A LINEARLY-CONSTRAINED APPROACH FOR FILTERED-X WIENER FILTERING

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ABSTRACT

This paper shows that the filtered-x Wiener filtering can be viewed as a linearly-constrained processing technique. For this purpose, the linearly-constrained Wiener filtering is formally introduced, as well as its GSC representation. As a result from this approach, a new set of linear constraints is defined and named as "convolution constraints". Simulation results validate the proposed methodology.

Index Terms**—** Filtered-x Wiener filtering, linearly-constrained processing, convolution constraints

1. INTRODUCTION

Active control of sound and vibration is an important application area for adaptive signal processing [1], [2], [3]. The filtering scheme for interference cancellation is known as filtered-x Wiener filtering (FXWF), since the controller is a causal and finite-duration impulse response (FIR) filter, and the reference input signal can be viewed as the output signal of an unknown linear system. The most widely used adaptive filtering algorithm in such active control systems is the least-mean-square (LMS) algorithm with the reference signal processed by a linear filter. This filter seeks to compensate for the effects of the secondary path from the output of the controller to the cancellation point. Because of this special arrangement, the algorithm is termed filtered-x LMS (FXLMS) algorithm. Interpolated FIR (IFIR) filtering is another example that can be modeled as a FXWF scheme [4], [5].

Linear constraints are widely applied in the optimization of temporal and/or spatial filtering. In general, the constraints are used when the second-order moment of the Wiener filter desired response or even its discrete-time samples are not available. The linear constraints enforce characteristics to the desired response in order to achieve the filter optimization [6], [7], [8]. The linearly-constrained minimum variance (LCMV) is a criterion that directly minimizes the average power of the filter output to which the linear constraints are applied. This technique has been successfully applied to adaptive beamforming and spectral analysis [6], [7], [8], [9]. The LCMV method may be alternatively implemented using an indirect structure called generalized sidelobe canceller (GSC), which modifies the constrained minimization problem to an unconstrained one [10]. It has been shown that the direct form and the GSC structures are equivalent [11].

In this paper, we consider the linearly-constrained Wiener filtering (LCWF), and its GSC representation, in order to show that the filtered-x Wiener filtering (FXWF) is equivalent to a linearly-constrained filtering. A new set of linear constraints, named "convolution constraints", is introduced, which can be directly imposed by either the constrained least-mean-square (CLMS) algorithm [6] or the constrained fast least-squares (CFLS) algorithm [12]. The FXWF-LCWF equivalence is illustrated by simulation.

2. LINEARLY-CONSTRAINED WIENER FILTERING

The LCWF problem may be stated as follows:

$$\begin{aligned} &\textit{minimize:} && J(\mathbf{w}) = E\{e^2(n)\} \\ & && = \sigma_d^2 - 2\mathbf{w}^t\mathbf{p} + \mathbf{w}^t\mathbf{R}\mathbf{w} && \text{(1a)} \\ &\textit{subject to:} && \mathbf{C}^t\mathbf{w} = \mathbf{f}\ , && \text{(1b)} \end{aligned}$$

where $e(n)$ is the estimation error defined as the difference between a desired response $d(n)$ and the transversal filter output $y(n)$. $\mathbf{w}=[w_0\ w_1\ \ldots\ w_{N-1}]^t$ and $\mathbf{x}(n)=[x(n)\ x(n-1)\ \ldots\ x(n-N+1)]^t$ are, respectively, the N-by-1 tap-weight and tap-input vectors of the $(N-1)$-order finite impulse response (FIR) filter. σ_d^2 denotes the variance of $d(n)$, $\mathbf{R}$ the N-by-N correlation matrix of $\mathbf{x}(n)$, and $\mathbf{p}$ the N-by-1 cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$. The N-by-K constraint matrix $\mathbf{C}$ and the K-element response vector $\mathbf{f}$ in (1b) establish the set of linear constraint equations imposed on the minimization of the mean-squared error (MSE). It is assumed that $K<N$ and $\mathbf{C}$ is a full column rank matrix.

Without loss of generality, all parameters and signals are assumed to be real-valued. Furthermore, both $x(n)$ and $d(n)$ sequences are modeled as wide-sense stationary zero-mean Gaussian discrete-time stochastic processes.

