MULTIPLE DESCRIPTION ANALOG JOINT SOURCE-CHANNEL CODING FOR PARALLEL CHANNELS WITH SIDE INFORMATION

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ABSTRACT

With the purpose of reducing the coding complexity and delay of the separation-based schemes, an analog joint source-channel coding scheme is proposed for transmissions through parallel AWGN channels with side information at the receiver. This scheme divides the bidimensional source space into a set of hexagons and transmits the relative position of the source vectors inside the corresponding hexagon by using two complimentary analog mappings. The results are satisfactory, specially taking into consideration the low complexity and delay of the proposed scheme.

Index Terms— Joint source-channel coding, analog mappings, side information, multiple description coding.

1. INTRODUCTION

Many sources (audio, video, etc.) can be modeled as sequences $\mathbf{S} = [S_1, S_2, ...S_n]$ of continuous alphabet symbols, i.e. $S_i \in \mathbb{R}$. In the digital era, the tendency has been to discretize this analog information by using source coding techniques, which basically consist in mapping the infinite set of source sequences into a finite set of descriptions. Obviously, once the discretization has taken place, the original source symbols $\mathbf{S}$ cannot be perfectly recovered, but estimated with an error that is quantified through a distortion measure $d : \mathcal{S}^n \times \widehat{\mathcal{S}}^n \to \mathbb{R}^+$, where $\mathcal{S}$ represents the alphabet of the source symbols and $\widehat{\mathcal{S}}$ the reconstruction alphabet. One of the advantages of applying source coding is that the source symbols can be manipulated digitally; for example, the descriptions can be transmitted free of errors to the receiver by using error-correcting codes.

During the late 70's, the research community went one step further and proposed to generate, not just one description for $\mathbf{S}$, but multiple descriptions, with the purpose of gaining robustness against the loss of descriptions. This solution, known as Multiple Description (MD) source coding

(see e.g. [1]), seeks to make possible the reconstruction the source symbols even when a small subset of the descriptions is available at decoder, although at a reduced quality. Let us focus now on the case of two descriptions. In this case, it is usual to define three different decoders or reconstruction functions, two of them to be employed when one of the descriptions is lacking, and the third one when both descriptions are available. When the source symbols are ergodic and have to be transmitted through two ergodic parallel channels, it was proven in [2] that the separation principle holds. This means that the same distortions can be achieved either by joint source-channel coding (JSCC) or by concatenating MD source coding with capacity-achieving channel codes, as long as the complexity is not a constraint.

Most of the works in the literature about MD coding have been focused on source coding, based in part on the the premise that the separation principle holds. However, separating the source and channel coding has its drawbacks. For instance, in order to achieve efficient transmissions, long channel codewords are required, with the consequent coding/decoding complexity and delay. Moreover, with the separation strategy, the transmission suffers from the cliff effect, i.e., small drops in the channels SNRs translate into big distortion increments. To overcome these problems, a joint source-channel coding (JSCC) MD scheme was proposed in [3, 4]. This scheme uses analog mappings [5, 6] to generate the channel symbols directly from the source symbols and it is characterized by its very low complexity and delay and by the progressive and smooth distortion degradation with sudden SNR drops. In addition, as shown in [3, 4], these features are not achieved at the expense of higher distortion than with the separation schemes.

In this work, we go one step further and assume that in the two parallel channels scenario, some side information is also available at the receiver, where by side information it is meant information correlated with the source symbols. The case of multiple description coding with side information at the decoders has been discussed in the literature from the source coding point of view. For instance, in [7], the rate-

distortion region for the case of two descriptions is given for Gaussian sources, a quadratic error distortion measure and Gaussian side information. Regarding practical schemes, in [8] a scheme that combines MD scalar quantization techniques [9] with Slepian-Wolf coding principles is proposed. After MD source coding, the resulting descriptions can be transmitted by using error-correcting codes, but again, this implies a complexity and delay that can be considerably reduced with analog JSCC techniques. With the purpose of reducing the complexity and delay inherent to the separation strategies, this paper proposes an analog MD JSCC scheme for the scenario of two parallel AWGN channels with side information at the receiver. This novel JSCC scheme is, in part, based on the MD JSCC scheme proposed in [3, 4] for parallel channels with no side information at the receiver.

