A Joint Precoding Scheme for Downlink Multi-user MIMO Systems

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***Abstract*—We address the problem of precoding in downlink multi-user MIMO communications using SDMA. In this set-up, multiple antennas are used at the base station and at the terminals. A double level of spatial multiplexing is present: the users are spatially multiplexed (SDMA) and each user receives spatially multiplexed symbol streams (SDM). The cumulative multi-user and multi-stream interference is a potential performance limitation in this scheme. To mitigate this, we propose a linear precoder that has the following properties: it can be computed analytically; it can be computed without iterations; it improves on the state-of-the-art solution based on the generalized eigenvectors; it is computed separately for each user. Although the formulation of the multi-user precoding can be separated from the multi-stream precoding, our solution optimizes the multi-user separation taking the multi-stream processing into account. The performance of our precoding scheme is assessed by simulations.**

***Index Terms*—multi-user MIMO, SDMA, SDM, spatial multiplexing, precoding**

I. Introduction

There is an increasing requirement for wireless systems to provide increased throughput. Based on the Shannon capacity formula for multiple-input multiple-output (MIMO) systems, system designers exploit multiple antennas at both the transmit and receive side of the links, which enables spatial division multiplexing (SDM). A further capacity enhancement is achievable thanks to spatial division multiple access (SDMA) whereby several user terminals communicate simultaneously with the access point. This combination of MIMO and SDMA is often referred to as multi-user MIMO (MU-MIMO). This technique is being adopted in emerging standards such as LTE-Advanced [1] and IEEE 802.11ac [2]. In MU-MIMO, there is a double level of interference within a cell: multi-user interference (MUI) and multi-stream interference (MSI). In downlink MU-MIMO, since the AP transmit to several users at the same time, the signals transmitted to a given user must be suitably precoded so that it creates little or no interference to all other users.

This issue has been identified and addressed by some authors. In [3] and [4], the precoding completely eliminates the MUI by using a zero-forcing criterion for the MUI. Although this technique, also referred to as block-diagonalization, conveniently decouples the MUI mitigation from the MSI mitigation, it suffers from some disadvantage in terms of performance and loss of degrees of freedom. Other authors have resorted to approaches whereby the precoder and decoder are optimally designed in order to optimize some criterion such as maximizing the output signal-to-interference plus-noise ratio (SINR) [5] or maximizing the overall system capacity under a diagonalization constraint [6]. In these schemes, the solution can only be computed iteratively due to the coupled nature of the corresponding optimization problem; the complexity of such solutions is also very high. In a third approach, the precoder for each user is computed separately so as to maximize a cost function that is, for each user, the ratio of the desired received signal strength over the total interference created onto other users. This has been proposed in the MU-MIMO context in [7] and in the cooperative transmission context in [8]. The optimization problem in [7] is solved by finding the solution of a generalized Rayleigh quotient. Although this solution provides satisfactory performance, there is still room for improvement because the solution is optimal for the MUI mitigation but is not optimized for the joint MUI-MSI mitigation.

The proposed solution is to compute a precoder that is jointly optimized to minimize the MUI and the MSI according to an MSE minimization criterion. Our approach has the following distinctive features: the precoder can be computed for each user directly (i.e. without iterations); the multi-user precoder optimization exploits the knowledge of the multi-stream precoder that will be used; the knowledge of the post-filter (MIMO equalizer at the RX side) is not needed at the TX side. These three features result in excellent performances.

The structure of the paper is as follows: Section II introduces the system model. In Section III, we describe the single user precoding scheme that will be part of the full precoder. Section IV provides the details of the joint MU and SDM precoder. Section V provides simulation results and Section VI concludes this paper.

Notational conventions: vectors and matrices are denoted by a single and double under-bar respectively ($\underline{a}$ and $\underline{\underline{A}}$); the superscript * is used to indicate the complex conjugate (a^*); the superscripts ^{T} and ^{H} denote the matrix transpose and complex conjugate transpose respectively ($\underline{\underline{A}}^T$ and $\underline{\underline{A}}^H$); the superscript † denotes the pseudo-inverse ($\underline{\underline{A}}^\dagger$); $\|\underline{\underline{A}}\|_F$ denotes the Frobenius norm of $\underline{\underline{A}}$; $\mathrm{tr}\left(\underline{\underline{A}}\right)$ denotes the trace of $\underline{\underline{A}}$.

