SOUND SPEED ESTIMATION FROM TIME OF ARRIVALS: DERIVATION AND COMPARISON WITH TDOA-BASED ESTIMATION

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ABSTRACT

In this paper we propose a method for the estimation of the propagation speed of sound waves from Time Of Arrivals (TOAs) observed at a sensor array. In our previous work we presented a speed estimation method based on Time Differences of Arrival (TDOAs), here we show that the same approach may be applied successfully to the TOA-based problem. The exploitation of TOA measurements is driven by applications where synchronization between source and receiver is available. The proposed method provides an estimate of the average propagation speed of the wave along the source-receiver paths. Such a speed estimate is evaluated and compared with the TDOA-based estimate by means of Cramer-Rao Bound analysis and simulations.

***Index Terms*—** Propagation speed, speed of sound, time of arrival, TOA, source localization

1. INTRODUCTION

The estimation of the propagation speed of a traveling wave is a problem of general interest in several research fields. Knowledge of the actual propagation speed not only may improve the accuracy of localization systems, e.g. in acoustic navigation [1], but also reveals important properties of the propagation medium, e.g. in seismic exploration [2].

In [3] we exploited the speed of sound estimation to infer the air temperature from Time Differences Of Arrivals (TDOAs) produced by an unknown acoustic source at a microphone array. However in many application scenarios, e.g. microphone array calibration [4] or loudspeaker localization [5], synchronization between source and receiver is available and the wave form of the source signal is also known. Thus also the absolute Time Of Arrivals (TOAs) observed at spatially distributed receivers can be exploited for the estimation. For example in [5] TDOA and TOA measurements are combined to achieve better loudspeaker localization accuracy, as they carry slightly different information about the source location. For this reason the authors deemed meaningful to assess the problem of estimating the propagation speed also from TOA measurements.

To this end, this paper reconsiders TOA-based source localization and shows how to use it as an intermediate step to obtain an estimate of the actual propagation speed from TOAs. Indeed accurate propagation speed estimates may also improve localization results when undesired temperature variations occur [6]. In fact the proposed method allows to track the resulting changes of the speed of sound without relying on local temperature measurements by means of dedicated sensors. Rather, it is based merely on acoustic measurements and provides an estimate of the average propagation speed of the wave along the whole source-receiver path. The paper is structured as follows; Section 2 reviews some standard TOA-based localization techniques since they serve as basis for the speed estimation described in Section 3. Section 4 presents the Cramer-Rao-Bound analysis of the TOA-based estimation. Section 5 includes simulation results which compare TOA- and TDOA-based estimations. Finally Section 6 draws the conclusions of this work.

2. SOURCE LOCALIZATION METHODS

In this section some standard localization techniques are reviewed. This is a necessary step to derive the estimate of the propagation speed. In contrast to our previous publications we assume here that synchronization between the unknown located source and the receivers is available. Therefore source localization is performed by exploiting the Time Of Arrivals (TOAs) from the source to different receivers.

2.1. TOA-Based Source Localization Problem

Consider the Euclidean space of $D = 2$ dimensions as depicted in Fig. 1 (the case $D = 3$ follows straightforwardly). The source to be localized lies in an unknown position $\boldsymbol{x} = [x\ y]^{\mathrm{T}}$ whereas the M sensors of the array are at the known positions $\boldsymbol{a}_i = [x_i\ y_i]^{\mathrm{T}}$ with $i = 1, \cdots, M$. Let t_i indicate the Time Of Arrival (TOA) between the source $\boldsymbol{x}$ and the reference sensor $\boldsymbol{a}_i$. Then the TOA-based localization problem

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is to find $\boldsymbol{x}$ given the sensor positions $\boldsymbol{a}_i$ and the TOA set $\boldsymbol{t} = [t_1 \ \cdots \ t_M]^{\mathrm{T}}$.