The rest of the paper is organized as follows. In Section II, the communication problem is stated and the analog mappings proposed in [5] and [6] are described. The proposed scheme description for the problem is given in Section III. In Section IV, the performance of the scheme is evaluated and compared to the theoretically achievable bounds. Finally, conclusions are given in Section V.
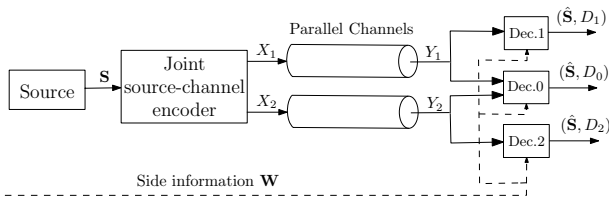
2. STATEMENT OF THE PROBLEM

Fig. 1. Communication Scenario

Let us consider the communication problem of Fig.1, where a sequence of n independent and identically distributed (i.i.d.) Gaussian symbols of zero mean and variance σ_S^2, i.e. $S_i \sim \mathcal{N}(0, \sigma_S^2)$, $i = 1, \ldots, n$, has to be transmitted through two parallel AWGN channels with side information $\mathbf{W}$ available at the decoder. For that purpose, the source sequence is encoded by two channel vectors $\mathbf{X}_i = [X_{i,1}, \ldots, X_{i,\beta_i n}]$, $i = 1, 2$, generated by two functions:

$$g_i(\cdot) : \mathcal{S}^n \longrightarrow \mathcal{X}_i^{\beta_i n}, \quad i = 1, 2, \quad (1)$$

where $\{\mathcal{X}_i\}_{i=1}^2$ denote the alphabets of the channel symbols and $\{\beta_i\}_{i=1}^2$, which we call processing gains, account for the number of channel symbols (channel i) used per source symbol. In this work, we focus on the case $\beta_1 = \beta_2 = 1/2$.

Let us consider that the side information vector $\mathbf{W} = [W_1, W_2, \ldots, W_n]$ is Gaussian. In order to model its correlation with the source vector $\mathbf{S}$, the vector $\mathbf{W}$ is defined as $\mathbf{W} = \mathbf{S} + \mathbf{U}$, where $\mathbf{U} = [U_1, \ldots, U_n]$ is a sequence of i.i.d Gaussian random variables, $U_i \sim \mathcal{N}(0, \sigma_u^2)$, independent to $\mathbf{S}$. This side information will be available at the receiver, but not at the transmitter.

The vectors received at the end of the AWGN channels are $\mathbf{Y}_i = \mathbf{X}_i + \mathbf{N}_i$, $i = 1, 2$, where the noise vectors $\mathbf{N}_i = [N_{i,1}, \ldots, N_{i,\beta_i n}]$ contain i.i.d. Gaussian variables, $N_{i,k} \sim \mathcal{N}(0, \sigma_n^2)$, $k = 1, \ldots, \beta_i n$. Like in other MD coding analysis in the literature, the possibility of one of the descriptions not reaching the other end of the channel is taken into account by defining three decoders at the receiver, as represented in Fig. 1. The decoder 0 (named *central decoder*) uses both $\mathbf{Y}_1$ and $\mathbf{Y}_2$ to estimate $\mathbf{S}$, whereas the two decoders 1 and 2 (called *side decoders*) use only $\mathbf{Y}_1$ or $\mathbf{Y}_2$, respectively. The distortions incurred at the three decoders are denoted by D_0 (*central distortion*) and D_1 and D_2 (*side distortions*). As most of the literature, we use the average squared error as the distortion measure:

$$D_m = \mathrm{E}\left[d(\mathbf{S}, \hat{\mathbf{S}}^{(m)})\right] = \frac{1}{n} \sum_{k=1}^n E\left[(S_k - \hat{S}_k^{(m)})^2\right], \quad (2)$$

$m = 0, 1, 2$, where $\mathrm{E}[\cdot]$ represents the expected value, and the estimation of S_i at decoder m is denoted by $\hat{S}_i^{(m)}$.