II. System model

We consider a downlink transmission in a single cell with the following devices: one base station (BS) or access point

(AP) with T transmit antennas and U simultaneous user terminals (UT) each having R_u antennas. The AP simultaneously transmits S symbol streams towards the U UTs: S_1 streams towards UT_1, S_2 streams towards UT_2, and so on ($S = \sum_{u=1}^{U} S_u$). Each user terminal UT_u receives the mixture of all symbol streams and attempts to recover its own S_u symbols streams. To this end, each UT must be fitted with a number of antennas R_u greater than or equal to S_u ($R_u \geq S_u$). The total number of receive antennas, summed over all users, is $R = \sum_{u=1}^{U} R_u$. This transmission scheme exploits both SDM and SDMA: SDMA achieves the user separation and SDM achieves the per-user stream separation. At each time instant k, the AP transmits the signal vector $\underline{s}(k)$ obtained by precoding the symbol vector $\underline{x}(k)$ (itself resulting from stacking the U symbol vectors $\underline{x}_u(k)$), with the matrix $\underline{\underline{F}}$ as follows:

$$\begin{aligned} \underline{s}(k) &= [s_1(k) \ \dots \ s_T(k)]^T \ = \ \underline{\underline{F}}\,\underline{x}(k) && (1)\\ \underline{x}(k) &= [\underline{x}_1(k)^T \ \dots \ \underline{x}_U(k)^T]^T && (2)\\ \underline{x}_u(k) &= [x_1(k) \ \dots \ x_{S_U}(k)]^T, \qquad u = 1 \dots U. && (3) \end{aligned}$$

In the sequel, we will assume that the transmitted symbols in every stream have unit variance ($E[x_i(k)x_i^*(k)] = 1$) and that the precoding matrix $\underline{\underline{F}}$ has unit Frobenius norm ($\|F\|_F^2 = 1$); these two conditions guarantee that the total average transmit power is always equal to 1, regardless of the number of users and streams per user. Assuming flat fading, the signal received by the uth terminal can be written as follows:

$$\underline{r}_u(k) = \underline{\underline{H}}_u\,\underline{s}(k) + \underline{n}_u(k) \tag{4}$$

where $\underline{\underline{H}}_u$ is the MIMO sub-channel from the AP to user u and $\underline{n}_u$ is a vector of additive complex white gaussian noise components with variance $\sigma_{n_u}^2$. The full channel matrix $\underline{\underline{H}}$ is obtained by stacking the U sub-channels:

$$\underline{\underline{H}} = [\underline{\underline{H}}_1^T \ \dots \ \underline{\underline{H}}_U^T]^T \tag{5}$$

Each user applies a linear post-filter $\underline{\underline{G}}_u$ to recover an estimate of the transmitted symbol vector $\underline{x}_u(k)$:

$$\begin{aligned} \hat{\underline{x}}_u &= \underline{\underline{G}}_u\,\underline{r}_u(k) && (6)\\ &= \underline{\underline{G}}_u\,\underline{\underline{H}}_u\,\underline{\underline{F}}\,\underline{x}(k) + \underline{\underline{G}}_u\,\underline{n}_u(k). && (7) \end{aligned}$$

Note that $\underline{x}(k)$ at the righthand side of (7) contains the symbols of all U users; hence, MUI can cause severe signal-to-noise ratio degradation if not properly dealt with by the precoding matrix $\underline{\underline{F}}$. The MU-MIMO matrices, together with their dimensions, are illustrated in Fig. 1 (the factorization $\underline{\underline{F}} = \underline{\underline{N}}\,\underline{\underline{E}}$ will be detailed in the next sub-section). From this point, we will drop the time parameter k for the sake of conciseness.