Typically the propagation speed c is assumed to be a known constant (though it might be unknown or known with large uncertainty), this allows to convert TOAs into source's ranges $r_i = ct_i$. From geometrical reasoning it can be stated

$$r_i = ct_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| \ , \qquad i = 1, \cdots, M \ , \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean vector norm. The unknown source position $\boldsymbol{x}$ must fulfill the above N equations.

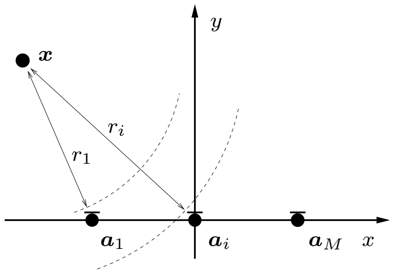

Fig. 1. Geometry of the source localization problem using a sensor array and TOA-measurements.

2.2. Least-Squares Solution

In what follows we use the approach called Squared Range Least Squares (SR-LS) [7]. First we take the square of Eq. (1), then after some calculations we obtain

$$\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{x} - \frac{\|\boldsymbol{x}\|^2}{2} = \frac{\|\boldsymbol{a}_i\|^2 - (ct_i)^2}{2}, \qquad i = 1, \cdots, M \ . \tag{2}$$

The above equation is quite similar to the one of the TDOA-based problem also known as Squared Range-Difference Least Squares (SRD-LS) [7]. Thus also the TOA-based problem can be addressed in much the same way. A change of variable, i.e.

$$\alpha = \|\boldsymbol{x}\|^2 \ , \tag{3}$$

allows an Unconstrained Least Squares (ULS) solution of the TOA-based problem by means of LS solution of the following linear system

$$\boldsymbol{\Phi}\boldsymbol{y} = \boldsymbol{b} \ , \tag{4}$$

where

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{x} \\ \alpha \end{bmatrix} \ , \quad \boldsymbol{\Phi} = \left[\boldsymbol{A} \,\middle|\, \boldsymbol{v}\right] = \begin{bmatrix} \boldsymbol{a}_1^{\mathrm{T}} & -\frac{1}{2} \\ \vdots & \vdots \\ \boldsymbol{a}_M^{\mathrm{T}} & -\frac{1}{2} \end{bmatrix} \ , \tag{5}$$

$$\boldsymbol{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_M \end{bmatrix} \ , \quad b_i = \frac{1}{2}\left(\|\boldsymbol{a}_i\|^2 - (ct_i)^2\right) \ . \tag{6}$$

As long as $M \geq D + 1$ the LS solution of the above system is given in terms of the pseudo-inverse $\boldsymbol{\Phi}^{\dagger}$

$$\hat{\boldsymbol{y}} = \begin{bmatrix} \hat{\boldsymbol{x}} \\ \hat{\alpha} \end{bmatrix} = \boldsymbol{\Phi}^{\dagger}\boldsymbol{b} = (\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{b} \ . \tag{7}$$

Usually the first vector $\hat{\boldsymbol{x}}$ comprised of the first D elements of $\hat{\boldsymbol{y}}$ is the *unconstrained* estimate of source position. The scalar estimate $\hat{\alpha}$ is considered a by-product of the estimation, though it is actually an independent LS estimate of the squared range $\|\boldsymbol{x}\|^2$.

3. PROPAGATION SPEED ESTIMATION

The above mentioned localization methods assume that the average propagation speed of the wave along the source-receiver paths is the same and is exactly known. Thus only the TOA-measurement noise impairs the localization and it is typically modelled as a zero mean, Gaussian random process as explained in Section 4. Under this assumption the residual function of the LS minimization compensates for such a random noise.

However in many practical cases the assumed propagation speed c has a variable degree of uncertainty depending on the specific application scenario. We may reasonably write the assumed speed as $c = c^{\circ} + \Delta c$ where Δc represents the deviation from the true average speed c°. Concerning the localization, this means that the ranges $r_i = ct_i = (c^{\circ} + \Delta c)t_i$ are all affected by the same deterministic scaling error that has to be taken into account. The propagation speed estimation method described in the following provides an estimate of the deviation Δc and thus an estimate of the true propagation speed $c^{\circ} = c - \Delta c$.