Fig. 1 illustrates the LCWF problem (1). The columns of $\mathbf{C}$ span a K-dimensional subspace $\mathscr{C}$, called constraint subspace, in the N-dimensional vector space $\mathscr{W}$ (plan of the sheet). Thus, matrix $\mathbf{C}^t$ in (1b) is a linear transformation from vector space $\mathscr{W}$ (domain space) into the vector space $\mathscr{C}$ (image space) that maps the tap-weight vector **w** into **f**. Hence, all **w** satisfying (1b) terminate on a (N–K)-dimensional hyperplane $\mathscr{V}$ defined by $\mathscr{V}=\{\mathbf{w}:\mathbf{C}^t\mathbf{w}=\mathbf{f}\}$, which is called constraint hyperplane.

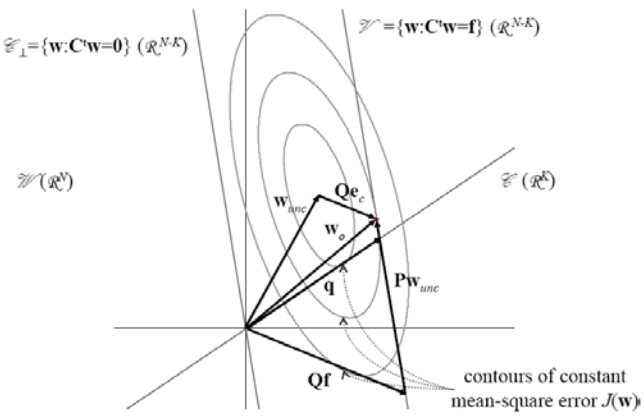

Fig. 1: Constraint and orthogonal constraint subspaces.

The homogeneous equation corresponding to (1b),

$$\mathbf{C}^t\mathbf{w} = \mathbf{0}\,, \tag{2}$$

defines a second (N–K)-dimensional hyperplane $\mathscr{C}_\perp$, which is orthogonal to the constraint subspace and passes through the origin (Fig. 1). This hyperplane is called orthogonal constraint subspace or orthogonal complement subspace.

Solving the linearly-constrained minimization problem (1) using Lagrange multipliers yields the optimal solution:

$$\begin{aligned}\mathbf{w}_o &= \mathbf{R}^{-1}\mathbf{p} + \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}(\mathbf{f} - \mathbf{C}^t\mathbf{R}^{-1}\mathbf{p}) \\ &= \mathbf{w}_{unc} + \mathbf{Q}\mathbf{e}_c = \mathbf{P}\mathbf{w}_{unc} + \mathbf{Q}\mathbf{f}\,,\end{aligned} \tag{3}$$

with

$$\mathbf{P}=\mathbf{I}-\mathbf{Q}\mathbf{C}^t, \tag{4}$$

where $\mathbf{w}_{unc}=\mathbf{R}^{-1}\mathbf{p}$ is the unconstrained Wiener filter [13] and $\mathbf{Q}\mathbf{e}_c$ is a modification imposed on $\mathbf{w}_{unc}$ to satisfy the constraints (see Fig. 1). $\mathbf{Q}=\mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}$ is a linear transformation applied to the K-dimensional constraint error vector $\mathbf{e}_c=\mathbf{f}-\mathbf{C}^t\mathbf{w}_{unc}$. Because $\mathbf{e}_c$ is an affine transformation on $\mathbf{w}_{unc}$, the imposition of constraints on the Wiener filtering scheme introduces a bias $\mathbf{Q}\mathbf{e}_c$ on the solution. Note that the N-by-K matrix **Q** is also dependent on **R**, a consequence of the fact that it results from the minimization of the MSE. Finally **Q** is a right pseudoinverse of $\mathbf{C}^t$ as $\mathbf{C}^t\mathbf{Q}=\mathbf{I}$.

