TENSOR-BASED PREPROCESSING OF COMBINED EEG/MEG DATA

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ABSTRACT

Due to their good temporal resolution, electroencephalography (EEG) and magnetoencephalography (MEG) are two often used techniques for brain source analysis. In order to improve the results of source localisation algorithms applied to EEG or MEG data, tensor-based preprocessing techniques can be used to separate the sources and reduce the noise. These methods are based on the Canonical Polyadic (CP) decomposition (also called Parafac) of space-time-frequency (STF) or space-time-wave-vector (STWV) data. In this paper, we analyse the combination of EEG and MEG data to enhance the performance of the tensor-based preprocessing. To this end, we consider the joint CP decomposition of two (or more) third order tensors with one or two identical loading matrices. We present the necessary modifications for several classical CP decomposition algorithms and examine the gain on performance in the EEG/MEG context by means of simulations.

Index Terms— Canonical polyadic decomposition, Parafac, EEG, MEG, STWV/STF analysis

1. INTRODUCTION

As non-invasive methods with a high temporal resolution, EEG and MEG play an important role in the analysis of brain signals. For example, they provide crucial insights on the location of epileptogenic zones in drug resistant epileptic patients, which can then be treated by surgery. In the past, a large variety of techniques has been developed to estimate the source positions based on EEG or MEG measurements [1]. In the context of usually low SNRs and several simultaneously active source regions, an important point as to the outcome of the source localisation process is the preprocessing. One preprocessing step consists in the separation of the sources, on which we concentrate in this paper. To this end, tensor-based techniques can be used, which separate the sources by applying the Canonical Polyadic (CP) decomposition to space-time-frequency (STF) (see [2, 3] and references therein) or space-time-wave-vector (STWV) [3] transformed EEG or MEG data. As has been shown in [3], these methods reduce the noise, extract the source time signals and improve the accuracy of source localisation estimates obtained with standard source localisation algorithms, which can then be applied separately to each source.

Since EEG and MEG measurements yield complementary information about the underlying sources and can be acquired simultaneously, several authors have examined the combination of EEG and MEG data in brain source localisation algorithms (see [4, 5] and references therein), reporting a gain on accuracy of the source position estimates compared to the results of each modality alone.

In this paper, we analyse the combination of EEG and MEG in tensor-based preprocessing, focusing on STF and STWV tensor analyses. This leads us to the problem of computing CP decompositions of third order tensors that have one or two loading matrices in common. In order to improve the estimates of the loading matrices of each of these tensors, we propose to apply a joint CP (JCP) decomposition that simultaneously computes the loading matrices that are identical for all tensors. This approach is comparable to the JCP decompositions proposed in the context of symmetric [6] or hermitian [7] tensors. We then present the modifications of several existing CP decomposition algorithms [8, 9, 10] that have to be carried out to this end. Finally, we examine the accuracy of the EEG and MEG lead field estimates that are obtained by applying the JCP decomposition to STF and STWV data, in comparison to the results achieved for a separate treatment of both modalities by means of simulations.

2. EEG/MEG DATA MODEL

Both EEG and MEG data are measured as a function of sensor position and time and can be stored into two real-valued data matrices, \( X_{\text{eeg}} \) and \( X_{\text{meg}} \) of sizes \( N_{r,\text{eeg}} \times N_{t,\text{eeg}} \) and \( N_{r,\text{meg}} \times N_{t,\text{meg}} \), respectively, where \( N_{r,\text{eeg}} \) and \( N_{r,\text{meg}} \) denote the number of EEG and MEG sensors and \( N_{t,\text{eeg}} \) and \( N_{t,\text{meg}} \) indicate the number of time samples recorded with the EEG and MEG systems.

Since the EEG and MEG measurements are generated by
the same sources and are generally sampled synchronously, the data can be stored into the larger EEG/MEG data matrix, which can, according to [11], be modelled as:

\[ X_{\text{mecg}} = \begin{bmatrix} X_{\text{eeg}} \\ X_{\text{meg}} \end{bmatrix} = \begin{bmatrix} G_{\text{eeg}} \\ G_{\text{meg}} \end{bmatrix} S + \begin{bmatrix} N_{\text{eeg}} \\ N_{\text{meg}} \end{bmatrix}. \]  

(1)

Here, \( G_{\text{eeg}} \in \mathbb{R}^{N_{t,\text{eeg}} \times R} \) and \( G_{\text{meg}} \in \mathbb{R}^{N_{t,\text{meg}} \times R} \) are the EEG and MEG lead field matrices, which are specific to the head model and the source positions, \( S \in \mathbb{R}^{R \times N_t} \) with \( N_t = N_{t,\text{eeg}} = N_{t,\text{meg}} \) denotes the signal matrix that contains the temporal activities of \( R \) equivalent dipole sources [1], and \( N_{\text{eeg}} \) and \( N_{\text{meg}} \) are the EEG and MEG noise matrices.

