TIME DELAY ESTIMATION FOR MOTION COMPENSATION AND BATHYMETRY OF SAS SYSTEMS

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ABSTRACT

Accurate and reliable time delay estimation methods play a key role in multiple receiver synthetic aperture systems to obtain precise motion error estimates required for the reconstruction of high-resolution imagery. Moreover, time delay estimates have a direct impact on the accuracy of height estimation in bathymetry. For both types of applications, it is crucial to estimate time delays with subsample precision, however, bathymetry additionally necessitates the use of small sample sizes to avoid detail loss in estimating height information of the sea bottom. We show for linear frequency modulated signals that time delay estimation based on adaptive filtering can be applied to obtain more accurate estimates of translational displacements in motion compensation techniques as well as of height estimates in bathymetry.

Index Terms— time delay estimation, synthetic aperture sonar, bathymetry, micronavigation

1. INTRODUCTION

The formation of high-resolution Synthetic Aperture Sonar (SAS) images requires precise position information of sensors over the entire synthetic aperture in order to coherently combine the reverberated signals [1]. Due to a lack of external reference, e.g. GPS as commonly used in Synthetic Aperture Radar (SAR), inertial navigation systems (INS) are solely used for autonomous underwater vehicles (AUVs).

The sonar of such AUVs typically consists of an array of hydrophone receivers, which increases the system's coverage rate while maintaining spatial sampling requirements. Moreover, it allows using redundant phase center information for compensating inaccurate position information. This technique known as the Displaced Phase Center Antenna (DPCA) exploits the temporal and spatial coherence of sea bottom reverberation to estimate time delays of redundant phase centers and relates them to translational and rotational displacements of successive pings [2]. Due to the geometrical relation between the imaging platform and the scene of interest, the time delay difference of redundant phase centers is range-variant. A similar geometry is found in bathymetry [3] with the difference that the displacements of receivers are known in order to extract height information.

Traditional time delay estimation (TDE) methods based on the generalized cross-correlation (GCC) function [4] are only applicable for jointly stationary signals. As a consequence, the reverberated signals have to be segmented into short-time windows to account for the time-variant delays by assuming quasi stationarity for each segment. Thus, there is a trade-off between the accuracy of the estimate and the compliance with the stationarity assumption. Contrarily, adaptive time delay estimation methods are able to cope with this shortcoming of traditional methods.

The contribution of this paper is the application of an adaptive filtering approach known as explicit time delay estimation (ETDE) [5] in the context of non-stationary time delay difference estimation in SAS processing. This allows for better displacement and height profile estimation for motion compensation and bathymetry, respectively. Based on synthetic data, we show that the use of adaptive filters for subsample time delay estimation is more appropriate compared to traditional cross-correlation techniques.

The remainder of the paper is organized as follows: In Section 2, we introduce the data model of echo signals and illustrate that the time delay difference between redundant phase centers is non-stationary. Section 3 provides a description of traditional subsample TDE methods based on the GCC. In Section 4 the TDE approach using adaptive filters is outlined. Section 5 links the time delay difference to displacement and height estimates for motion compensation and bathymetry, respectively. Finally, Section 6 shows the performance gain using adaptive filters for TDE.

2. PROBLEM FORMULATION

Consider a one-target scenario where the echo signal $X_i[n]$ of an i^{th} transceiver sampled at a rate T_s is given by

$$X_i[n] = s\left[n - D_{i,1}\right] + W_i\left[n\right], \;\; n = 0, \ldots, N-1 \quad (1)$$

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with $s\left[n-D_{i,1}\right]$ representing the delayed version of the transmitted signal $s[n]$ and where $W_i[n]$ is a stationary zero-mean white Gaussian noise signal. The round-trip delay $D_{i,1}$ is related to the distance between the single target located at position $\mathbf{v}_1 = [x_1, 0]^T$ and a transceiver at location $\mathbf{u}_i = [x_i, z_i]^T$. It can be written as

$$D_{i,1} = \frac{2R_{i,1}}{cT_s} \quad \text{with } R_{i,1} = \|\mathbf{v}_1 - \mathbf{u}_i\|_2 \tag{2}$$

where c is the speed of propagation, e.g. of sound in water. Now, let us consider a scenario as depicted in Fig. 1, showing the geometrical relation between two transceivers located at positions $\mathbf{u}_i$ with $i = 1, 2$. In both applications, motion compensation and bathymetry, one is interested in the time delay difference $\Delta\tau = 2\left(R_{1,1} - R_{1,2}\right)/c$. Note that there is no time dependence on the time delay difference for a single target scenario.