Premultiplication of any vector by **P** annihilates any components in the constraint subspace spanned by the columns of **C**. Thus, $\mathbf{P}\mathbf{w}_{unc}$ keeps only the part of $\mathbf{w}_{unc}$ orthogonal to this subspace. The bias term **Qf** satisfies the constraints and, consequently, terminates on the constraint hyperplane (Fig. 1). It corresponds to the optimal solution of the LCMV filter for the same set of constraints (1b). As the LCMV does not work with a reference (desired) signal, the imposition of constraints on its parameters leads to a non-elementary optimal solution. When the LCMV problem is transformed into the LCWF problem by the presence of $d(n)$ as in Fig. 1, the optimal LCMV filter is corrected taking into account the cross-correlation between $x(n)$ and $d(n)$.

The minimum MSE is given by

$$J_{min} = J_{unc} + \mathbf{e}_c^t(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{e}_c\,, \tag{5}$$

where

$$J_{unc} = \sigma_d^2 - \mathbf{p}^t\mathbf{w}_{unc} \tag{6}$$

denotes the minimum MSE produced by the unconstrained Wiener filter [13]. Thus, the second term in (5) can be viewed as an excess mean-squared error due to the constraints.

The expression for the estimation error at time n when the filter operates in its optimum condition is given by:

$$e_o(n) = d(n) - \mathbf{x}^t(n)\mathbf{w}_{unc} - \mathbf{x}^t(n)\mathbf{Q}\mathbf{e}_c\,. \tag{7}$$

Pre-multiplying both sides of the above equation by $\mathbf{x}(n)$ and taking the expected value, we get the result

$$E\{\mathbf{x}(n)e_o(n)\} = -\mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{e}_c\,. \tag{8}$$

It means that the orthogonality principle does not apply to the linearly-constrained optimal Wiener filtering problem, since the unconstrained optimal Wiener filter does not satisfy the constraints ($\mathbf{e}_c\neq\mathbf{0}$). Thus, the minimum error signal $e_o(n)$ is correlated with the input signal samples $\mathbf{x}(n)$. The constraints prevent the filter from attaining the minimum MSE of the unconstrained optimal Wiener filter. From (8), we also have that

$$E\{y_o(n)e_o(n)\} = -\mathbf{f}^t(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{e}_c\,, \tag{9}$$

where $y_o(n)$ denotes the output produced by the optimal filter $\mathbf{w}_o$ at time n.

For the LCMV filtering case, we obtain from (5) that the minimum average output power of the filter is given by:

$$J_{LCMV} = \mathbf{f}^t(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{f}\,, \tag{10}$$

and that

$$E\{\mathbf{x}(n)y_o(n)\} = \mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{f}\,. \tag{11}$$

The direct LCWF structure can alternatively be implemented via GSC as represented in block diagram form in Fig. 2.

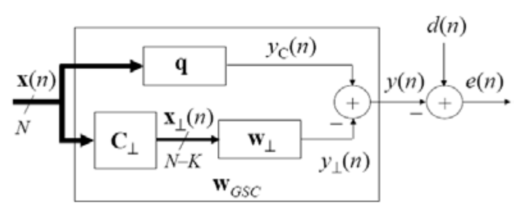

Fig. 2: GSC representation of LCWF.

The coefficient vector

$$\mathbf{q}=\mathbf{C}(\mathbf{C}^t\mathbf{C})^{-1}\mathbf{f} \tag{12}$$

represents a filter that satisfies the constraints ($\mathbf{C}^t\mathbf{q}=\mathbf{f}$), called quiescent filter. It depends only on the constraints and corresponds to the minimum-norm solution (Fig. 1) of the underdetermined system in (1b), obtained from the right pseudoinverse of $\mathbf{C}^t$:

$$(\mathbf{C}^t)^+=\mathbf{C}(\mathbf{C}^t\mathbf{C})^{-1}. \quad (13)$$

In contrast, the $(N–K)$-element vector $\mathbf{w}_\perp$ is an unconstrained filter in the orthogonal constraint subspace $\mathscr{C}_\perp$ and provides the degrees of freedom in the MSE minimization. The column vectors of the N-by-$(N–K)$ matrix $\mathbf{C}_\perp$ span the subspace $\mathscr{C}_\perp$ and represent a basis for the orthogonal complement of the subspace spanned by columns of $\mathbf{C}$ ($\mathbf{C}^t\mathbf{C}_\perp=\mathbf{0}_{K\text{x}N-K}$). Together the column vectors of $\mathbf{C}$ and $\mathbf{C}_\perp$ span the entire N-dimensional space $\mathscr{W}$. $\mathbf{C}_\perp$ is termed the signal-blocking matrix in the sense of rejecting the signal in the pass-band of $\mathbf{q}$.