3.1. TOA-Based Speed Estimation

The basic idea of the proposed speed estimation method is the same used in [8] to address the TDOA-based problem: a scalar value $\hat{c}$ is determined which best fits the model given the observed TOAs.

As usual we express the ULS solution (7) separately for $\hat{\boldsymbol{x}}$ and $\hat{\alpha}$, respectively, and dependent on the assumed propagation speed c,

$$\hat{\boldsymbol{x}}(c) = \boldsymbol{\Gamma}\boldsymbol{b}(c) \ , \quad \hat{\alpha}(c) = \boldsymbol{\theta}\boldsymbol{b}(c) \tag{8}$$

where

$$\boldsymbol{\Gamma} = (\boldsymbol{P}_{\boldsymbol{v}}^{\perp}\boldsymbol{A})^{\dagger} \ , \quad \boldsymbol{\theta} = (\boldsymbol{P}_{\boldsymbol{A}}^{\perp}\boldsymbol{v})^{\dagger} \tag{9}$$

are a known matrix and a known vector, respectively. They follow from the orthogonal projectors

$$\boldsymbol{P}_{\boldsymbol{A}}^{\perp} = \mathbf{I} - \boldsymbol{A}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}} \ , \quad \boldsymbol{P}_{\boldsymbol{v}}^{\perp} = \mathbf{I} - \frac{1}{\|\boldsymbol{v}\|^2}\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}} \ . \tag{10}$$

The scalar error function $\delta(c)$ follows by using (8) and (3)

$$\delta(c) = \|\hat{\boldsymbol{x}}(c)\| - \sqrt{\hat{\alpha}(c)} = \|\boldsymbol{\Gamma}\boldsymbol{b}(c)\| - \sqrt{\boldsymbol{\theta}\boldsymbol{b}(c)} \ . \tag{11}$$

The above function represents, dependent on the assumed speed c, the error between the source's ranges obtained from the two ULS estimates $\hat{\boldsymbol{x}}(c)$ and $\hat{\alpha}(c)$, respectively. The speed estimate has to be a zero of $\delta(c)$, as it provides compatible estimates $\hat{\boldsymbol{x}}(c)$ and $\hat{\alpha}(c)$. Given a reasonable guess $\bar{c}$ for the propagation speed, the first order Taylor expansion

$$\delta(c) \approx \delta_{\text{lin}}(c) = \bar{\delta} + \bar{\delta}'(c - \bar{c}) \ , \tag{12}$$

with

$$\bar{\delta} = \delta(\bar{c}) \quad \text{and} \quad \bar{\delta}' = \left. \frac{\mathrm{d}\delta(c)}{\mathrm{d}c} \right|_{c=\bar{c}} \tag{13}$$

strongly simplifies the problem. For instance in acoustic applications a good initial value is $\bar{c} = 343.4 \ \mathrm{m/s}$ which is the speed of sound in dry air at $20 \ ^\circ\mathrm{C}$. The propagation speed estimate $\hat{c}$ is obtained from the zero-crossing of the linearized function $\delta_{\text{lin}}(c)$ as

$$\hat{c} = \bar{c} - \Delta\hat{c} \quad \text{with} \quad \Delta\hat{c} = \frac{\bar{\delta}}{\bar{\delta}'} \ . \tag{14}$$

The value of the first order derivative at $\bar{c}$ can be calculated with derivation rules from (11)

$$\bar{\delta}' = \frac{\hat{\boldsymbol{x}}(\bar{c})^{\mathrm{T}}}{\|\hat{\boldsymbol{x}}(\bar{c})\|} \boldsymbol{\Gamma} \bar{\boldsymbol{b}}' - \frac{1}{2} \frac{\boldsymbol{\theta} \bar{\boldsymbol{b}}'}{\sqrt{\hat{\alpha}(\bar{c})}} \ , \tag{15}$$

where $\hat{\boldsymbol{x}}(\bar{c})$ and $\hat{\alpha}(\bar{c})$ are unconstrained estimates given the speed guess $\bar{c}$ while the vector $\bar{\boldsymbol{b}}'$ contains the derivatives of (6) evaluated at $c = \bar{c}$.