As mentioned in Section 1, the rate-distortion region for the MD source coding problem with Gaussian side information is derived in [7]. To the best of our knowledge, it has not been proved that the separation principle holds in the scenario depicted in Fig.1. However, the fact that the separation principle holds in the case of two parallel channels and three decoders and in the case of point-to-point communications with side information at the receiver [2], suggests that in the scenario of Fig.1, the separation principle will also hold. Under this assumption, it is straightforward to derive the minimum achievable distortion by combining the expression of the rate-distortion region with the capacity expression. If the SNR of channel i is denoted by ρ_i, $i = 1, 2$, the minimum achievable side distortions D_1 and D_2 are given by:

$$D_i(\rho_i, \lambda_i) = \sigma_F^2 (1 + \rho_i)^{-\beta_i(1-\lambda_i)}, \quad 0 \le \lambda_i \le 1, \quad (3)$$

where $\sigma_F^2 = \frac{\sigma_S^2 \sigma_u^2}{\sigma_S^2 + \sigma_u^2}$ and $i = 1, 2$. The minimum achievable central distortion D_0 is, in turn, given by:

$$D_0(\rho_i, \lambda_i) = \sigma_F^2 \cdot (1 + \rho_1)^{-\beta_1} \cdot (1 + \rho_2)^{-\beta_2} \cdot$$
$$\cdot \gamma_D (D_1(\rho_1, \lambda_1), D_2(\rho_2, \lambda_2)), \quad (4)$$

where $\gamma_D = 1$ if $D_1 + D_2 > \sigma_F^2 + \sigma_F^2 \eta$ and $\eta = (1 + \rho_1)^{-\beta_1} \cdot (1 + \rho_2)^{-\beta_2}$, whereas

$$\gamma_D = \frac{1}{1 - \left(\sqrt{\left(1 - \frac{D_1}{\sigma_F^2}\right)\left(1 - \frac{D_2}{\sigma_F^2}\right)} - \sqrt{\frac{D_1 D_2}{\sigma_F^4} - \eta}\right)^2}$$

otherwise. It can be observed that D_0 decreases as λ_1 and λ_2 grow (as long as the condition $D_1 + D_2 > \sigma_F^2 + \sigma_F^2 \eta$ is not satisfied), as opposed to D_1 and D_2, which grow with λ_1 and λ_2. This means that, for fixed $\{\rho_i\}_{i=1}^2$ and $\{\beta_i\}_{i=1}^2$, there is not a unique triplet (D_0, D_1, D_2) of minimum distortions,

but a set of triplets of pareto-minimum distortions. This set is usually called Optimum Performance Theoretically Attainable (OPTA) set.

2.1. Analog Mappings for bandwidth-compression

Analog bandwidth-reduction mappings arise as a particular solution for bandwidth-reduction JSCC schemes, where by bandwidth-reduction it is meant that the ratio between the channel and source bandwidths is lower than one, assuming idealized sampling of the source and ideal Nyquist channels (see e.g. [5]). For the particular case of a compression ratio 2:1 ($\beta = \frac{1}{2}$), these analog mappings consist of projecting a two-dimensional vector formed by two source symbols, $\mathbf{S} = (S_k, S_{k+1})^\top$, onto a subset consisting of a parametric curve in the two-dimensional space and then, translating this projection into a real value to be used as the channel symbol X.
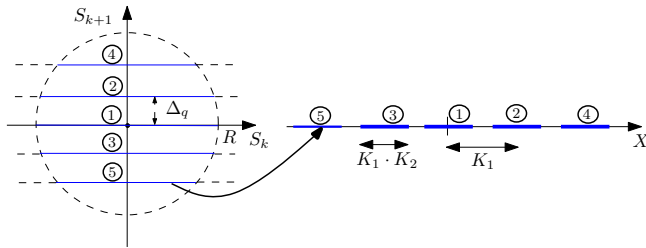
Fig. 2. The analog mapping proposed in [6].

In point-to-point communications, the best-known analog bandwidth-reduction mapping is the curve based on the Archimedes' spirals [5]. However, as shown in [3, 4], in the context of MD coding, the mapping proposed in [6] and named as *Lines mapping*, outperforms the spirals. For this reason, in this work we only consider this second mapping. Basically, as shown in Fig.2, the curve used in this mapping is formed by horizontal segments confined inside a circumference of radius R. First, the vector $\mathbf{S} = (S_k, S_{k+1})^T$ is projected into the closest horizontal line. Then, this point is mapped onto a real value X through a function that maps horizontal segments into equal-width intervals of the real axis. A deeper insight on this mapping can be found in [6].