Now, the error signal is given by:

$$e(n)=d(n)-(\mathbf{q}-\mathbf{C}_\perp\mathbf{w}_\perp)^t\,\mathbf{x}(n)\,. \quad (14)$$

In the mean-square-error sense, the vector $\mathbf{w}_\perp$ is chosen to minimize the following cost function:

$$J(\mathbf{w}_\perp)=\sigma_d^2-2(\mathbf{q}-\mathbf{C}_\perp\mathbf{w}_\perp)^t\mathbf{p}+ \\ +(\mathbf{q}-\mathbf{C}_\perp\mathbf{w}_\perp)^t\mathbf{R}(\mathbf{q}-\mathbf{C}_\perp\mathbf{w}_\perp) \quad (15)$$

The optimum solution is

$$\mathbf{w}_{\perp o}=(\mathbf{C}_\perp^t\mathbf{R}\mathbf{C}_\perp)^{-1}\mathbf{C}_\perp^t(\mathbf{R}\mathbf{q}-\mathbf{p}) \quad (16)$$

and

$$\mathbf{w}_{GSC_o}=\mathbf{q}-\mathbf{C}_\perp\mathbf{w}_{\perp o}\,. \quad (17)$$

Substituting (16) in (15) yields the minimum MSE of the GSC representation in Fig. 2:

$$J_{GSC}=\sigma_d^2+(\mathbf{R}\mathbf{q}-\mathbf{p})^t\,\mathbf{w}_{GSC_o}-\mathbf{p}^t\mathbf{q}\,. \quad (18)$$

When the GSC operates in its optimum condition, the expression for the estimation error at time n is given by

$$e_o(n)=d(n)-\mathbf{x}^t(n)\mathbf{q}+\mathbf{x}^t(n)\mathbf{C}_\perp\mathbf{w}_{\perp o}\,. \quad (19)$$

Pre-multiplying (19) by $\mathbf{x}(n)$ and taking the expected value yields

$$E\{\mathbf{x}(n)e_o(n)\}=\mathbf{P}_\perp(\mathbf{p}-\mathbf{q})\,, \quad (20)$$

where

$$\mathbf{P}_\perp=\mathbf{I}-\mathbf{R}\mathbf{C}_\perp(\mathbf{C}_\perp^t\mathbf{R}\mathbf{C}_\perp)^{-1}\mathbf{C}_\perp^t\,. \quad (21)$$

Based on (20), it is also verified that the orthogonality principle does not apply to the GSC representation of LCWF. However, checking in the GSC structure the orthogonality between $\mathbf{x}_\perp(n)$ and $e_o(n)$, it is easily verified that:

$$E\{\mathbf{x}_\perp(n)e_o(n)\}=\mathbf{0}\,. \quad (22)$$

So, from the point of view of $\mathbf{w}_\perp$, the MSE minimization is unconstrained and the orthogonality principle does now apply. From (22), we also have that

$$E\{y_{\perp o}(n)e_o(n)\}=\mathbf{0}\,, \quad (23)$$

where $y_{\perp o}(n)$ denotes the output produced by the optimal filter $\mathbf{w}_{\perp o}$ at time n.

Both the direct form and GSC structures of LCWF are equivalent. This can be proved by showing that $\mathbf{w}_o=\mathbf{w}_{GSCo}$, or:

$$\mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}=(\mathbf{I}-\mathbf{C}_\perp(\mathbf{C}_\perp^t\mathbf{R}\mathbf{C}_\perp)^{-1}\mathbf{C}_\perp^t\mathbf{R})\mathbf{C}(\mathbf{C}^t\mathbf{C})^{-1} \quad (24)$$

and

$$(\mathbf{I}-\mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{C}^t)\mathbf{R}^{-1}=\mathbf{C}_\perp(\mathbf{C}_\perp^t\mathbf{R}\mathbf{C}_\perp)^{-1}\mathbf{C}_\perp^t\,. \quad (25)$$

The veracity of (24) has already been proven in [12], since Buckley's sufficient conditions are verified, i.e. $\mathbf{C}^t_\perp\mathbf{C}=\mathbf{0}$ and rank$(\mathbf{C}_\perp)=N–K$. Now, post-multiplying both sides of (25) by $\mathbf{R}\mathbf{C}(\mathbf{C}^t\mathbf{C})^{-1}$ leads to the equality in (24). It also means that J_{min} in (5) is equal to J_{GSC} in (18).