However, assuming M targets as a model of the sea-bottom, located at $\mathbf{v}_m = [x_m, 0]^T$ with $m = 1, \ldots, M$ yields a time delay difference of

$$\begin{aligned}\Delta\tau(n) &= 2/c \cdot \Delta R(m)|_{m=g(n)} \\ \text{with } \Delta R(m) &= R_{1,m} - R_{2,m} \\ &= \|\mathbf{v}_m - \mathbf{u}_1\|_2 - \|\mathbf{v}_m - \mathbf{u}_2\|_2. \end{aligned}\tag{3}$$

Note that for a multiple target scenario the time delay difference $\Delta\tau(n)$ depends on the discrete time-index n in the slant-range dimension and is linked to the ground-range index m via the non-linear geometry mapping function $g(\cdot)$. Thus, both problems, motion compensation and bathymetry, can be cast into an estimation problem of time-varying time delay differences. For the remainder, we assume the subsequent signal model to ease notation on the subscripts:

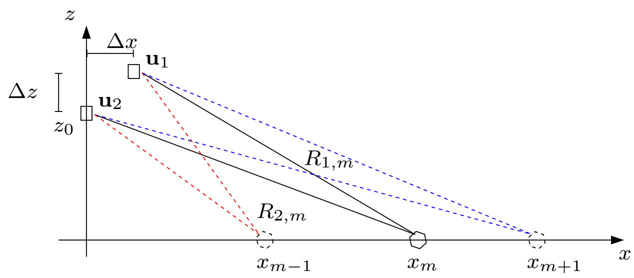

Fig. 1. Scenario of transceiver and phase center geometry.

$$\begin{aligned}X[n] &= s[n - D_1(n)] + W_1[n] \\ Y[n] &= s[n - D_2(n)] + W_2[n]. \end{aligned}\tag{4}$$

Assuming that the transmitted signal $s[n]$ as well as the noise processes $W_i[n]$ with $i = 1, 2$ are wide-sense stationary with zero-mean and mutually uncorrelated, the auto-covariance function of the echo signal is given by

$$\begin{aligned}c_{XX}(\kappa) &= E\left\{X[n+\kappa]X[n]^*\right\} \\ &= c_{ss}(\kappa - (D_1(n+\kappa) - D_1(n))). \end{aligned}\tag{5}$$

While (2) demonstrates that the sampling rate T_s limits the accuracy of the time delay estimates and makes it necessary to apply estimation techniques with subsample precision, (5) clarifies that the echo signals are not stationary due to the nonlinear mapping between ground and slant-range. This requires short-time windowing of the echo signals to be able to apply traditional cross-correlation techniques and simultaneously motivates the application of adaptive filtering.

To complete the data model, we introduce the transmitted signal $s[n]$, which will be used later in Section 6. It is a linear-frequency modulated (LFM) or chirp signal, which is mostly used in sonar and radar imaging and is expressed as

$$\begin{aligned} s[n] &= \exp\{j\pi\alpha n^2\} \cdot \exp\{j\omega_c n\} \cdot a[n] \\ \text{with } a[n] &= \begin{cases} 1 & -N_p/2 \le n \le N_p/2 - 1 \\ 0 & \text{elsewhere} \end{cases} \end{aligned}\tag{6}$$

where $\alpha = B/N_p$ is the chirp rate with bandwidth B, N_p is the pulse length in samples and ω_c the carrier frequency.

3. GENERALIZED CROSS-CORRELATION TECHNIQUES FOR TIME DELAY ESTIMATION

Traditional techniques for time delay estimation are based on the generalized cross-correlation (GCC), which differs from a standard cross-correlation by introducing pre-filters. Depending on the kind of pre-filter, they either enhance or attenuate frequency bands according to the signal-to-noise ratio of these bands or pre-whiten the input signals. Various types of pre-filters have been suggested, e.g. the smoothed coherence transform (SCOT), the phase transform (PHAT) and the weighting functions of the maximum likelihood (ML) estimator [4]. All pre-filters have in common that they require knowledge of the cross spectrum or the magnitude squared coherence of the received echoes, which are typically unknown in practice and consequently need to be estimated from finite data length.