3. PROPOSED COMMUNICATIONS SCHEME

In this section, we introduce the proposed JSCC scheme for the communication scenario depicted in Fig.1. The i.i.d. source symbols are first grouped in pairs. For the sake of simplicity, let us assume $k = 1$ and denote the pair of symbols $\mathbf{S} = (S_1, S_2)$. Then, this pair is encoded by the channel symbols $X_{1,1}$ and $X_{2,1}$ (one per channel) by using two analog mappings g_1 and g_2 (explained below).

Both g_1 and g_2 are based on the analog mapping described in Section 2.1. However, they are not directly applied to the pair (S_1, S_2). First, we partition $\mathbb{R}^2$ into a lattice of regular hexagons of side length δ, as shown in Fig.3. We denote the
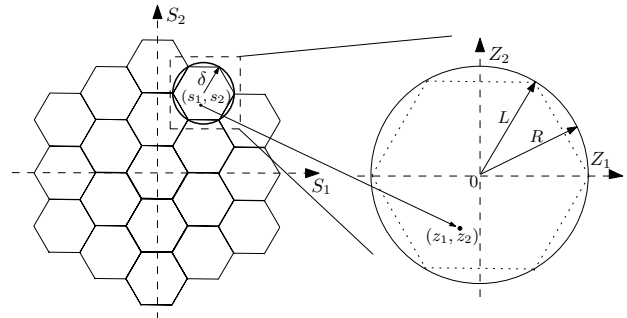
Fig. 3. The nested Hexagons $\{\mathcal{H}_i\}$

lattice by Λ and by $\{\mathcal{H}_i\}$ the set of hexagons forming the lattice. Then, a pair of new variables (Z_1, Z_2) is defined:

$$(Z_1, Z_2) = ((S_1, S_2) \mod \Lambda) \cdot \frac{L}{\delta}, \qquad (5)$$

where $(\cdot) \mod \Lambda$ is the modulo-lattice operation. The multiplication by $\frac{L}{\delta}$ aims to normalize (Z_1, Z_2) such that $\|(Z_1, Z_2)\|_2 \leq L$, where L is an arbitrary constant.

After this transformation, we apply two analog mappings, let us say $\breve{g}_1$ and $\breve{g}_2$, to the vector $\mathbf{z} = (Z_1, Z_2)^\top$. In order to minimize the central distortion, it is necessary to make the channel symbols X_1 and X_2 as independent as possible. With this purpose, we follow the same strategy as in [3, 4] and $\breve{g}_1$ and $\breve{g}_2$ are built from the analog mapping g described in Section 2.1 as follows:

$$\breve{g}_1(\mathbf{z}) = g(\mathbf{z}) \text{ and } \breve{g}_2(\mathbf{z}) = g(\mathbf{H} \cdot \mathbf{z}), \ \mathbf{H} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

where $\mathbf{H}$ is an orthogonal rotation matrix introduced to make the two mappings as independent as possible. The rotation angle θ does not influence on the side distortions (S_1 and S_2 are i.i.d.), whereas the maximum independence between X_1 and X_2, and consequently the minimum central distortion, is achieved with $\theta = \pi/2$. Notice that the distribution of (Z_1, Z_2) is not Gaussian. Due to this circumstance, in order not to leave areas of (Z_1, Z_2) points with relevant probability outside the circumference of radius R used in the mapping g, the radius R is chosen as $R = L$.

The rationale behind this coding procedure is that, at the receiver, channel symbols Y_1 and Y_2 will give information about the relative position of (S_1, S_2) inside the hexagon whereas the side information will provide information about which hexagon (S_1, S_2) belongs to. Thus, the availability of the side information at the receiver allows us to focus on a smaller area of $\mathbb{R}^2$ at the coding step, and consequently, the channel symbols X_1 and X_2 define the source symbols (S_1, S_2) more accurately. The optimum δ varies almost linearly with σ_u. As the goal is to minimize the distortion given by the quadratic error, the optimal estimator of $\mathbf{S} = (S_1, S_2)$ is the minimum mean square error (MMSE) estimator, that is $\widehat{\mathbf{S}}^{(\text{dec } i)} = \mathrm{E}[\mathbf{S}|Y_i, \mathbf{W}]$, $i = 1, 2$ for the side decoders, and $\widehat{\mathbf{S}}^{(\text{dec } 0)} = \mathrm{E}[\mathbf{S}|Y_1, Y_2, \mathbf{W}]$ for the central decoder.