It is worth noting that the equivalence between the direct form and its GSC representation for LCMV filtering has been well studied. However, the linearly-constrained Wiener filtering scheme has not been formally displayed in the literature. This allows us to see filtered-x Wiener filtering as a linearly-constrained filtering.

3. FILTERED-X WIENER FILTERING

Fig. 3 illustrates the filtered-x Wiener filtering problem in block diagram form using vector notation. The request is to find the filter $\mathbf{h}$ that minimizes the estimation error $e(n)$ in the mean-square sense.

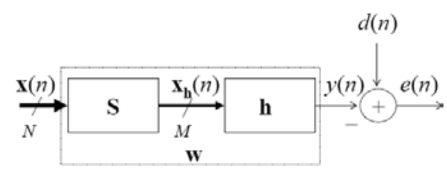

Fig. 3: Filtered-x Wiener filtering

The filter $\mathbf{w}$ arises from the convolution of a predefined filter $\mathbf{s}=[s_0\ s_1\ \ldots\ s_{L-1}]^t$ and the unknown filter $\mathbf{h}=[h_0\ h_1\ \ldots\ h_{M-1}]^t$:

$$\mathbf{w}=\mathbf{S}\mathbf{h}, \quad (26)$$

where $\mathbf{S}$ is a N-by-M non-symmetric Toeplitz matrix having the vector $[\mathbf{s}^t,\ \mathbf{0}^t_{N-L}]^t$ as its first column and the vector $[s_0,\ \mathbf{0}^t_{M-1}]$ as its first row ($N=L+M–1$). For instance, $\mathbf{S}$ has the following form for $M=5$ and $L=3$ ($N=7$):

$$\mathbf{S}=\begin{bmatrix} s_0 & 0 & 0 & 0 & 0 \\ s_1 & s_0 & 0 & 0 & 0 \\ s_2 & s_1 & s_0 & 0 & 0 \\ 0 & s_2 & s_1 & s_0 & 0 \\ 0 & 0 & s_2 & s_1 & s_0 \\ 0 & 0 & 0 & s_2 & s_1 \\ 0 & 0 & 0 & 0 & s_2 \end{bmatrix}. \quad (27)$$

By inspection, $\mathbf{S}$ is a full column rank matrix with rank equal to M. Note that the reference signal vector $\mathbf{x}_h(n)$ of filter $\mathbf{h}$ is a filtered version of $\mathbf{x}(n)$ by filter $\mathbf{s}$:

$$\mathbf{x}_h(n)=\mathbf{S}^t\mathbf{x}(n), \quad (28)$$

giving rise to the name filtered-x Wiener filtering for the scheme in Fig. 3.

Comparing Fig. 3 with Fig. 2, we can visualize the filtered-x Wiener filtering as a linearly-constrained Wiener filtering employing the GSC structure. For the case in point: **q**=**0**, $\mathbf{C}_\perp$=**S** and $\mathbf{w}_\perp$=–**h** (K=N–M=L–1). Substituting **0** for **q** and $\mathbf{C}_\perp$ for **S** in (16) and (18) leads, respectively, to the optimal solution and to the minimum MSE:

$$\mathbf{h}_o = -\mathbf{w}_{\perp o} = (\mathbf{S}^t\mathbf{R}\mathbf{S})^{-1}\mathbf{S}^t\mathbf{p} \tag{29}$$

and

$$J_{GSC} = \sigma_d^2 - \mathbf{p}^t\mathbf{w}_{GSC_o}, \tag{30}$$

where

$$\mathbf{w}_{GSC_o} = \mathbf{S}(\mathbf{S}^t\mathbf{R}\mathbf{S})^{-1}\mathbf{S}^t\mathbf{p}. \tag{31}$$

So, what is the set of linear constraints that imposes the above condition? Naturally, the answer for this question corresponds to determine the N-by-(L–1) constraint matrix **C** and the (L–1)-element response vector **f** in (1b).

