scheme is decentralized. In this context, we ask ourselves whether there exists an optimal number of training timeslots, and about the optimal split for the training and data transmission periods. To answer this question, we analytically derive upper bounds of the resulting beamforming gain in two scenarios of interest: ideal, and noisy phase shift estimation. Computer simulation results are mainly given in terms of (normalized) beamforming gain and achievable throughput.

## 1. INTRODUCTION

In recent years, research in Wireless Sensor Networks (WSN) has attracted considerable attention. Nowadays, WSN are used in many industrial and civilian application areas, this including process control, environmental monitoring, health-care applications and home automation. Energy consumption is one of the main issues in design of WSN. Sensor nodes, which are mainly battery operated, are supposed to work for long periods of time without having their batteries replaced or its energy source renewed. Data transmission is one of the most energy consuming tasks and, thus, specific communication schemes need to be investigated. The problem is aggravated by the fact that, in many cases, sensors need to convey their measurements to a distant Base Station (BS). Sensor networks typically consist of a *large* number of sensors deployed in a given geographical area. This allows us to resort to (energy efficient) collaborative beamforming (CBF) schemes by which sensors cooperate in the transmission of a common message signal.

CBF schemes demand carrier frequency and phase synchronization in order to form a beam pattern with a stable mainlobe. The authors in [1] proposed a simple CBF scheme capable of achieving phase synchronization using only one bit of feedback from the BS. This approach was generalized in [2] by including simultaneous frequency and phase synchronization. Another approach for CBF is given in [3], where the authors show that by opportunistically *selecting* a subset of sensor nodes whose carrier signals combine in a quasi-coherent manner at the BS one can get substantial gains with respect to single node direct transmission. In [4, 5] the authors show that the main lobe does not depend on the actual node locations.

On the one hand, the main drawback of the *iterative* algorithms proposed in [1, 2] is that the number of itera-

tions needed to attain a given percentage of the maximum beamforming gain scales in the number of sensors. For large networks, this can lead to substantial signalling overhead (in particular, if phase synchronization needs to be carried out periodically). On the other, the opportunistic selection schemes proposed in [3] might lead to unequal energy consumption over sensors which, in general, is not practical.

In this paper, we propose a decentralized phase synchronization scheme in which sensors randomly choose their respective training timeslots. The total duration of the training plus data transmission period is fixed irrespectively of the number of sensors, which makes the iterative schemes proposed [1, 2] unsuitable. Instead, we ask ourselves whether there exists an optimal number of training timeslots, and about the optimal split for the durations of the training and data transmission periods. In our scheme, we let all sensors participate in the beamforming process since, unlike in [3], we count with mechanisms to pre-compensate the oscillator phase offset and the channel phase shift.

## 2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider a WSN consisting of  $N$  sensor nodes randomly placed over a disk of radius  $R$  according to a uniform distribution. The goal is to collaboratively transmit a common message  $m(t)$  with  $\mathbb{E}[|m(t)|^2] = 1$ , to a BS located on the XY plane at a distance  $D \gg R$  (i.e. far field conditions) (Fig. 1). In order to save energy, sensors are in the sleep

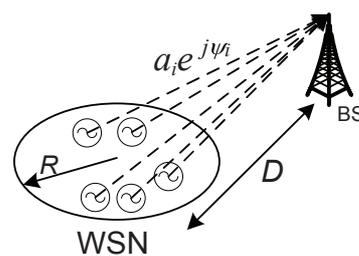


Figure 1: Wireless sensor network setup.

state (i.e. transceiver circuits are turned off) for most of the time. When new data needs to be collected, the BS broadcasts an RF signal. This beacon activates the energy detectors in the sensor nodes and wakes them up again (see [6] for details). After the sleep period, all transmitters are assumed to remain frequency-locked to the reference carrier frequency  $f_c$  (i.e., negligible frequency drift). The phase offset of each oscillator, however, is unknown, of a random nature, i.i.d. over sensors and uniformly distributed. The analytical signal transmitted by the  $i$ -th sensor node reads:

$$s_i(t) = w_i^* m(t) e^{j(2\pi f_c t + \gamma_i)}, \quad (1)$$

This work is partly supported by the EC-funded project NEWCOM++ (216715), JUNTOS, and the Spanish Ministry of Science and Innovation (FPU grant AP2008-03952).