Due to the time-variant delays, the echo signals in (4) have to be segmented using a short-time window. Assuming a window size of $\tilde{N}$ samples for which both delays are considered to be constant, the echoes of the l^{th} segment with $l = 1, \ldots, \lfloor N/\tilde{N} \rfloor$ can be expressed as

$$X[\tilde{n}, l] = X[\tilde{n} + (l-1)\cdot\tilde{N}], \quad \tilde{n} = 0, \ldots, \tilde{N}-1. \tag{7}$$

An estimate of the GCC function of the l^{th} segment is then given by

$$\hat{c}_{XY}(\kappa; l) = \sum_{\nu=0}^{\tilde{N}-1} \hat{\Psi}(e^{j\omega_\nu})\hat{C}_{XY}(e^{j\omega_\nu}; l)e^{j\omega_\nu\kappa} \tag{8}$$

where $\hat{\Psi}(e^{j\omega_\nu})$ is an estimate of the frequency response of the pre-filter, $\hat{C}_{XY}(e^{j\omega_\nu})$ of the cross-spectrum and $\omega_\nu = 2\pi\nu/\tilde{N}$ is the discrete frequency. Without pre-filters, i.e. $\hat{\Psi}(e^{j\omega_\nu}) = 1$, one obtains the estimate of the standard cross-correlation function as

$$\hat{c}_{XY}(\kappa; l) = \hat{c}_{ss}\left(\kappa - \Delta D_s^{(l)}; l\right) \quad (9)$$

where $\Delta D_s^{(l)} = [\Delta\tau^{(l)}/T_s]$, with $[\cdot]$ being the rounding operator, is the discrete time delay difference of the l^{th} segment and can be estimated by

$$\Delta\hat{\tau}_s^{(l)} = T_s \cdot \Delta\hat{D}_s^{(l)} = T_s \cdot \arg\max_{\kappa} \ |\hat{c}_{XY}(\kappa; l)| \quad (10)$$

since $c_{ss}(0) \geq c_{ss}(\kappa)$ for $\kappa > 0$. Note that $\Delta\tau^{(l)}$ can only be estimated up to sample precision using this approach. In the following two subsections, we list extensions to the cross-correlation method to obtain subsample precision of the time delay estimates. For the sake of readability, the segment notation will be omitted.

3.1. Sub-sample TDE using Parabolic Peak Interpolation

A wide spread technique due to its simplicity is a parabolic peak interpolation of the cross-correlation function [6] by determining the location of the extremum, $\kappa_{\max} = -a_1/(2a_2)$, of the parabola

$$y(\kappa) = a_0 + a_1\kappa + a_2\kappa^2. \quad (11)$$

Its coefficients can be determined by solving the linear equation system consisting of the values of the maximum of the sample covariance function, i.e. $\hat{c}_{XY}(\kappa_s)$ for $\kappa_s = \Delta\hat{D}_s$, and its direct neighbors. Plugging the coefficients into (11) yields the subsample estimate of the time delay difference as

$$\hat{\tau}_p = \Delta\hat{\tau}_s + \frac{T_s \cdot (c_{XY}(\kappa_s - 1) - c_{XY}(\kappa_s + 1))}{2c_{XY}(\kappa_s - 1) - 4c_{XY}(\kappa_s) + 2c_{XY}(\kappa_s + 1)}. \quad (12)$$

3.2. Sub-sample TDE using Phase Information

Apart from fitting a parabola to the cross-correlation peak using only magnitude information, one can exploit phase information to obtain a subsample time delay difference estimate. After down conversion, the complex baseband signal is given by

$$\begin{aligned}\tilde{X}[n] &= (s\left[n - D_1\right] + W_1[n]) \cdot \exp\{-j\omega_c n\} \\ &= \tilde{s}\left[n - D_1\right] \exp\left\{-j\omega_c D_1\right\} + \tilde{W}_1[n]. \quad (13)\end{aligned}$$

where $\tilde{s}[n]$ and $\tilde{W}_1[n]$ denote the baseband versions of the transmitted and additive noise signal, respectively. Substituting (13) into (9) yields the cross-correlation function in the baseband as

$$c_{\tilde{X}\tilde{Y}}(\kappa) = c_{\tilde{s}\tilde{s}}\left(\kappa - \Delta D_s\right) \cdot \exp\left\{-j\omega_c \Delta\tau/T_s\right\}. \quad (14)$$

As can be seen from (14) the time delay difference $\Delta\tau$ is also contained in the phase information of the complex cross-correlation function, which can be exploited to obtain a subsample estimate

$$\Delta\hat{\tau}_f = -\frac{\theta}{\Omega_c} + \frac{\lambda_f}{f_c} \quad (15)$$

with $\theta = \angle\left(c_{\tilde{X}\tilde{Y}}(\kappa)\right)$, $\Omega_c = 2\pi f_c$ and $\lambda_f \in \mathbb{Z}$ represents the ambiguity number due to modulo 2π phase wraps. This ambiguity is solved by minimizing the magnitude difference between the sample and subsample precision delays, given by

$$\lambda_f = \arg\min_{\lambda} |\Delta\hat{\tau}_s - \Delta\hat{\tau}_f(\lambda)|\,, \lambda \in \mathbb{Z}.$$

We refer to this method as the fine time delay estimation technique.