When visualizing the FXWF (Fig. 3) as a LCWF-GSC structure (Fig. 2), we have verified that **q**=**0**. So, based on (12), we can conclude that **f**=**0**. Since the filter **w** must satisfy the constraints, we have the set of linear constraint equations:

$$\mathbf{C}^t\mathbf{w}=\mathbf{0}, \tag{32}$$

which corresponds to the homogeneous system in (2). It means that any solution **w** (there are infinite solutions) lies in the M-dimensional null space (or kernel) of the linear transformation $\mathbf{C}^t$, and such a set of linear constraints imposes the implicit convolution condition on the filter **w**, but only one solution minimizing the MSE.

As far as the constraint matrix **C** is concerned, let us start remembering that

$$\mathbf{C}^t\mathbf{S}=\mathbf{0}_{L-1\text{x}M}. \tag{33}$$

Now, making use of the singular value decomposition (SVD) of matrix **S**, and taking into account that it is a full column rank matrix (rank equal to M), we have that:

$$\mathbf{U}^t\mathbf{S}\mathbf{V} = \begin{bmatrix} \mathbf{\Sigma}_M \\ \cdots \\ \mathbf{0}_{L-1\text{x}M} \end{bmatrix}, \tag{34}$$

where $\mathbf{\Sigma}_M$ is a M-by-M diagonal matrix whose diagonal elements correspond to the M singular values of **S**, $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M > 0$, i.e. $\mathbf{\Sigma}_M = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_M)$. **U**=[$\mathbf{u}_1$ $\mathbf{u}_2$ … $\mathbf{u}_N$] and **V**=[$\mathbf{v}_1$ $\mathbf{v}_2$ … $\mathbf{v}_M$] are N-by-N and M-by-M unitary matrices ($\mathbf{U}^t\mathbf{U}$=**I** and $\mathbf{V}^t\mathbf{V}$=**I**), respectively. From (34), it results that

$$\mathbf{U}^t\mathbf{S} = \begin{bmatrix} \mathbf{\Sigma}_M\mathbf{V}^t \\ \cdots \\ \mathbf{0}_{L-1\text{x}M} \end{bmatrix}. \tag{35}$$

Partitioning the unitary matrix $\mathbf{U} = [\mathbf{U}_1 \;\vdots\; \mathbf{U}_2]$, where $\mathbf{U}_1$=[$\mathbf{u}_1$ $\mathbf{u}_2$ … $\mathbf{u}_M$] and $\mathbf{U}_2$=[$\mathbf{u}_{M+1}$ $\mathbf{u}_{M+2}$ … $\mathbf{u}_N$], (35) becomes

$$\begin{bmatrix} \mathbf{U}_1^t\mathbf{S} \\ \cdots \\ \mathbf{U}_2^t\mathbf{S} \end{bmatrix} = \begin{bmatrix} \mathbf{\Sigma}_M\mathbf{V}^t \\ \cdots \\ \mathbf{0}_{L-1\text{x}M} \end{bmatrix}. \tag{36}$$

From the above equation, we readily see that

$$\mathbf{U}_2^t\mathbf{S} = \mathbf{0}_{L-1\text{x}M}, \tag{37}$$

and compared to (33) that

$$\mathbf{C}=\mathbf{U}_2. \tag{38}$$

It means that the constraint matrix **C** can be determined by taking the last L–1 column vectors of the unitary matrix **U** coming from the SVD of matrix **S**:

$$\mathbf{C}=[\mathbf{u}_{M+1}\ \mathbf{u}_{N+2}\ \ldots\ \mathbf{u}_N]. \tag{39}$$

Constructed in this way, **C** is a full column rank matrix (rank equal to L–1) and the set of linear constraints in (32) is named as "convolution constraints". Thus, taking into account this set of constraints in the LCWF structure yields:

$$\mathbf{w}_o = \mathbf{P}\mathbf{w}_{unc}, \tag{40}$$

and

$$J_{min} = J_{unc} + \mathbf{w}_{unc}^t\mathbf{C}(\mathbf{C}^t\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{C}^t\mathbf{w}_{unc}. \tag{41}$$

In the adaptive context, the convolution constraints can be directly imposed by both CLMS and CFLS algorithms, making **f**=**0** in their update equations.