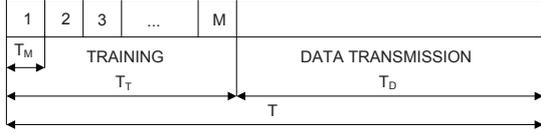


Figure 2: Training and data transmission phases.

where  $w_i = b_i e^{j\theta_i}$  denotes the corresponding complex transmit weight (to be designed), and  $f_c$  and  $\gamma_i \sim \mathcal{U}(-\pi, \pi)$  stand for the carrier frequency and the phase offset, respectively. The complex channel from the  $i$ -th sensor to the BS is denoted by  $h_i = a_i e^{j\psi_i}$ , where  $a_i$  and  $\psi_i$  account for the channel gain and phase shift associated to the Euclidean distance between the sensor and the BS. Accordingly, the signal received at the BS can be expressed as

$$r(t) = m(t) e^{j2\pi f_c t} \sum_{i=0}^{N-1} a_i b_i e^{j(\gamma_i + \psi_i - \theta_i)} + w(t) \quad (2)$$

with  $w(t) \sim \mathcal{CN}(0, \sigma_w^2)$ . Due to hardware limitations, the transmit power at each sensor node is assumed to be constant and, hence, the transmit weights read  $w_i = e^{j\theta_i}$ . We also assume line-of-sight conditions between the sensor and the (distant) BS which leads to  $a_i = 1$  for all sensors. Clearly, the received signal strength (RSS) at the BS is maximized when the individual signals are coherently combined, namely,  $\gamma_i + \psi_i - \theta_i = C; \forall i$  (where  $C$  is a constant, for example 0). To that aim, sensors must pre-compensate the (unknown) oscillator and channel phase offsets by properly adjusting the  $\theta_i$  term ( $\theta_i = \xi_i$  in Sec. 3, while  $\xi_i$  is estimated using ML phase estimator in Sec. 4). Consequently, a training period prior to data transmission is needed.

## 2.1 Communication protocol

Upon BS request, nodes wake up for  $T$  seconds during which a data packet will be transmitted. Typically,  $T$  is predefined and turns out to be a small percentage of the time elapsed between consecutive requests (i.e. low duty-cycle). Within this period of time, sensors need to (i) estimate  $\theta_i$ ; (ii) share the common message  $m(t)$ ; and (iii) actually transmit the message. For simplicity, we assume that (ii) is carried out transparently to (i) and (iii) and, hence, the packet consists of one training block and one data transmission block only. Their respective durations are  $T_T$  and  $T_D$ , with  $T = T_T + T_D$ . The training block, in turn, consists of  $M$  timeslots of duration  $T_M$  (see Fig.2). Each timeslot is used by a sensor (or group of sensors) in order to estimate the corresponding pre-compensation phase. In order to relieve the BS from the burden of allocating sensors to timeslots<sup>1</sup>, we allow sensors to randomly choose training timeslots according a uniform distribution, namely,  $p_j = 1/M; j = 0 \dots M-1$ . Let  $\mathcal{S}_j$  denote the subset of sensors in timeslot  $j$  of cardinality  $|\mathcal{S}_j| = N_j$ . Clearly,  $N_j$  is a binomial random variable and it fulfills  $\sum_{j=0}^{M-1} N_j = N$ . Whenever  $N_j > 1$ , the phase pre-compensation will be carried out for the *group* of sensors rather than for *individual* ones. Because of that, the overall received signal strength in the subsequent data transmission period will be lower. However, arbitrarily increasing the number of timeslots  $M$  (to avoid sensors to overlap) is detrimental, as well. The amount of information conveyed (i.e., throughput) in the transmit period is given by:

$$R(T_D) = T_D \log_2 \left( 1 + \frac{\text{RSS}^2(T_T)}{\sigma_w^2} \right). \quad (3)$$

<sup>1</sup>Note that, in realistic settings, the BS should first learn about which sensors woke up. Since the number is potentially large, the associated signalling needs would also be.