3. TENSOR-BASED PREPROCESSING

The objective of the preprocessing techniques considered in this paper consists in recovering the EEG and MEG lead field and signal matrices from the measurement data, yielding separated sources with an estimate of the temporal activity and the lead field vector of each source. This can be achieved by means of the Canonical Polyadic (CP) decomposition, which is introduced in the next section.

3.1. CP decomposition

Each element of a third order tensor \( X \) of size \( I \times J \times K \) can be written in the form:

\[ X_{i,j,k} = \sum_{p=1}^{P} a_{i,p} b_{j,p} c_{k,p}. \]  

(2)

where \( P \) is the rank of the tensor and \( a_{i,p}, b_{j,p}, \) and \( c_{k,p} \) are elements of three loading matrices \( A \in \mathbb{C}^{I \times P}, B \in \mathbb{C}^{J \times P}, \) and \( C \in \mathbb{C}^{K \times P} \), respectively. This trilinear representation is referred to as the CP model. There is almost surely a finite number of decompositions of tensor \( X \) into the three loading matrices \( A, B \) and \( C \), up to scale and permutation indeterminacies, if \( P < \frac{IJK}{I+J+K-2} \) [12]; tighter bounds also give sufficient conditions for uniqueness. This is an important advantage over matrix decompositions.

3.2. STF analysis

In case of the STF analysis [2], a 3-dimensional tensor \( W \) is constructed from the 2-dimensional EEG or MEG data matrix \( X \) by computing a Wavelet transform over time. Under the assumption of oscillatory signals, the tensor \( W \) is approximately trilinear and can be decomposed using the CP model into space, time and frequency characteristics, \( a_r(r_i), b_r(t_j), \) and \( c_r(f_k), r = 1, \ldots, R, \) respectively:

\[ W_{i,j,k} \approx \sum_{r=1}^{R} a_r(r_i) b_r(t_j) c_r(f_k). \]  

(3)

where \( r_i \) is the location of the \( i \)-th sensor, \( t_j \) denotes the \( j \)-th time sample, and \( f_k \) is the \( k \)-th frequency sample. The loading matrix \( A \) containing the spatial characteristics provides a good estimate of the lead field matrix, \( \hat{G} = A. \) To accurately estimate the source activities, the signal matrix is computed in a second step from the pseudo inverse \( \hat{G}^+ \) of the estimated lead field matrix and the data matrix:

\[ \hat{S} = \hat{G}^+ X. \]  

(4)

3.3. STWV analysis

The idea of the STWV analysis [3] consists in building a 3-dimensional tensor \( F \) from the EEG or MEG measurement data by computing a local Fourier transform over space. For superficial sources, this tensor can then be approximated by the CP model and decomposed into space, time, and wave vector characteristics, \( a_r(r_i), b_r(t_j), \) and \( c_r(k_k), r = 1, \ldots, R, \) respectively:

\[ F_{i,j,k} \approx \sum_{r=1}^{R} a_r(r_i) b_r(t_j) c_r(k_k). \]  

(5)

Here, \( k_k \) denotes the \( k \)-th wave vector sample. This permits us to obtain an estimate of the signal matrix \( \hat{S} \), which is approximated by the loading matrix \( B \) containing the temporal characteristics of the tensor \( F \). An estimate of the lead field matrix can then obtained from:

\[ \hat{G} = X \hat{S}^+. \]  

(6)

The pertinency of this approach has been demonstrated by means of simulations in [3] whereas its theoretical validation will be the subject of future work.