4. EXPLICIT TIME DELAY ESTIMATION

A completely different approach for time delay estimation is based on adaptive filtering and has been presented in [5], [7]. In contrast to the previous methods, adaptive filters have the advantage of avoiding the need for spectral estimation from finite data segments. Moreover, they provide tracking capabilities of the delays, which is highly desirable for both applications of interest. These benefits motivate the use of adaptive filters in the context of motion compensation and bathymetry. In the sequel, we briefly review the concept of explicit time delay estimation by means of adaptive filtering [7].

The principle idea for applying an adaptive filter for time delay estimation is depicted in Fig. 2. The impulse response $h[n, \Delta D(n)]$ is parametrized by the desired subsample time delay difference $\Delta D(n) = \Delta\tau(n)/T_s$ in order to time shift the input signal $X[n]$ and therefore to approximate the reference signal $Y[n]$. The corresponding time delay difference $\Delta D(n)$ is found adaptively by minimizing the square error $\epsilon[n, \Delta D(n)]^2$ between input and reference signal.

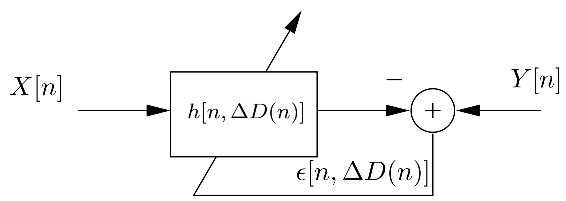

Fig. 2. Block diagram of explicit time delay estimation (ETDE) technique using an adaptive filter [5].

Let us for simplicity assume that there are no additive noise sources in (4), i.e. $W_i[n] \equiv 0$ for $i = 1, 2$. Then, based on the ideal interpolation, the echo signal of the second receiver can be expressed as

$$Y[n] = s[n - D_2(n)] = \sum_{l=-\infty}^{\infty} h[l, \Delta D(n)] X[n - l] \quad (16)$$

where the impulse response is given by $h[n, \Delta D(n)] = \mathrm{sinc}[n - \Delta D(n)]$. However, in practice the received signals are superimposed by noise, i.e. $W_i[n]$ for $i = 1, 2$ exists, and moreover the impulse response $h[n, \Delta D(n)]$ has to be realized by a finite impulse response (FIR) filter of order P. Consequently, only an estimate of $\Delta D(n)$ and therefore of $Y[n]$ is available, which results in an error term denoted by

$$\begin{aligned} \epsilon[n, \Delta\hat{D}(n)] &= Y[n] - \hat{Y}[n] \\ &= Y[n] - \sum_{l=-P/2}^{P/2} h[l, \Delta\hat{D}(n)] X[n-l]. \quad (17) \end{aligned}$$

Minimizing $\epsilon[n, \Delta\hat{D}(n)]^2$ with respect to the time delay difference $\Delta\hat{D}(n)$ using the steepest descent approach given by

$$\Delta\hat{D}(n+1) = \hat{D}(n) - \frac{\mu}{2} \frac{\delta}{\delta \Delta\hat{D}(n)} \epsilon[n, \Delta\hat{D}(n)]^2 \quad (18)$$

leads to the explicit time delay estimation (ETDE) technique:

$$\Delta\hat{D}(n+1) = \hat{D}(n) - \mu\epsilon[n, \Delta\hat{D}(n)] \cdot \tilde{X}[n] \quad (19)$$

$$\text{with } \tilde{X}[n] = \sum_{l=-P/2}^{P/2} g[l, \Delta\hat{D}(n)] X[n-l].$$

Here, $g[n, \Delta\hat{D}(n)]$ denotes the derivative of the impulse response $h[n, \Delta\hat{D}(n)]$ with respect to $\Delta\hat{D}(n)$ and μ is the step size of the steepest descent method.