If  $T_M$  is pre-defined, then increasing  $M$  results into a shorter data block ( $T_D = T - M \cdot T_M$ ) and, consequently, lower throughput. If, on the contrary, the duration of the training period is fixed, then timeslots become shorter ( $T_M = \frac{T_T}{M}$ ) which results into poorer phase estimates (and, thus, lower RSS in the data transmission period). Consequently, the optimal split between the training and data transmission periods, and the optimal number of timeslots in the training phase should be identified. A more detailed analysis follows.

## 3. RSS ANALYSIS: IDEAL PHASE SHIFT ESTIMATION

In the training period, sensors merely transmit an unmodulated carrier. From (2) and by defining  $\phi_i = \gamma_i + \psi_i$ , the received signal in the  $j$ -th timeslot reads<sup>2</sup>:

$$r_j(t) = e^{j2\pi f_c t} \sum_{i \in \mathcal{S}_j} e^{j\phi_i} + w(t); \quad t \in [jT_M, (j+1)T_M) \quad (4)$$

The summation term can be conveniently expressed as

$$\sum_{i \in \mathcal{S}_j} e^{j\phi_i} = \text{RSS}_j e^{j\xi_j} \quad (5)$$

with  $\text{RSS}_j \geq 0$  denoting the received signal strength in  $j$ -th

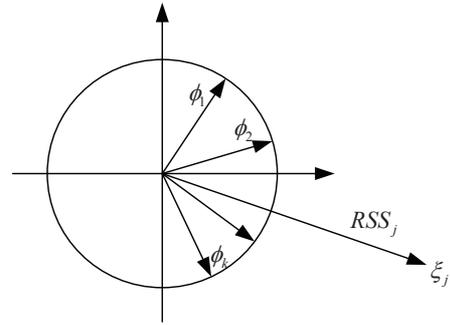


Figure 3: Aggregated phase shift in a training slot.

timeslot<sup>3</sup> and  $\xi_j$  standing for the aggregated phase shift (see Fig. 3). For the time being, we assume that  $\xi_j$  can be *perfectly* estimated and feedback to all the  $N_j$  sensors in timeslot  $j$ . Upon completion of the training period, each sensor node pre-compensates its carrier phase by setting  $\theta_i = \xi_j; \forall i \in \mathcal{S}_j$  and, hence, the received signal strength during the data transmission period yields:

$$\text{RSS} = \left| \sum_{j=0}^{M-1} \sum_{i \in \mathcal{S}_j} e^{j(\phi_i - \xi_j)} \right|, \quad (6)$$

and the expected value of the *normalized* RSS then reads

$$\overline{\text{RSS}} = \frac{1}{N} \mathbb{E}_{\{N_j\}_{j=0}^{M-1}, \{\phi_i\}_{i=0}^{N-1}} [\text{RSS}]. \quad (7)$$

From (5), it follows that  $\sum_{i \in \mathcal{S}_j} e^{j(\phi_i - \xi_j)} = \text{RSS}_j \in \{\mathbb{R}^+, 0\}$ . If, in addition, we define  $\Phi_N \triangleq \{\phi_i\}_{i=0}^{N-1}$  then we can write:

$$\overline{\text{RSS}} = \frac{1}{N} \sum_{j=0}^{M-1} \mathbb{E}_{N_j, \Phi_N} [\text{RSS}_j], \quad (8)$$

<sup>2</sup>Time synchronization is already achieved.

<sup>3</sup>For simplicity, the contribution of the additive noise to the resulting RSS will be neglected throughout this paper.