3.4. Combination of EEG and MEG

Since the signal matrices of EEG and MEG are identical, in case of the STF analysis, the Wavelet transform can be computed simultaneously for both EEG and MEG by applying it to the extended data matrix \( X_{\text{mecg}}, \) yielding the tensor \( W = [W_{\text{eeg}} \| W_{\text{meg}}], \) where \( \| \) denotes a concatenation along the first dimension. The tensors \( W_{\text{eeg}} \) and \( W_{\text{meg}} \) can be decomposed using the CP model and exhibit two different loading matrices \( A_{\text{eeg}} \) and \( A_{\text{meg}} \) for the spatial characteristics of EEG and MEG. However, the two loading matrices \( B \) and \( C \) that contain the time and frequency characteristics are the same for EEG and MEG due to the identical signal matrices. Therefore, in order to improve the results of the CP decomposition, we propose to exploit this property by jointly decomposing the tensors using the JCP decomposition for two common loading matrices as described in Section 4.1. To this end, the tensors should be normalised to \( W'_{\text{eeg}} = w \sqrt{\sum_{r=1}^{R} \frac{w_{r,\text{eeg}}}{||W_{r,\text{eeg}}||_F}}, W'_{\text{meg}} = \sqrt{\sum_{r=1}^{R} \frac{w_{r,\text{meg}}}{||W_{r,\text{meg}}||_F}}, \) where \( || \cdot ||_F \) denotes the Frobenius norm and \( w \) is a weighting factor.
factor to account for different separability and SNR of EEG and MEG.

On the other hand, the STWV tensors need to be constructed separately for both modalities because EEG and MEG yield physically different measurements and their lead field matrices differ. In the next step of the STWV analysis, the resulting tensors $F_{\text{eeeg}}$ and $F_{\text{meg}}$ can be decomposed individually using the CP model. However, in this case, we do not exploit the fact that both modalities are generated by the same sources. In fact, due to the identical EEG and MEG signal matrices, the loading matrices $B_{\text{eeeg}}$ and $B_{\text{meg}}$ containing the temporal characteristics of the tensors $F_{\text{eeeg}}$ and $F_{\text{meg}}$ should be equal, whereas the loading matrices associated to the space and wave vector characteristics generally differ. To achieve this, we propose to apply a JCP decomposition to the normalised tensors $F_{\text{eeeg}} = w\sqrt{N_{r,\text{eeeg}}|F_{\text{eeeg}}|^T}$ and $F_{\text{meg}} = \sqrt{N_{r,\text{meg}}|F_{\text{meg}}|^T}$ that enforces one loading matrix (in this case the matrix $B$) to be the same for both tensor decompositions while allowing different loading matrices $A$ and $C$ for the two tensors. This technique is described in detail in Section 4.2.

4. JOINT CP DECOMPOSITION

In this section, we describe some algorithms for the Joint CP (JCP) decomposition of third order tensors that have one or two loading matrices in common.

4.1. Two common loading matrices

Consider $M$ tensors $W_m \in \mathbb{C}^{I_m \times J \times K}$, $m = 1, \ldots, M$, ($M = 2$ for the STF analysis of EEG/MEG data), with common loading matrices $B$ and $C$ in the second and third mode and $M$ different loading matrices $A_m$ in the first mode. These tensors can be stacked into a larger tensor $W = W_1 \cup \ldots \cup W_M$ of size $(I_1 + \ldots + I_M) \times J \times K$. The JCP decomposition of the $M$ tensors can then be achieved by solely decomposing the tensor $W$ using any existing algorithm to fit the CP model, e.g., [8, 9, 10]. This yields common loading matrices $B$ and $C$ for all tensors and the loading matrix $A = [A_1^T, \ldots, A_M^T]^T$ that contains all individual mode-1 loading matrices.

4.2. One common loading matrix

In the following, we consider $M$ tensors $F_m \in \mathbb{C}^{I_m \times J \times K_m}$, $m = 1, \ldots, M$, ($M = 2$ for the STWV analysis of EEG/MEG data). We assume that these tensors have one common loading matrix $B$ in the second mode and different loading matrices $A_m$ and $C_m$ in the first and third mode, respectively. The objective consists in decomposing the tensors simultaneously such that the loading matrix $B$ is computed jointly for all tensors while allowing different loading matrices for each tensor in the first and third mode. Subsequently, we present modified versions of several CP decomposition algorithms that meet these specifications.

4.2.1. ALS

Starting from an initial setting, the classical ALS algorithm [8] iteratively updates the three loading matrices $A_m$, $B_m$, and $C_m$ of the tensor $F_m$, $m = 1, \ldots, M$, until convergence or a certain number of iterations is reached:

$$A_m = [F_m]_1 (C_m \odot B_m)^T$$  \hspace{1cm} (7)

$$B_m = [F_m]_2 (C_m \odot A_m)^T$$  \hspace{1cm} (8)

$$C_m = [F_m]_3 (B_m \odot A_m)^T.$$  \hspace{1cm} (9)

Here, $[F_m]_n$ denotes the mode-$n$ unfolding matrix, as defined in [8], for instance, and $\odot$ denotes the Khatri-Rao product.