5. MOTION COMPENSATION AND BATHYMETRY

This section briefly outlines the basic concepts of motion compensation and bathymetry based on the obtained time delay estimates and will be used in Section 6 to assess the performance of the time delay estimation techniques. Due to inaccurate position information of navigation systems, it is essential in SAS to correct unknown ping-to-ping displacements by exploiting redundant information provided by overlapping phase center pairs, e.g. as shown at positions $\mathbf{u}_1$ and $\mathbf{u}_2$ in Fig. 1, which are displaced by $\boldsymbol{\theta} = [\Delta x, \Delta z]^T$ but have the same along-track position.

Using a non-linear least-squares approach [8], it is feasible to estimate these displacements, assuming a flat sea-bottom, by minimizing

$$\Lambda(m, \boldsymbol{\theta})^2 = \frac{1}{2} \left| \Delta\hat{R}(m, \boldsymbol{\theta}) - \Delta R(m, \boldsymbol{\theta}) \right|^2 \quad (20)$$

using the Gauss-Raphson method where $\Delta R(m, \boldsymbol{\theta}) \equiv \Delta R(m)$ as stated in (3) and $\Delta\hat{R}(m, \boldsymbol{\theta})$ are the scaled time delay difference estimates. The recursive rule to determine the displacement estimates is then given by

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) - \left(\boldsymbol{J}^T \boldsymbol{J} + \boldsymbol{S} \right)^{-1} \cdot \boldsymbol{J}\boldsymbol{\Lambda}$$

$$\text{with } \boldsymbol{S} = \sum_{m=1}^{M} \Lambda(m, \boldsymbol{\theta}) \boldsymbol{H}, \quad (21)$$

where $\boldsymbol{J}$ and $\boldsymbol{H}$ represent the Jacobian and Hessian matrix, respectively, and $\boldsymbol{\Lambda}$ is a vectorized representation of $\Lambda(m, \boldsymbol{\theta})$ for $m = 1, \ldots, M$.

The scenario of Fig. 1 can also be applied to bathymetry after replacing the transmitter and receivers by their respective phase centers at positions $\mathbf{u}_1$ and $\mathbf{u}_2$. In contrast to motion compensation, in bathymetry the displacements $\boldsymbol{\theta}$ are known but the height profile $z_h(r)$ of the sea bottom is not. Using the geometric relations, one can determine the height of a scatterer in slant-range r using $\mathbf{u}_1$ as a reference point by

$$z_h(r) = z_0 - r \left[\sqrt{1 - a^2} \cos(\xi) - a \sin(\xi) \right]$$

$$\text{with } a = \frac{[r + \Delta R(r)]^2 - r^2 - d^2}{2rd}, \quad (22)$$

where $d = \|\boldsymbol{\theta}\|_2$ and $\xi = \tan^{-1}(\Delta z / \Delta x)$. Note that in practice a combination of bathymetry with motion compensation may significantly improve the ping-to-ping displacement estimates by overcoming the implicit assumption of a flat sea bottom when compensating motion errors.

6. SIMULATION RESULTS

In order to assess the performance of different time delay estimation techniques, $\mathcal{M} = 500$ Monte Carlo simulations have been evaluated using the parameters listed in Table 1. Exemplary results of range difference estimates used for motion compensation and height profile estimation are depicted in Fig. 3 and 4, respectively. In both cases, the phase centers have been displaced by $\Delta x = 2$ cm and $\Delta z = -2$ cm. It is observable from Fig. 3 that the maximum likelihood GCC method performs worse than the standard cross-correlation method combined with phase information for subsample precision and using the pulse compressed echo data. As a consequence, we focus on the latter and the adaptive filter approach, which already shows superior performance in both examples. Note that the parabolic peak interpolation technique is used implicitly in the fine time delay method but has shown poor performance if used for subsample estimation by itself.

PARAMETER	VARIABLE	VALUE
Sampling Frequency	f_s	80 kHz
Carrier Frequency	f_c	20 kHz
Bandwidth	B	4 kHz
Pulse Length	T_p	5 ms

Table 1. Simulation Parameters

The results of the Monte Carlo simulations are depicted in Fig. 5 and 6 for estimating the displacements in case of motion compensation as well as for height profile estimation. While the adaptive filter approach is superior to the cross-correlation technique for displacement estimation, it only shows an improved performance for SNR values larger than 5dB in case of height profile estimation.