where:

$$\begin{aligned}
\mathbb{E}_{N_j, \Phi_N} [\text{RSS}_j] &= \\
&= \mathbb{E}_{N_j, \Phi_N} \left[ \sum_{k \in \mathcal{S}_j} e^{j(\phi_k - \xi_j)} \right] \\
&= \mathbb{E}_{N_j, \Phi_N} \left[ \sqrt{\sum_{k \in \mathcal{S}_j} e^{j(\phi_k - \xi_j)} \sum_{l \in \mathcal{S}_j} e^{-j(\phi_l - \xi_j)}} \right] \\
&= \mathbb{E}_{N_j, \Phi_N} \left[ \sqrt{N_j + \sum_{k \in \mathcal{S}_j} \sum_{\substack{l \in \mathcal{S}_j \\ l \neq k}} e^{j(\phi_k - \phi_l)}} \right] \quad (9) \\
&= \mathbb{E}_{N_j, \Phi_N} \left[ \sqrt{N_j + \sum_{k \in \mathcal{S}_j} \sum_{\substack{l \in \mathcal{S}_j \\ l > k}} 2\mathcal{R}\{e^{j(\phi_k - \phi_l)}\}} \right] \\
&= \mathbb{E}_{N_j, \Phi_N} \left[ \sqrt{N_j + \sum_{k \in \mathcal{S}_j} \sum_{\substack{l \in \mathcal{S}_j \\ l > k}} 2 \cos(\phi_k - \phi_l)} \right]
\end{aligned}$$

The expectation in this last expression is difficult to compute in closed form. Hence, we resort to the following upper bound:

$$\begin{aligned}
\mathbb{E}_{N_j, \Phi_N} [\text{RSS}_j] &\leq \\
&\leq \mathbb{E}_{N_j} \left[ \sqrt{N_j + \sum_{k \in \mathcal{S}_j} \sum_{\substack{l \in \mathcal{S}_j \\ l > k}} 2\mathbb{E}_{\phi_k, \phi_l} [\cos(\phi_k - \phi_l)]} \right], \quad (10)
\end{aligned}$$

which follows from Jensen's inequality and the fact that  $N_j$  is statistically independent of  $\Phi_N$ . With the change of variables  $z = \phi_k - \phi_l$ , the inner expectation term yields

$$\mathbb{E}_{\phi_k, \phi_l} [\cos(\phi_k - \phi_l)] = \mathbb{E}_z [\cos(z)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(z) dz = 0. \quad (11)$$

which follows from the fact that  $z$  is uniformly distributed in  $[-\pi, \pi]$  since so are both  $\phi_k$  and  $\phi_l$ <sup>4</sup>.

From (10) and (11), we conclude that the contribution of the sensors in the  $j$ -th slot to the resulting RSS can be upper-bounded as follows:

$$\mathbb{E}_{N_j, \Phi_N} [\text{RSS}_j] \leq \mathbb{E}_{N_j} [\sqrt{N_j}]. \quad (12)$$

and, hence, the *normalized* RSS of (8) can in turn be upper-bounded by:

$$\overline{\text{RSS}} \leq \frac{M}{N} \mathbb{E}_{N_j} [\sqrt{N_j}]. \quad (13)$$

Unfortunately, this expectation can not be computed in closed-form and, as such, is not very informative. We can gain some insight by letting  $M$  and  $N$  grow without bound at a constant ratio  $\alpha = \frac{M}{N}$ . In this case, the (binomial) random variable  $N_j$  is well approximated by a Poisson r.v. of

<sup>4</sup>The pdf of  $z = \phi_k - \phi_l$  is given by the convolution of the individual pdfs and, thus, it exhibits a triangular shape within  $[-2\pi, 2\pi]$ . Phase wrapping effects, render this pdf equivalent to a uniform one within  $[-\pi, \pi]$ .

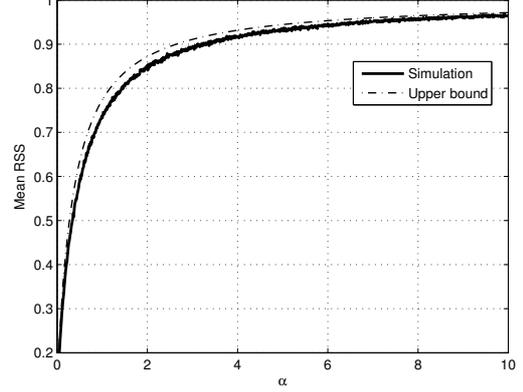


Figure 4: Normalized RSS vs.  $\alpha = \frac{M}{N}$  ( $N = 100$ ).