3.2. Multiple Observation Approach

In this section we show how to exploit multiple TOA observations for estimating the propagation speed. This approach applies whenever the average propagation speed can be considered constant independently of time or source position. This is true for instance for consecutive TOA observations of the same acoustic source as long as they are made in a short time interval with no significant temperature variations. Another example are experiments in small spaces with homogeneous temperature distribution, then TOA observations of differently positioned sources still carry the same speed information. Following this idea the scalar function in (11) is built using TOA measurements of N different sources, $\boldsymbol{x}_n$ with $n = 1, 2, \cdots, N$ (actually the index n could represent the same source but at different measurement times). In noisy conditions a speed estimate can be found from the minimization in the least-squares sense of the obtained functions $\delta_n(c)$. The corresponding cost function is given by

$$\sum_{n=1}^{N} \delta_n^2(c) = \sum_{n=1}^{N} \left(\|\boldsymbol{\Gamma} \boldsymbol{b}_n(c)\| - \sqrt{\boldsymbol{\theta} \boldsymbol{b}_n(c)} \right)^2 \ , \tag{16}$$

where the index n indicates that the vectors $\boldsymbol{b}_n(c)$ from (6) are obtained using the TOA-observations corresponding to the source $\boldsymbol{x}_n$. Again the linear approximation in (12) can be applied to perform the minimization efficiently.

3.3. Comparison with TDOA-based Estimation

Table 1 represents similarities and differences between the quantities involved in the TOA- and TDOA-based LS estimation. For the latter case an extra sensor $\boldsymbol{a}_0$ is assumed to be positioned at the origin of the reference system to obtain M measurements relative to that extra sensor (spherical TDOA set). As already noted in [7] we see that the TOA-based problem presents a quadratic constraint while the TDOA-based a linear one. In addition the system matrix of the TOA-based problem does not contain measured data, rather it is formed by the sensor positions and the constant vector $\boldsymbol{v}$. As a consequence, the matrix $\boldsymbol{\Gamma}$ and the vector $\boldsymbol{\theta}$ are free of measurements noise.

4. CRAMER-RAO BOUND ANALYSIS

To assess the statistical performance of the proposed estimation method we consider the Cramer-Rao Bound (CRB). The CRB is a lower bound on the variance of *any* unbiased estimator, we already used it as a benchmark for the TDOA-based propagation speed estimation [3]. The derivation here will be sketchy, more details on the CRB can be found in [9].

The following error model for the measured TOA vector is used

$$\boldsymbol{t} = \boldsymbol{t}^\circ + \boldsymbol{\epsilon} = \frac{1}{c} \boldsymbol{r}^\circ(\boldsymbol{x}) + \boldsymbol{\epsilon} \ , \tag{17}$$

where $\boldsymbol{t}^\circ$ is the vector of ideal TOAs, $\boldsymbol{r}^\circ(\boldsymbol{x})$ the vector of exact ranges derived from geometry and $\boldsymbol{\epsilon}$ a random vector with covariance matrix $\boldsymbol{\Sigma} = \sigma^2 \mathbf{I}$ independent of $\boldsymbol{x}$ and c. The vector $\boldsymbol{t}$ carries information about the unknowns of the problem, i.e. the position of the source $\boldsymbol{x}$ and the propagation speed c. Such an information, degraded by the noise vector $\boldsymbol{\epsilon}$ is quantified by the Fisher's Information Matrix (FIM). The inverse of the FIM gives the CRB on the variance of *any* unbiased estimator. For the TOA-based localization and speed estimation problem the following matrix can be derived