It is interesting to see that the precoding matrix $\underline{\underline{F}}$ can be split into two matrices, $\underline{\underline{N}}$ and $\underline{\underline{E}}$ which represents the MU separation precoder and the per user SDM precoder, respectively:

$$\underline{\underline{F}}\,\underline{x} = \underline{\underline{N}}\,\underline{\underline{E}}\,\underline{x} = \sum_{u=1}^{U} \underline{\underline{N}}_u\,\underline{\underline{E}}_u\,\underline{x}_u \tag{8}$$

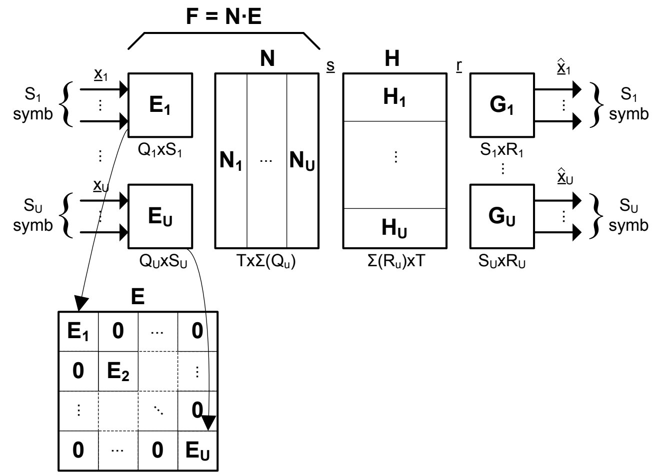

Fig. 1. Matrices of the downlink MU-MIMO System.

where $\underline{\underline{N}}_u$ and $\underline{\underline{N}}$ have dimension $T \times Q_u$ and $T \times \sum_{u=1}^{U} Q_u$, respectively, and $\underline{\underline{E}}$ is a block-diagonal matrix, whose blocks are the U SDM precoders. In both $\underline{\underline{N}}_u$ and $\underline{\underline{E}}_u$, the dimension Q_u is defined as follows:

$$Q_u = T - \sum_{k=1, k \neq u}^{U} R_u \geq S_u, \qquad u = 1 \dots U, \tag{9}$$

which states that the available degrees of freedom for a given user is Q_u and that this number must be greater than or equal to S_u, for all users u. This sets the requirement on the values of the number of streams and transmit and receive antennas (this must be verified for $u = 1 \dots U$). Note that we will ensure that $\| \underline{\underline{N}}_u\,\underline{\underline{E}}_u \|_F^2 = S_u/S$ so that each user receives an amount of power proportional to its number of streams and that the total average transmit power, summed over all streams and users, is unity. In the sequel, we will refer to $\underline{\underline{N}}_u$ as the *MU precoder* and to $\underline{\underline{E}}_u$ as the *SDM precoder*. In [4], $\underline{\underline{N}}$ is designed to block-diagonalize the channel, which already provides good performance. The main merit of the (zero-forcing) block-diagonalization design is to allow to completely separate the multi-user separation from the SDM precoding. A better solution, from the multi-user point of view, is detailed in [7] where, for each user separately, the columns of $\underline{\underline{N}}_u$ are computed as the generalized eigenvectors of a generalized Rayleigh quotient. In this quotient for user u, the numerator represents the signal power received by user u whereas the denominator contains the sum of the noise power and the interference caused to all the other $U-1$ users. Unfortunately, this solution does not include per se an SDM precoder for user u, which results in a performance penalty when the SDM precoder is computed on top of the generalized eigenvector solution. Our solution, detailed in Section IV, also builds on the precoder design for each user separately as in [7] or [8] to avoid an iterative computation, but optimizes the MU precoder taking the SDM precoder into account. This improved SDM precoding is first detailed in Section III.