mean  $\alpha^{-1}$  [7](Ch.3). From all the above, the upper bound for *normalized* RSS in (13) yields:

$$\overline{\text{RSS}} \leq e^{-\frac{1}{\alpha}} \sum_{k=0}^{\infty} \alpha^{1-k} \frac{\sqrt{k}}{k!}. \quad (14)$$

This expression reveals that, with perfect phase-shift estimation, the normalized RSS exclusively depends on  $\alpha$ , that is, the ratio of the number of available timeslots to the number of sensors. In Fig. 4, we depict the actual RSS (for a scenario with  $N = 100$  sensors) along with the corresponding upper bound. The bound is particularly tight for large  $\alpha$  (i.e.  $M \gg N$ ) since, in this case, the upper bound in (10) is tight as well (essentially, the cross terms in the summations vanish). Besides, we also realize that, for large  $\alpha$ , the system achieves full beamforming gain. This follows from the fact that, for large  $M$ , the probability of having more than one sensor in a time slot is low. Consequently, the phase shift can be ideally estimated and pre-compensated for each individual sensor rather than for the whole group.

#### 4. RSS ANALYSIS: NOISY PHASE SHIFT ESTIMATION

Here, we assume that the duration of the training period is fixed (e.g. defined as a percentage of the data transmission time). Consequently, the higher the number of timeslots, the shorter their duration. This has an impact on the quality of the corresponding phase estimates  $\hat{\xi}_j$  that we analyze next.

Let  $f_s$  be the sampling frequency. Consequently, the total number of samples in the training period and in each timeslot are  $L_T = f_s T_T$  and  $L = L_T/M$ , respectively. The maximum-likelihood (ML) estimate of the aggregated phase shift in the  $j$ -th slot is given by [8](Ch.7)

$$\hat{\xi}_j = -\arctan \frac{\sum_{n=0}^{L-1} r_j[n] \sin(2\pi f_c n)}{\sum_{n=0}^{L-1} r_j[n] \cos(2\pi f_c n)}. \quad (15)$$

where  $r_j[n]$  denotes the sampled version of the received signal. For large  $L$ , the estimation error  $\Delta \xi_j = \xi_j - \hat{\xi}_j$  turns out to be a zero-mean Gaussian r.v. of variance

$$\sigma_{\Delta \xi_j}^2 = \frac{2\sigma_w^2}{L \cdot \text{RSS}_j^2} = \frac{2M\sigma_w^2}{L_T \cdot \text{RSS}_j^2}. \quad (16)$$

which indicates that the quality of the estimate is a function of the number of samples  $L$  in the timeslot. To recall, the instantaneous RSS at the BS after phase pre-compensation

by all sensors reads

$$\text{RSS} = \left| \sum_{j=0}^{M-1} \sum_{i \in \mathcal{S}_j} e^{j(\phi_i - \xi_j)} \right|, \quad (17)$$

and, from (5), we can write

$$\frac{1}{N} \text{RSS} = \frac{1}{N} \left| \sum_{j=0}^{M-1} \text{RSS}_j e^{j\Delta\xi_j} \right| = \alpha \left| \frac{1}{M} \sum_{j=0}^{M-1} \text{RSS}_j e^{j\Delta\xi_j} \right|. \quad (18)$$

From the weak law of large numbers, for large  $M$  and  $N$  we have that

$$\alpha \left| \frac{1}{M} \sum_{j=0}^{M-1} \text{RSS}_j e^{j\Delta\xi_j} \right| \xrightarrow{P} \alpha \left| \mathbb{E}_{N_j, \Phi_N, \Delta\xi_j} [\text{RSS}_j e^{j\Delta\xi_j}] \right|, \quad (19)$$

where  $P$  denotes convergence in probability. With a slight abuse of notation, the normalized RSS asymptotically reads:

$$\overline{\text{RSS}} = \frac{M}{N} \left| \mathbb{E}_{N_j, \Phi_N, \Delta\xi_j} [\text{RSS}_j e^{j\Delta\xi_j}] \right| \quad (20)$$

or, equivalently,

$$\overline{\text{RSS}} = \frac{M}{N} \left| \mathbb{E}_{N_j, \Phi_N} \left[ \text{RSS}_j \mathbb{E}_{\Delta\xi_j | N_j, \Phi_N} [e^{j\Delta\xi_j}] \right] \right|. \quad (21)$$