$$\boldsymbol{F}^{-1} = (\sigma \, c)^2 \left(\begin{bmatrix} \boldsymbol{U}^{\mathrm{T}} \boldsymbol{U} & -\boldsymbol{U}^{\mathrm{T}} \boldsymbol{t}^\circ \\ -\boldsymbol{t}^{\circ \mathrm{T}} \boldsymbol{U} & \|\boldsymbol{t}^\circ\|^2 \end{bmatrix} \right)^{-1} \ , \tag{18}$$

where $\boldsymbol{U}$ is a matrix whose rows are the unit vectors $\boldsymbol{u}_i = (\boldsymbol{x} - \boldsymbol{a}_i)/\|\boldsymbol{x} - \boldsymbol{a}_i\|, \ i = 0, \cdots, N$ pointing from the sensors to the source.

4.1. CRB on Propagation Speed Estimation

The $(D+1)$-th diagonal element of $\boldsymbol{F}^{-1}$ is a theoretical variance bound on the estimation of the propagation speed, e.g. for the two-dimensional case $D = 2$

$$\sigma_c^2 \geq [\boldsymbol{F}^{-1}]_{33} = (\sigma \, c)^2 \|\boldsymbol{P}_{\boldsymbol{U}}^{\perp} \boldsymbol{t}^\circ\|^{-2} \tag{19}$$

with the orthogonal projection matrix $\boldsymbol{P}_{\boldsymbol{U}}^{\perp}$.

	TOA-Based	**TDOA-Based** (spherical set)
Measurements	$t_i \quad i = 1, \cdots, M$	$\tau_{i,0} \quad i = 1, \cdots, M$
Basic Equation	$\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{x} - \frac{\lVert\boldsymbol{x}\rVert^2}{2} = b_i$	$\boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{x} + c\tau_{i,0}\lVert\boldsymbol{x}\rVert = b_i$
System Matrix	$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{a}_1^{\mathrm{T}} & -\frac{1}{2} \\ \vdots & \vdots \\ \boldsymbol{a}_M^{\mathrm{T}} & -\frac{1}{2} \end{bmatrix}$	$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{a}_1^{\mathrm{T}} & -c\tau_{1,0} \\ \vdots & \vdots \\ \boldsymbol{a}_M^{\mathrm{T}} & -c\tau_{M,0} \end{bmatrix}$
Unknowns	$\boldsymbol{y} = \begin{bmatrix}\boldsymbol{x} & \alpha\end{bmatrix}^{\mathrm{T}}$	$\boldsymbol{y} = \begin{bmatrix}\boldsymbol{x} & r\end{bmatrix}^{\mathrm{T}}$
Known Term	$b_i = \frac{1}{2}\left(\lVert\boldsymbol{a}_i\rVert^2 - (ct_i)^2\right)$	$b_i = \frac{1}{2}\left(\lVert\boldsymbol{a}_i\rVert^2 - (c\tau_{i,0})^2\right)$
Constraint	$\alpha = \lVert\boldsymbol{x}\rVert^2$	$r = \lVert\boldsymbol{x}\rVert$
Error Function	$\delta(c) = \lVert\hat{\boldsymbol{x}}(c)\rVert - \sqrt{\hat{\alpha}(c)}$	$\delta(c) = \lVert\hat{\boldsymbol{x}}(c)\rVert - \hat{r}(c)$

Table 1. Comparison between TOA- and TDOA-based estimation. The TOA relations are taken from (1)-(11), the corresponding TDOA relations can be found in [8, 3].