III. PER USER SDM PRECODING

In a single-user MIMO (SU-MIMO) system with SDM, transmitter precoding has been shown to provide some performance improvement. The transmit and receive beamforming matrices can be optimized according to several criteria: max sum-capacity, minimum mean squared error (MSE), minimum bit error rate (BER), etc.. ([9], Chapter 6). A constraint on the transmit power is usually imposed such that the precoder keeps the total transmit power (summed over all TX antennas) constant. Criteria based on capacity are appealing because they allow to make the best possible use of the channel. However, they are difficult to implement because they assume gaussian symbols and most often different "loading" of the channel modes. For these reasons, practical systems (such as the one being adopted in [2]) usually require to transmit the same constellation on all streams of a given user. Therefore, we focus our attention on the joint TX-RX Minimum Mean Square Error (MMSE) optimization: for a given channel $\underline{\underline{H}}$ with singular value decomposition (SVD) $\underline{\underline{H}} = \underline{\underline{U}}\,\underline{\underline{\Sigma}}\,\underline{\underline{V}}^H$, the solution is given by the precoder $\underline{\underline{E}}$ and post-filter $\underline{\underline{G}}$ [10]:

$$\underline{\underline{E}} = \underline{\underline{V}}\,\underline{\underline{\Sigma}}_t \tag{10}$$

$$\underline{\underline{\Sigma}}_t^2 = \left(\frac{\sigma_n}{\sqrt{\lambda}}\,\underline{\underline{\Sigma}}^{-1} - \sigma_n^2\,\underline{\underline{\Sigma}}^{-2}\right)_+ \tag{11}$$

$$\underline{\underline{G}} = \frac{\sqrt{\lambda}}{\sigma_n}\,\underline{\underline{\Sigma}}_t\,\underline{\underline{U}}^H \tag{12}$$

where σ_n is the receiver noise and $(.)_+$ indicates that only the non-negative values are acceptable. This solution is in fact an inverse water-filling strategy. At moderate to high SNR (the second term vanishes in (11)), it tends to

$$\underline{\underline{\Sigma}}_t = \frac{1}{\sqrt{\mathrm{tr}\,(\Sigma^{-1})}}\,\Sigma^{-1/2}. \tag{13}$$

Simulations (not shown here) of the BER performance of the joint TX-RX MMSE and its simplification (13) over a flat fading Rayleigh channel indicate that the simplified scheme comes very close to the exact solution (10 to 12). Because there is little loss of performance with the simplified solution (13), we will use it for the per user SDM precoding in the derivation of the complete multi-user precoding in Section IV. We will refer to the power allocation as in (10) and (13) as the *inverse trace* precoding. Its associated post-filter $\underline{\underline{G}}$ can be shown to be:

$$\underline{\underline{G}} = \sqrt{\mathrm{tr}\,(\Sigma^{-1})}\,\Sigma^{-1/2}\,\underline{\underline{U}}^H. \tag{14}$$

It is easy to verify that (13) guarantees $\|\underline{\underline{E}}\|_F^2 = \|\underline{\underline{V}}\,\underline{\underline{\Sigma}}_t\|_F^2 = 1$ and, combined with (14), results in a total MSE equal to the trace of the error auto-correlation matrix $\underline{\underline{R}}_{ee}$:

$$\mathrm{tr}\left(\underline{\underline{R}}_{ee}\right) = \sigma_n^2\,\mathrm{tr}\left(\underline{\underline{\Sigma}}^{-1}\right)^2. \tag{15}$$

IV. JOINT MULTI-USER AND SDM PRECODING

A. Multi-user Precoding

In order to develop our MU precoder (i.e. matrix $\underline{\underline{N}}_u$) for user u, it is necessary to express $\underline{\underline{N}}_u$ in its most general way as a linear combination of vectors from an orthonormal basis. We select on purpose this orthonormal basis as consisting of two parts: the first part consist of the null space of $\underline{\underline{H}}_u^C$, which is the "complementary" channel of user u. $\underline{\underline{H}}_u^C$ is obtained by removing from $\underline{\underline{H}}$ the R_u rows corresponding to user u (so $\underline{\underline{H}}_u^C$ has $\sum_{k=1,k\neq u}^{U} R_k = R - R_u$ rows). The second part is made of unitary vectors spanning the range space of $\underline{\underline{H}}_u^C$. The null space and range space bases $\underline{\underline{B}}_u^N$ and $\underline{\underline{B}}_u^S$ of $\underline{\underline{H}}_u^C$ are made of the right singular vectors of $\underline{\underline{H}}_u^C$ corresponding to the zero and non-zero singular values of $\underline{\underline{H}}_u^C$, respectively. Eventually, we can express the columns of the MU precoder for user u as linear combinations of vectors from the two orthonormal bases $\underline{\underline{B}}_u^N$ and $\underline{\underline{B}}_u^S$ as follows:

$$\underline{\underline{N}}_u = \underline{\underline{B}}_u^N\,\underline{\underline{D}}_u + \underline{\underline{B}}_u^S\,\underline{\underline{M}}_u \tag{16}$$

where $\underline{\underline{D}}_u$ and $\underline{\underline{M}}_u$ allow to span the range of the bases and have dimensions $Q_u \times R$ and $(T - Q_u) \times R$. The reason for expressing $\underline{\underline{N}}_u$ in this way becomes apparent when we apply the precoder (16) to the full channel $\underline{\underline{H}}$:

$$\underline{\underline{H}}\,\underline{\underline{N}}_u = \begin{bmatrix} \underline{\underline{H}}_u \\ \underline{\underline{H}}_u^C \end{bmatrix} (\underline{\underline{B}}_u^N \underline{\underline{D}}_u + \underline{\underline{B}}_u^S \underline{\underline{M}}_u) \tag{17}$$

$$= \begin{bmatrix} \underline{\underline{H}}_u\,(\underline{\underline{B}}_u^N \underline{\underline{D}}_u + \underline{\underline{B}}_u^S \underline{\underline{M}}_u) \\ \underline{\underline{H}}_u^C\,\underline{\underline{B}}_u^S \underline{\underline{M}}_u \end{bmatrix}. \tag{18}$$

We observe in (18) that there are two parts: the top part for the given user (user u) and the second part for the interference to all users other than user u. In the interference part, there is only one term because the basis $\underline{\underline{B}}_u^N$ is orthogonal to $\underline{\underline{H}}_u^C$ by definition. Hence, only matrix $\underline{\underline{M}}_u$ controls the interference. On the other hand, both matrices $\underline{\underline{D}}_u$ and $\underline{\underline{M}}_u$ contribute to the desired desired part. This is the key idea of this paper: matrix $\underline{\underline{M}}_u$, although being the sole contributor to MUI, can also be optimized to increase the SDM performance through its combination with matrix $\underline{\underline{D}}_u$ in the top part of (18).

We now describe in detail our approach to the design of the MU precoder for user u, taking the SDM precoder for user u into account. It must satisfy the following criteria:

1) Matrix $\underline{\underline{M}}_u$ must be optimized to contribute a minimal noise plus interference term in the bottom part of (18). Similarly to [7], the interference term is expressed as:

$$\mathrm{tr}\left((\underline{\underline{H}}_u^C\,\underline{\underline{B}}_u^S\,\underline{\underline{M}}_u)^H\,\underline{\underline{H}}_u^C\,\underline{\underline{B}}_u^S\,\underline{\underline{M}}_u\right) = \frac{R - R_u}{U - 1}\sigma_n^2. \tag{19}$$

The product inside the trace must have all its diagonal terms equal so that the interference is equally spread across all streams other than those of user u. The factor $R - R_u$ accounts for the fact that the interference is generated on the $R - R_u$ antennas of the other users and the division by $U - 1$ accounts for the fact that $U - 1$ users will contribute to the interference.

2) We want to ensure that the two contributions to the desired signal optimize the single user SDM precoder for user u. From Section III, we know that $\underline{\underline{D}}_u$ and $\underline{\underline{M}}_u$ must be optimized so that the total MSE (15) is

minimized. Defining $\underline{\underline{\tilde{H}}}_u = \underline{\underline{H}}_u \, \underline{\underline{N}}_u = \underline{\underline{\tilde{U}}}_u \, \underline{\underline{\tilde{\Sigma}}}_u \, \underline{\underline{\tilde{V}}}_u^H$, the optimization becomes

$$\min_{\underline{\underline{D}}_u, \underline{\underline{M}}_u} \text{tr}\left(\underline{\underline{\tilde{\Sigma}}}_u^{-1}\right). \tag{20}$$

3) The MU precoder must have unit power gain: $\text{tr}\left(\underline{\underline{N}}_u^H \, \underline{\underline{N}}_u\right) = 1$.