Since the phase estimation errors are Gaussian-distributed, this implies that

$$\mathbb{E}_{\Delta\xi_j | N_j, \Phi_N} [e^{j\Delta\xi_j}] = e^{-\frac{1}{2}\sigma_{\Delta\xi_j}^2} = e^{-\frac{M\sigma_w^2}{L_T \cdot \text{RSS}_j^2}}. \quad (22)$$

From the last two equations and by resorting again to Jensen's inequality, we can thus write

$$\overline{\text{RSS}} = \frac{M}{N} \left| \mathbb{E}_{N_j, \Phi_N} \left[ \text{RSS}_j e^{-\frac{M\sigma_w^2}{L_T \cdot \text{RSS}_j^2}} \right] \right| \quad (23)$$

$$= \frac{M}{N} \mathbb{E}_{N_j, \Phi_N} \left[ \text{RSS}_j e^{-\frac{M\sigma_w^2}{L_T \cdot \text{RSS}_j^2}} \right] \quad (24)$$

$$\leq \frac{M}{N} \mathbb{E}_{N_j} \left[ \sqrt{N_j} e^{-\frac{M\sigma_w^2}{L_T \cdot N_j}} \right] \quad (25)$$

where in the second equality we have exploited the fact that all the terms in the expectation are real-valued and positive. Again, for  $M, N \rightarrow \infty$  and  $\alpha = \frac{M}{N}$  constant, the random variable  $N_j$  is approximately Poisson distributed and, thus,

$$\overline{\text{RSS}} \leq e^{-\frac{N}{M}} \sum_{k=0}^{\infty} \left( \frac{M}{N} \right)^{1-k} \frac{\sqrt{k}}{k!} e^{-\frac{M\sigma_w^2}{L_T k}} \quad (26)$$

Interestingly, the exponential term in the summation models the decrease in RSS resulting for the use of imperfect phase estimates.

## 5. NUMERICAL RESULTS

In this section, we present some computer simulation and numerical results which illustrate the behavior of the proposed distributed beamforming scheme. In all cases, the duration of the training slots is inversely proportional to  $M$  and, hence, we address the scenario with imperfect (noisy)

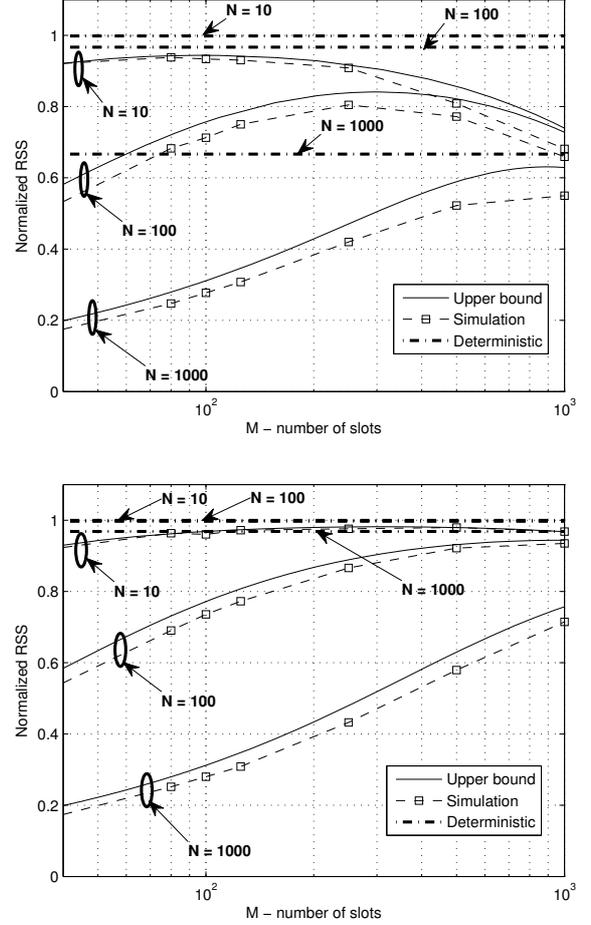


Figure 5: Normalized RSS vs. number of timeslots  $M$  ( $\sigma_w^2 = 3$ , Top:  $T_T = 10^4$ , Bottom:  $T_T = 10^5$ )

phase estimates. Without loss of generality, time is normalized to the sampling period (i.e.  $T_s = 1$ ).