4.2. CRB with Multiple Observations

Here we show how to extend the CRB to take multiple TOA observations into account. The extension is quite straightforward, we consider the error model in (17) but with the following modification: the vector $\boldsymbol{t}$ is now obtained by stacking TOAs corresponding to N different sources $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N$,

$$\boldsymbol{t} = \left[\boldsymbol{t}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{t}_N^{\mathrm{T}}\right]^{\mathrm{T}} = \frac{1}{c}\boldsymbol{r}^{\circ}(\boldsymbol{x}_1, \ \cdots, \ \boldsymbol{x}_N) + \boldsymbol{\epsilon}\,, \tag{20}$$

where $\boldsymbol{\epsilon}$ is an $(M \cdot N)$-vector. As a consequence the range vector becomes

$$\boldsymbol{r}^{\circ}(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N) = \left[\boldsymbol{r}_1^{\circ\mathrm{T}} \ \boldsymbol{r}_2^{\circ\mathrm{T}} \ \cdots \ \boldsymbol{r}_N^{\circ\mathrm{T}}\right]^{\mathrm{T}}, \tag{21}$$

$$\text{with} \quad \boldsymbol{r}_n^{\circ} = \begin{bmatrix} \lVert\boldsymbol{x}_n - \boldsymbol{a}_1\rVert \\ \lVert\boldsymbol{x}_n - \boldsymbol{a}_2\rVert \\ \vdots \\ \lVert\boldsymbol{x}_n - \boldsymbol{a}_M\rVert \end{bmatrix} \quad \text{and} \quad n = 1, \cdots, N\,. \tag{22}$$

The above vector leads to a new matrix

$$\boldsymbol{U}_{\mathrm{multi}} = \left[\boldsymbol{U}_1^{\mathrm{T}} \quad \boldsymbol{U}_2^{\mathrm{T}} \quad \cdots \quad \boldsymbol{U}_N^{\mathrm{T}}\right]^{\mathrm{T}}\,, \tag{23}$$

where the rows of the submatrices $\boldsymbol{U}_n$ are unit vectors pointing from the array sensors to the source $\boldsymbol{x}_n$. The CRB for multiple TOA observations is given by using (23) and (20) in (18).

5. SIMULATION RESULTS

The theoretical bounds derived in the previous section will be used to evaluate the performance of the proposed method. The simulations consider a two-dimensional scenario and an emitting source positioned sequentially at 48 different bearing angles around the sensor array. The source range is fixed to 1.5 m from the array center as the considered methods are thought for the near-field case (spherical wave propagation), anyway the TOA-based estimate is independent of the source distance under the assumptions of Section 4. For each source's bearing angle bias and variance of the estimation are obtained over 1000 Monte Carlo trials. To demonstrate that the bias of the proposed method is substantially independent of the array geometry, a random sensor distribution has been used, i.e. the coordinates of $M = 8$ sensors that are not positioned at the origin were randomly generated from a uniform distribution over a square with side length of 70 cm. The standard deviation of the error corrupting the simulated TOAs is set to be $\sigma = 10\ \mu$s as in [3, 9], which corresponds to an error of about one sample at 96 kHz.

Figure 2 shows the results of the TOA-based speed estimation against the source's bearing angle together with the CRB from (19) and the CRB of the TDOA-based estimation [3]. First we emphasize that, given the SNR, the variance of TOA and TDOA measurements it is not likely to be the same as they rely on different estimation techniques. Nevertheless, we used the same value of standard deviation σ to make the two approaches comparable at least from a theoretical point of view. In practical cases a relation between TOA and TDOA variances may be derived statistically from measured data as shown in [5]. From the figure we see that the proposed method performs well: its bias is always less than $0,5$ m/s and its variance is close to the corresponding CRB. However, it turns out that the two CRBs are quite close to each other, this means that TOA and TDOA measurements carry virtually the same information about the propagation speed. In contrast it can be shown that the CRB of the position estimation decreases dramatically when using TOAs instead of TDOAs (these numerical results are not reported since they are beyond the scope of this paper). In both cases an accurate estimation of the speed of sound turns out to be an hard task; a small standard deviation $\sigma = 10\ \mu$s in the measurements yields a relatively high CRB on the speed estimation (around $4 - 6\ \mathrm{m}^2/\mathrm{s}^2$ in this case).