4. SIMULATION RESULTS

To certify that FXWF is equivalent to LCWF for the convolution constraints, we first consider an active noise control (ANC) system with the following characteristics. The primary and secondary (**s**) paths are bandpass (300 Hz – 5 kHz) FIR filters of order J–1=69 and L–1=19, respectively. We have assumed that the secondary path is perfectly estimated. The controller **h** is a filter of order M–1=50 (N=70 and K=19). White and colored Gaussian processes of zero-mean and unitary-variance are used as the input signal $x(n)$. The colored Gaussian process is governed by the difference equation $x(n)=0.1x(n-1)+0.8x(n-2)+v(n)$, where $v(n)$ is a zero-mean white Gaussian process of variance σ_v^2=0.27 (chosen to make σ_x^2=1). The differences between the optimal solution and minimum MSE of both approaches are summarized in Table I. Fig. 4 compares the amplitude responses of the systems.

As a second example, the identification of a system by an IFIR filter is considered. The plant to be identified is a twentieth order lowpass ($f_{3\text{dB}}$=3.1kHz) FIR filter, and the interpolated filter is composed of the cascade of the interpolator **s**=[0.5 1 0.5]^t and a tenth order unknown filter (N=13, M=11, L=3 and K=2). Again, white and colored Gaussian processes of zero-mean and unitary-variance are used as the input signal $x(n)$. The results are summarized in Table II, and the amplitude responses of the systems are plotted in Fig. 5.

The differences in both tables are insignificant and prove the equivalence of FXWF and LCWF.

Table I: ANC system

	white process	colored process
$\lvert\mathbf{w}_o-\mathbf{w}_{GSCo}\rvert^2$	1.8852x10^{-24}	1.3989x10^{-22}
$\lvert J_{min}-J_{GSC}\rvert$	2.1021x10^{-13}	4.4176x10^{-12}

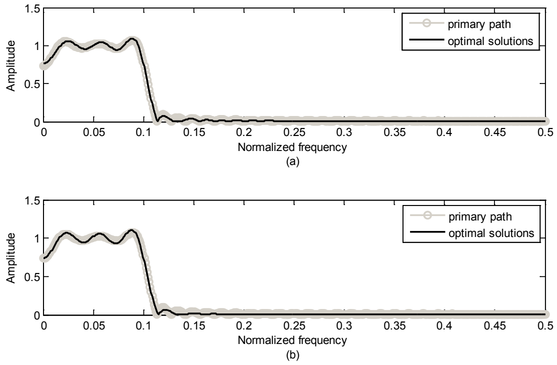

Fig. 4: ANC system: (a) white and (b) colored Gaussian process

Table II: Interpolated FIR filter (system identification)

	white process	colored process
$\|\mathbf{w}_o - \mathbf{w}_{GSCo}\|^2$	$7.2409\text{x}10^{-30}$	$2.5633\text{x}10^{-27}$
$\|J_{min} - J_{GSC}\|$	$9.7145\text{x}10^{-17}$	$3.4656\text{x}10^{-14}$

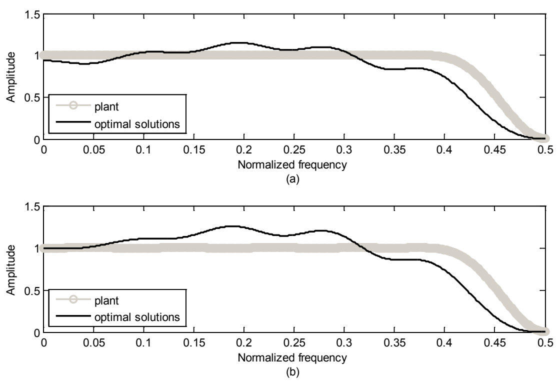

Fig. 5: IFIR filter: (a) white and (b) colored Gaussian process

5. CONCLUSION

A novel methodology of filtered-x Wiener filtering has been proposed using a linearly-constrained approach. It has been shown that FXWF is equivalent to LCWF for a particular set of constraints, named as “convolution constraints”. Studies concerning the application of the proposed method and convolution constraints are in development.

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