Figure 5 (top) depicts the normalized RSS as a function of the number of timeslots  $M$ . For benchmarking purposes, we also indicate the RSS attainable when the BS allocates sensors to timeslots in a centralized manner ( $M = N$  case, curves labeled as 'deterministic') and the corresponding upper bounds of (26). The plot reveals that the upper bound is, in general, tight. However, it is worth noting that it is not valid for very small or large values of  $M$ . For small  $M$ , on the one hand, the bound (25) is loose and, besides, the approximation of a binomial distribution by a Poisson one is not accurate. For large  $M$ , on the other, the assumption of large  $L = L_T/M$  in the computation of the asymptotic variance of (16) does not hold. We also observe that the optimal number of timeslots increases for larger networks (see maxima for the curves with  $N = 10, 100$  and  $1000$  sensors). Intuitively, the higher the number of sensors, the higher the number of timeslots needed to minimize the risk of having more than one sensor in one timeslot (even if this comes at a price of experiencing higher variance in the phase estimates). Besides, we observe that the loss (in terms of normalized RSS) for schemes with decentralized allocation of timeslots is moderate (some 5% to 15%). In other words, the lack of coordination can be in part compensated with a sufficiently high (and optimally designed) number of training timeslots. Complementarily, Fig. 5 (bottom) shows some additional results in scenarios with longer training periods ( $T_T = 10^5$ )

vs.  $T_T = 10^4$ ). Unsurprisingly, the resulting RSS is higher and so is the optimal number of timeslots.

Next, in Fig. 6, we plot the optimal number of timeslots  $M^*$  as a function of the variance of the channel noise  $\sigma_w^2$ . Clearly,  $M^*$  is a monotonically decreasing function of  $\sigma_w^2$ : in noisy scenarios, more samples per timeslot ( $L = L_T/M$ ) are needed to average out channel noise. It also reveals that the optimal number of timeslots increases for an increasing duration of the training period  $T_T$ . Since the number of sensors is fixed ( $N = 100$ ), for larger  $T_T$  the probability of having more than one sensor per timeslot decreases without impairing too much the quality of the estimates. As a remark, the curves saturate in the low-noise region because the lowest possible number of samples per timeslot (namely,  $L = 1$ ) is reached there.

Finally, in Fig. 7 we depict the throughput given by (3) vs.

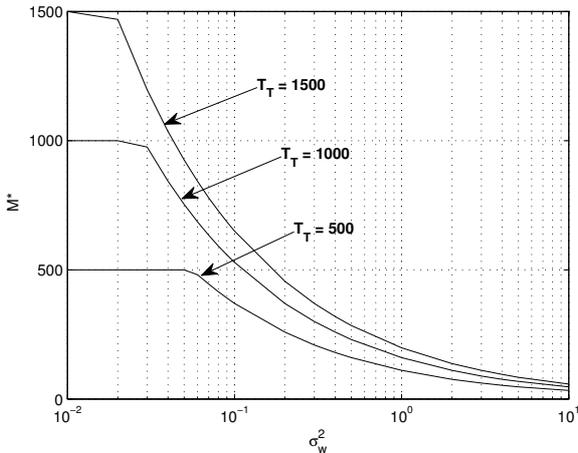


Figure 6: Optimal number of timeslots vs. variance of the channel noise ( $N = 100$ )

the total duration of the data transmission period (for fixed  $T = 10^4$ ). For the training period, we (numerically) optimize on the number of timeslots. For benchmarking purposes, we also include data on the theoretical throughput achievable if the (channel and oscillator) phase shifts were known and, consequently, no training period were needed (i.e.  $T_D = T$  and  $RSS = N$ , dashed lines). As expected, there exists an optimal operating point for each curve where the best trade-off in terms of the beamforming gain after phase adjustments vs. time left for data transmission is reached. Interestingly, the duration of the training period amounts to less than 10% of the wake up time  $T$  only. Moreover, the gap with respect to the highest theoretical throughput is on the order of some 25%, in all cases.