Under these conditions the use of the TOAs instead of TDOAs does not yield a significant improvement of the estimation accuracy. However the proposed speed estimate ben-

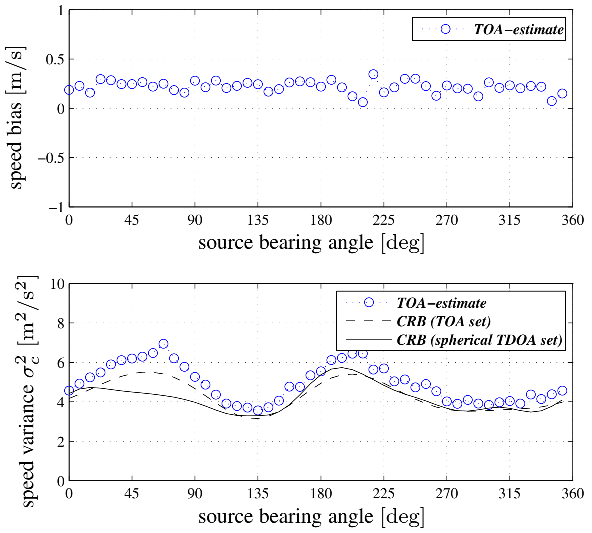

Fig. 2. Bias and variance of the TOA-based propagation speed estimation against the source's bearing angle (random sensor array and measurement standard deviation $\sigma = 10$ μs).

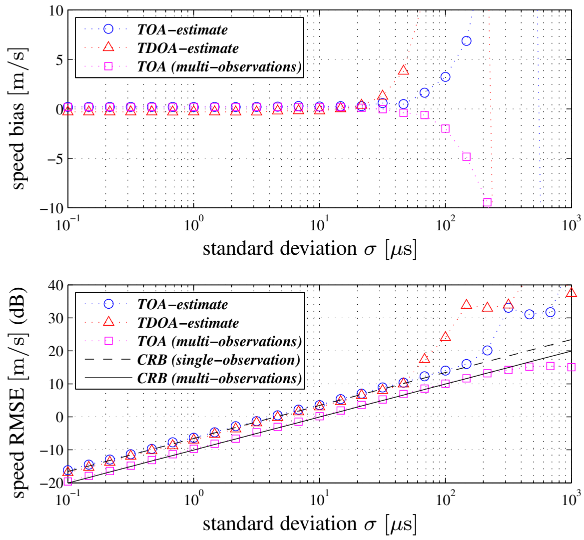

Fig. 3. Bias and root mean square error (RMSE) of the speed estimation for different values of the standard deviation σ.

efits from a TOA-approach in terms of robustness, as visible in Fig. 3 the TOA-based estimate tolerates a higher level of TOA estimation error. The figure shows bias and variance of the estimation dependent on the measurement variance and given a single source positioned at 0°; the estimate based on TOAs is optimal up to $\sigma = 100$ μs whereas the TDOA-based estimate [3, 8] is optimal up to $\sigma = 40$ μs. The figure also shows the estimation results and the CRB for multiple observations (in this case 5 observations of the same source); it is clear that a multiple observation approach lowers the estimation variance and so the estimation accuracy improves.

6. CONCLUSION

A method for estimating the propagation speed from TOAs observed at a sensor array is presented. The estimate of the propagation speed is obtained with the same approach that we used to address the TDOA-case. The statistical properties of such a speed estimate are evaluated by means of simulations and compared with the TDOA-case. The proposed TOA-based estimate provides substantially bias-free results independent of the source position and attains the CRB. Given the same measurement variance, the accuracy of the speed estimate does not improve significantly by using TOAs instead of TDOAs. However the TOA-based estimate gives better results in terms of robustness and its accuracy can be still improved by using a multiple observation approach. Thus it is recommended when source-receiver synchronization is available. Currently the authors are working on further comparisons by means of experimental measurements.

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