4. PERFORMANCE ANALYSIS

In this section, we evaluate the performance of the proposed scheme and compare it with the theoretical bounds defined in Section 2. We plot the signal-to-distortion ratios (SDR) defined as $\text{SDR} = \frac{\sigma_S^2}{d}$, where the distortions rendered by the proposed scheme are denoted by lower case, (d_0, d_1, d_2), and the OPTA distortions by upper case. We also assume equal SNRs in both channels, i.e. $\rho_1 = \rho_2 = \rho$, which implies $d_1 = d_2$ (both g_1 and g_2 based on g) and $D_1 = D_2$.
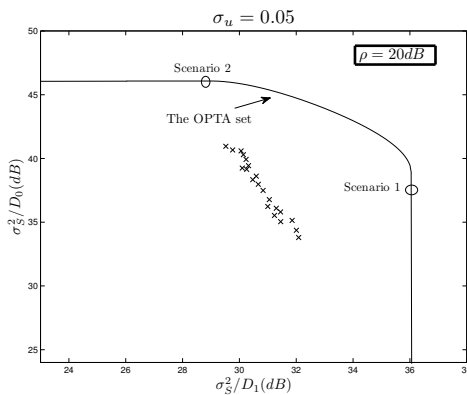

Fig. 4. The central distortion d_0 (its SDR) versus the side distortions $d_1 = d_2$ (their SDRs) achieved by the proposed scheme represented by ×'s. The OPTA set is represented by the continuous line.

First, in Fig.4, the central distortion versus the side distortion is plotted. The continuous line corresponds to the OPTA set (D_0 versus D_1), whereas the ×-points plot some pairs of distortions (d_1, d_1) achieved with the proposed scheme by varying the design parameters. The OPTA curve corroborates what we mentioned previously in the theoretical analysis: minimizing the central and side distortions are opposite requirements. Regarding the pairs achieved with the proposed scheme, it can be observed that a tradeoff between d_0 and d_1 exists, although not so marked as in the OPTA set.

Next, we analyze how the channel SNR and the correlation between the source and the side information influence on the performance. However, there are as many different goals to be pursued as there are OPTA points, i.e. infinite. In the following, we only evaluate the proposed scheme in relation to the goals corresponding to the two extreme points of the OPTA set. We call these two situations **scenario 1** and **scenario 2**; in the first, the objective is to minimize d_1 and d_2 irrespective of d_0, whereas in the second, we aim to minimize the central distortion d_0.

Fig.5 (a) and (b) plot the SDRs (in dB) corresponding to the distortions achieved with the proposed scheme versus the standard deviation σ_u of the variable **U**, for $\rho = 20$dB and for scenarios 1 and 2, respectively. In Fig.6 (a) and (b), on the other hand, the achieved SDRs are plotted versus the channel SNR for $\sigma_u = 0.05$ and scenario 1 and 2, respectively.

The distortions corresponding to the scenario 1 and 2 of the OPTA set are also plotted in the figures; in Fig.5 (a) and Fig.6 (a), $D_1(\rho, 0)$ and $D_0(\rho, 0)$, and in Fig.5 (b) and Fig.6 (b), $D_0(\rho, \lambda^*)$ and $D_1(\rho, \lambda^*)$, where λ^* is the value that minimizes the expression of D_0 in (6). As d_0 does not reach the minimum theoretical central distortion $D_0(\rho, \lambda^*)$ in the second scenario, the side distortion d_1 can be, and it is indeed, lower than the minimum side distortion $D_1(\rho, \lambda^*)$ in scenario 2. For this reason, in order to show that d_1 does not exceed the absolute minimum side distortion, $D_1(\rho, 0)$ is also included in Fig.5 (b) and Fig.6 (b).

It can be observed in Fig.5 that the achieved distortions and the minimum theoretical distortion curves vary with σ_u at a similar rate. Unfortunately, there is a gap between the theoretical and the achieved distortions that grows with the channel SNR (see Fig.6). Furthermore, this gap is larger than in the case of the MD JSCC scheme proposed in [3, 4] for the case of parallel channels without side information at the receiver, where the gap does not exceed 2dB in the range of analyzed SNRs. However, when compared with separation schemes of similar complexity, the proposed scheme clearly has a very satisfactory performance. For example, if we focus on the side distortion of the first scenario [1], the source coding scheme based on nested lattices [10] of two dimensions, which is comparable to the proposed scheme in complexity and delay, renders a distortion 6.7dB[2] higher than the minimum theoretically achievable bound for the rate ($R \approx 1.7b/s$) that corresponds to a SNR of 20dB for the channels. In the cited work, this gap is reduced to 1.5dB by employing Slepian-Wolf channel coding schemes, but also increasing considerably the complexity and delay by the use of LDPC codes. In [11], a source coding scheme of moderate complexity based on trellis codes is proposed, which renders distortions around 3dB away from the Wyner-Ziv bound. Notice that the 3dB and the moderate complexity does not take into account the channel coding, which will add an extra distortion loss respect to the bound and a complexity increment.