## 6. CONCLUSIONS

In this paper, we have proposed a phase synchronization scheme for collaborative beamforming with wireless sensor networks. The scheme does not require the BS to coordinate the allocation of sensors to the training timeslots or poll them individually and, in that sense, it is decentralized. We have derived (in general, tight) closed-form expressions for the upper bound of the resulting received signal strength (i.e., beamforming gain) in two scenarios of interest: ideal, and noisy phase shift estimates arising from a finite duration of the training period. For the first scenario, we have learnt that, asymptotically, the normalized received signal strength exclusively depends on the ratio of the number of timeslots to sensors in the network (and it is a monotonically increasing function). For the latter, numerical results reveal that there

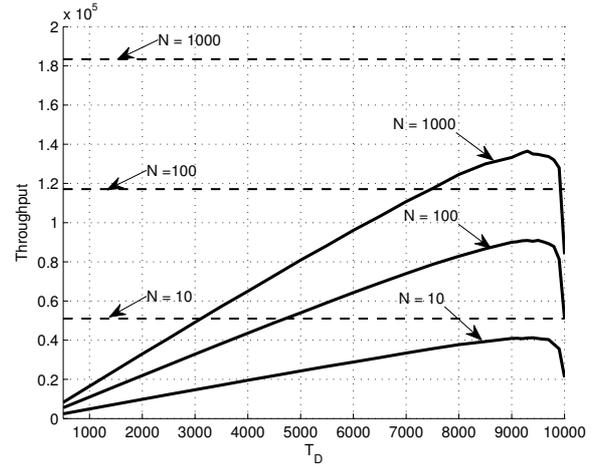


Figure 7: Throughput vs. data transmission time ( $\sigma_w^2 = 3$ ,  $T = 10^4$ ). Dashed lines indicate the throughput with known channel and oscillator phase shifts.

exists an optimal number of timeslots maximizing the overall received signal strength for the data transmission period. The optimal number of timeslots increases with the number of sensors in the network and the duration of the training period, and it decreases with the variance of the channel noise. The loss, in terms of normalized RSS, with respect to centralized schemes is moderate (some 5% to 20%). In terms of achievable throughput, there also exists an optimal split for the duration of the training and data transmission periods which attains the best trade-off in terms of beamforming gain vs. time left for data transmission. The gap with respect to the theoretical throughput with ideal knowledge on channel and oscillator phase shifts is on the order of 25%.

## REFERENCES

- [1] R. Mudumbai, J. Hespanha, U. Madhow, and G. Barriac, "Distributed transmit beamforming using feedback control," *IEEE Trans. Information Theory*, vol. 56, pp. 411–426, jan. 2010.
- [2] M. Seo, M. Rodwell, and U. Madhow, "A feedback-based distributed phased array technique and its application to 60-ghz wireless sensor network," *Microwave Symposium Digest, 2008 IEEE MTT-S International*, pp. 683–686, jun. 2008.
- [3] M.-O. Pun, D. Brown, and H. Vincent Poor, "Opportunistic collaborative beamforming with one-bit feedback," *IEEE 9th Workshop SPAWC 2008.*, pp. 246–250, jul. 2008.
- [4] H. Ochiai, P. Mitran, H. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," *IEEE Trans. Signal Processing*, vol. 53, pp. 4110–4124, nov. 2005.
- [5] M. F. Ahmed and S. A. Vorobyov, "Collaborative beamforming for wireless sensor networks with gaussian distributed sensor nodes," *IEEE Trans. Wireless Communications*, vol. 8, pp. 638–643, feb. 2009.
- [6] G. Mergen, Q. Zhao, and L. Tong, "Sensor networks with mobile access: Energy and capacity considerations," *IEEE Trans. Communications*, vol. 54, no. 10, p. 1896, 2006.
- [7] A. Papoulis, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, Inc., 1991.
- [8] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall PTR, 1993.