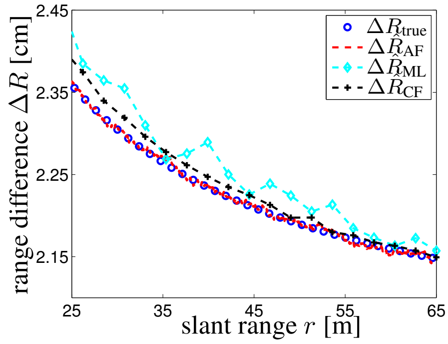

Fig. 3. Range difference estimation example for different time delay estimation methods, i.e. adaptive filter (AF), maximum likelihood GCC (ML) and standard cross-correlation with pulse compressed echoes (CF), for a SNR $= 10$dB .

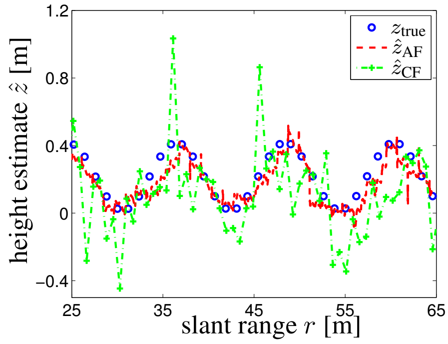

Fig. 4. Height profile estimation for the adaptive filter approach (AF) and the fine time delay estimation technique (CF) for a SNR $= 15$dB.

7. CONCLUSION

We compared the classical cross-correlation techniques for time delay estimation with an adaptive filter approach in the context of motion compensation for synthetic aperture sonar high-resolution imaging as well as for bathymetry. Simulation results demonstrate that the adaptive filter approach outperforms the subsample cross-correlation technique for displacement estimation, especially for high signal-to-noise ratios. In case of height profile estimation the performance gain depends on the SNR. For future work, a validation on real sonar data is required to prove the performance gain of the explicit time delay estimation technique.

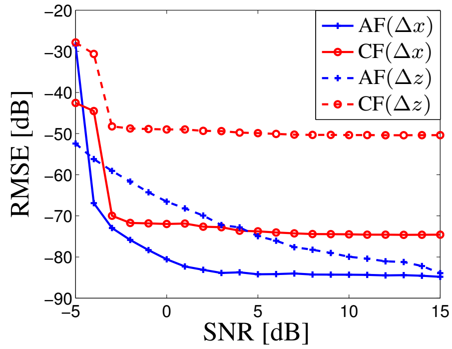

Fig. 5. Simulation results of displacement estimates.

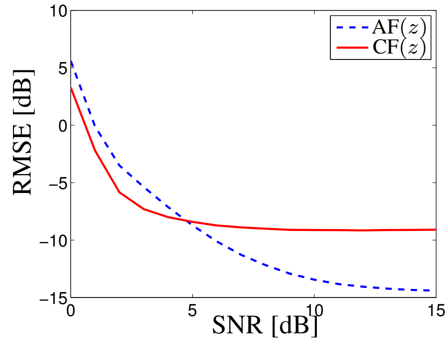

Fig. 6. Simulation results of height profile estimation.

8. REFERENCES

[1] M. P. Hayes and P. T. Gough. Synthetic Aperture Sonar: A Review of Current Status. *IEEE Journal of Oceanic Engineering*, 34(3):207–224, July 2009.

[2] A. Bellettini and M. A. Pinto. Theoretical Accuracy of Synthetic Aperture Sonar Micronavigation Using a Displaced Phase-Center Antenna. *IEEE Journal of Oceanic Engineering*, 27(4):780–789, October 2002.

[3] H. D. Griffiths, T. A. Rafik, Z. Meng, C. F. N. Cowan, H. Shafeeu, and D. K. Anthony. Interferometric synthetic aperture sonar for high resolution 3-D mapping of the seabed. *IEE Proceedings -Radar, Sonar and Navigation*, 144(2):96–103, April 1997.

[4] G. Carter. Time delay estimation for passive sonar signal processing. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 29(3):463–470, 1981.

[5] S. R. Dooley and A. K. Nandi. Adaptive subsample time delay estimation using Lagrange interpolators. *IEEE Signal Processing Letters*, 6(3):65–67, 1999.

[6] R. Moddemeijer. On the determination of the position of extrema of sampled correlators. *IEEE Transactions on Signal Processing*, 39(1):216–219, 1991.

[7] D. Youn, N. Ahmed, and G. Carter. On using the LMS algorithm for time delay estimation. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 30(5):798–801, 1982.

[8] D. A. Cook, D. C. Brown, and J. E. Fernandez. Synthetic aperture sonar motion estimation using nonlinear least squares. *Proceedings of the Institute of Acoustics*, 28:176–182, 2006.