A joint update of the loading matrix $B = B_m$, $m = 1, \ldots, M$, of the $M$ tensors $F_m$ can hence be incorporated by replacing equation (8) by

$$B = [F]_2 D^+$$  \hspace{1cm} (10)

where $D = [C_1 \odot A_1, \ldots, C_M \odot A_M]^T$ and $[F]_2 = [F_{[1]}_2, \ldots, F_{[M]}_2]$. The other loading matrices are updated separately according to equations (7) and (9).

4.2.2. SALT and CFP

Contrary to the ALS technique, the CFP [10] and SALT [9] algorithms belong to the class of semi-algebraic methods. Indeed, they algebraically formulate the CP problem as the combination of classical matrix decomposition problems for which efficient numerical solutions exist. For instance, CFP resorts to several Joint EigenValue Decompositions (JEVDs) [9] while SALT makes use of only one JEVD and several singular value decompositions. Although the sets of matrices jointly diagonalized by CFP and SALT are totally different for tensors of order greater than or equal to four, for third order tensors the unique JEVD computed in SALT corresponds to one of the JEVDs computed by CFP. Nevertheless, a more straightforward way of determining the matrices to be jointly diagonalized is used by SALT.

More particularly, the loading matrix of the $n$-th mode, e.g., $B_m$ for the second mode, can be obtained by multiplying the left signal subspace matrix of the $n$-th mode, $U_{[n]}^m$, obtained from a singular value decomposition of the mode-$n$ unfolding, by a projection matrix $T_{n,m}$, e.g., $B_m = U_{[n]}^m T_{2,m}$. The estimation of the mode-$n$ loading matrix is thus replaced by the determination of the projection matrix $T_{n,m}$. Only the tallest loading matrix is determined in this way by SALT whereas the other loading matrices...
are obtained from rank-1 decompositions of the matrices $T_{n,m}^{-1} (U_{n,m}^{[s]} )^H [F_m]_{[n]}$. On the contrary, for CFP, all the unfolding matrices are required to determine the three projection matrices and therefore the three loading matrices. The projection matrix $T_{n,m}$ can be determined by computing the JEVD of several matrices:

$$
\Theta_{m}(k_m,l_m) = T_{n,m} \Lambda_{m}(k_m,l_m) T_{n,m}^{-1}
$$

with $\Lambda_{m}(k_m,l_m) = \text{diag}(\phi_{m,k_m}) \text{diag}(\phi_{m,l_m})$, $1 \leq k_m \neq l_m \leq K_m$, where $\phi_{m,k}$ corresponds to the $k$-th row of a loading matrix that does not belong to the mode $n$, e.g., for $n = 2$, $C_{m}$. As previously mentioned, SALT uses a cheaper way of determining the matrices $\Theta_{m}(k_m,l_m)$ than CFP.

In SALT, the matrices $\Theta_{m}(k_m,l_m)$ are obtained from the blocks $\Gamma_{m}^{[k_m,l_m]} \in \mathbb{C}^{R \times L_m}$ of the matrices $\Gamma_{m} = (U_{n,m}^{[s]} )^H [F_m]_{[n]} = \left[ \Gamma_{m}^{(1)} \cdots \Gamma_{m}^{(K_m)} \right]$ as $\Theta_{m}(k_m,l_m) = \Gamma_{m}^{(k_m)} (\Gamma_{m}^{(l_m)})^H + [9]$. To reduce the computational complexity [13], only one matrix $\Gamma_{m}^{(l_m)}$ can be used, e.g. the best conditioned. For the determination of the matrices $\Theta_{m}(k_m,l_m)$ using the CFP algorithm, the reader is referred to [10].

Now, based on SALT’s strategy, let’s present our way of computing the JCP of tensors sharing only one loading matrix. To enforce an identical loading matrix $\mathbf{B}$ for all tensors, we can only consider joint diagonalisation problems for the mode-2 projection matrix $T_2$, which has to be equal for all tensors. Consequently, the mode-2 subspace $U_2$, which is identical for all tensors, needs to be computed jointly to prevent different representations:

$$
[F_1]_{[2]} \cdots [F_M]_{[2]} = U_2 \Sigma_2 V_2^H.
$$

The matrix $U_2^{[s]}$ then corresponds to the columns of $U_2$ that are associated with the $R$ largest singular values. We then extend the joint diagonalization problem for $T_2$ by simultaneously diagonalizing all matrices $\Theta_{m}(k_m,l_m)$, $m = 1, \ldots, M$, in the following way to combine all tensors:

$$
\Theta_{m}(k_m,l_m) = T_2 \cdot \Lambda_{m}(k_m,l_m) \cdot T_2^{-1}.
$$

Once an estimate of the matrix $T_2$ has been obtained, the matrix $\mathbf{B}$ can be computed. The other loading matrices can be obtained either from rank-1 decompositions of the matrices $T_1^{-1} (U_2^{[s]} )^H [F_m]_{[2]}$ according to the SALT algorithm [9] or by recovering, for each tensor $F_m$, one loading matrix from the entries of the diagonal matrices $\Lambda_{m}(k_m,l_m)$ and computing the third loading matrix by ALS from the tensor $F_m$ and the two already known loading matrices. In fact, both latter strategies could be combined in order to jointly use the different estimates of the same loading matrix [13]. Eventually note that our JCP semi-algebraic algorithm requires that two dimensions of each tensor, namely the second dimension corresponding to the number of rows of $T_2$ and one of both other dimensions, are larger than the tensor rank.

5. SIMULATION RESULTS

In order to analyse the gain in accuracy of the lead field estimates that can be achieved by combining EEG and MEG data in the tensor-based preprocessing using the JCP decomposition, we performed some computer simulations. To this end, EEG and MEG data were generated for two dipole sources located at [6.33, -1.35, 4.70] cm and [6.33, 1.35, 4.70] cm with dipole moment vectors [0.98, -0.21, -0.07] cm and [0.98, 0.21, -0.07] cm and 64 EEG electrodes as well as 148 MEG sensors (magnetometers) in a 3-shell spherical head model. The radii of the three shells representing the brain, the skull, and the scalp were 8 cm, 8.5 cm, and 9.2 cm with conductivities $3.3 \cdot 10^{-3}$ S/cm, $8.25 \cdot 10^{-5}$ S/cm, and $3.3 \cdot 10^{-3}$ S/cm, respectively. The MEG sensors were positioned on a sphere with radius 10.5 cm. Epileptogenic signals were obtained using the Jansen model [14] with parameters $\tau = [7.6, 1.35, 10 \cdot 100, 50]$, $A = [7.6]$, $B = [46.6, 40]$, and $C_c = 135$ for two sources and $N_f = 100$ time samples that were acquired at a sample rate of 125 Hz. White Gaussian noise was added to the EEG and MEG data according to a given SNR, which was assumed to be equal for EEG and MEG.

The STF tensors were built by computing a Wavelet transform of the EEG and MEG data using a real-valued Morlet wavelet with a centre frequency of 35 Hz and $N_f = 100$ frequency samples. The STWV tensors were constructed separately for EEG and MEG by calculating a discrete local Fourier transform over space of data selected by a spherical Blackman window function. For both modalities, we considered 63 wave vector samples. Each of the resulting tensors was then decomposed individually using a slightly modified version of the SALT algorithm, yielding the lead field matrices of the separately treated data. Moreover, we computed the JCP decompositions of the EEG and MEG tensors using the same modified SALT algorithm. For the present source configuration, we used a weighting factor of $w = 4$ for the EEG tensor, which was chosen because of the high associated core consistency (cf. [8]) of the decomposed tensors. To ensure a real-valued loading matrix for the temporal characteristics of the STWV tensors, one iteration of ALS was applied after the SALT decomposition. For all cases, we assumed that the number of sources and thereby the number of CP components is known.

In Figure 1, we plotted the average correlation coefficient of the original and estimated EEG (top) and MEG (bottom) lead field vectors depending on the SNR. It can be seen that for the STWV analysis, the JCP decomposition of the EEG and MEG tensors generally results in better estimates for the lead field matrices of both modalities, whereas in case of the STF technique, we observe only a small improvement of the MEG lead field estimate. This can be explained by the fact that for STWV preprocessing the JCP decomposition improves the temporal characteristics and therefore the sig-
and MEG improves especially the MEG lead field because the electric potential is more focused than the magnetic field, which facilitates the source separation based on EEG measurements. In case of the STWV analysis, the combination of EEG and MEG improves especially the MEG lead field because the electric potential is more focused than the magnetic field, which facilitates the source separation based on EEG measurements.

6. CONCLUSIONS

We have shown that, due to the approximately identical signal matrices for EEG and MEG, the two modalities can be combined in tensor-based STF and STWV preprocessing. This can be accomplished by simultaneously decomposing EEG and MEG data tensors using the JCP decomposition introduced in Section 4, and described for ALS, SALT and CFP algorithms. As we have demonstrated by simulations, the application of the JCP decomposition to STWV EEG/MEG data leads to clearly improved lead field estimates, whereas in case of the STF analysis, only a slight amendment of the MEG lead field can be achieved. Based on the promising results for the STWV method, the next step of our studies will consist in verifying the results for a realistic head model and actual EEG and MEG measurement data.

7. REFERENCES