5. CONCLUSIONS

In this article, a novel multiple description analog joint source-channel coding scheme for parallel channels when side information is available at the receiver has been presented. The performance of this new scheme is not as satisfactory as the performance of the scheme in [3, 4] when no side information is available. However, we think that this new scheme can pave the way for future research with the definition of a strategy that takes advantage of the side information in an efficient way. For instance, it would be interesting to

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[1] The central distortion and the results in scenario 2 are not compared because the only practical MD source coding scheme (for the case of side information at the receiver) proposed in [8] does not focus on scenarios 1 and 2.

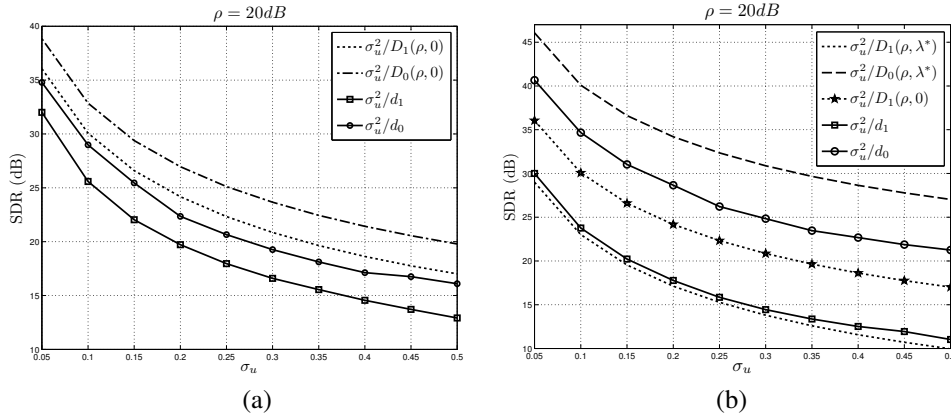[2] With lattices D_4 and Λ_{24} around 5.5dB and with E_8 around 4.7dB.

Fig. 5. The OPTA and the achieved SDRs versus σ_u for $\rho = 20$dB (a) in the first and (b) second scenarios.
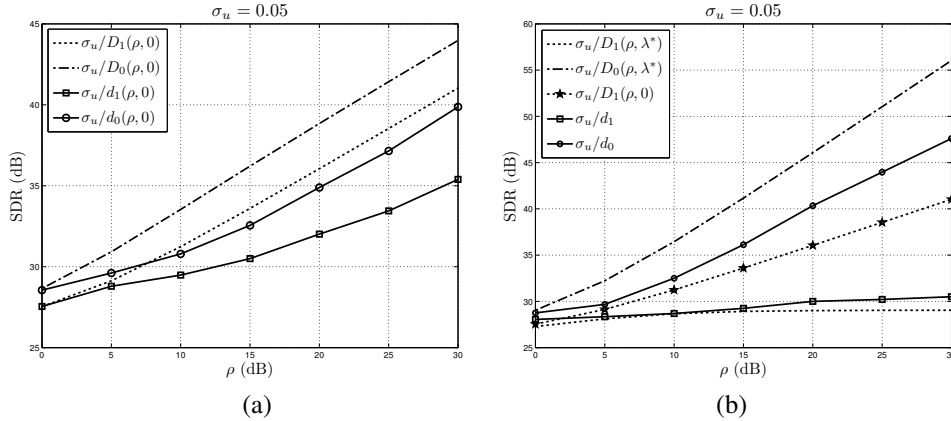

Fig. 6. The OPTA and the achieved SDRs versus ρ for $\sigma_u = 0.05$ (a) in the first and (b) second scenarios.

find analog mappings that are more suitable for mapping (Z_1, Z_2) into X, since the analog mapping described in Section 2.1 is specifically designed for transmissions of Gaussian sources. Another possibility to reduce the gap could be to use a nested structure with a variable size of the cells.

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