The second optimization criterion is difficult to solve at first sight. However, exploiting the following lemma, we will derive an elegant solution.

Lemma 1: Given two square matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$, their product $\underline{\underline{C}} = \underline{\underline{A}} \, \underline{\underline{B}}$ and their SVDs:

$$\begin{aligned} \underline{\underline{A}} &= \underline{\underline{U}}_A \, \underline{\underline{\Sigma}}_A \, \underline{\underline{V}}_A^H && (21)\\ \underline{\underline{B}} &= \underline{\underline{U}}_B \, \underline{\underline{\Sigma}}_B \, \underline{\underline{V}}_B^H && (22)\\ \underline{\underline{C}} &= \underline{\underline{U}}_C \, \underline{\underline{\Sigma}}_C \, \underline{\underline{V}}_C^H = \underline{\underline{A}} \, \underline{\underline{B}}, && (23) \end{aligned}$$

the matrix $\underline{\underline{B}}$ that minimizes $\text{tr}\left(\underline{\underline{\Sigma}}_C^{-1}\right)$ constrained to $\|B\|_F^2 = 1$ is such that

$$\begin{aligned} \underline{\underline{U}}_B &= \underline{\underline{V}}_A && (24)\\ \underline{\underline{\Sigma}}_B &= \frac{1}{\sqrt{\text{tr}\left(\underline{\underline{\Sigma}}_A^{-2/3}\right)}} \underline{\underline{\Sigma}}_A^{-1/3} \qquad \blacksquare && (25) \end{aligned}$$

Lemma 1 teaches us that, to minimize the trace of $\underline{\underline{\Sigma}}_C^{-1}$, the left singular vectors of $\underline{\underline{B}}$ must exactly compensate the right singular vectors of $\underline{\underline{A}}$ (so that $\underline{\underline{V}}_A^H \, \underline{\underline{U}}_B = \underline{\underline{I}}$) and that the singular values of $\underline{\underline{B}}$ must be computed according to (25). Building on that, we can optimize (20) by "aligning" the two contributions to the desired signal (top part of (18)). To this end, we rewrite the desired contribution as follows:

$$\begin{aligned} &\underline{\underline{H}}_u \, \underline{\underline{N}}_u \\ &= \underline{\underline{H}}_u \, (\underline{\underline{B}}_u^N \underline{\underline{D}}_u + \underline{\underline{B}}_u^S \underline{\underline{M}}_u) \\ &= \underline{\underline{H}}_u \, \underline{\underline{B}}_u^N \underline{\underline{D}}_u + \underline{\underline{H}}_u \, \underline{\underline{B}}_u^S \, (\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S)^\dagger \, \underline{\underline{H}}_u \, \underline{\underline{B}}_u^N \underline{\underline{\tilde{D}}}_u && (26)\\ &= \underline{\underline{H}}_u \, \underline{\underline{B}}_u^N \, (\underline{\underline{D}}_u + \underline{\underline{\tilde{D}}}_u) && (27) \end{aligned}$$

which means that we have redefined the MU precoder $\underline{\underline{N}}_u$ as[1]:

$$\underline{\underline{N}}_u = \underline{\underline{B}}_u^N \underline{\underline{D}}_u + \underline{\underline{B}}_u^S \, (\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S)^\dagger \, \underline{\underline{H}}_u \, \underline{\underline{B}}_u^N \underline{\underline{\tilde{D}}}_u. \tag{28}$$

Lemma 1 can now be exploited for the product in (27), where $\underline{\underline{H}}_u \underline{\underline{B}}_u^N$ and $(\underline{\underline{D}}_u + \underline{\underline{\tilde{D}}}_u)$ take the role of A and B, respectively, in Lemma 1. We define the SVD of $\underline{\underline{H}}_u \, \underline{\underline{B}}_u^N$

$$\underline{\underline{H}}_u \, \underline{\underline{B}}_u^N = \underline{\underline{U}}_u^D \, \underline{\underline{\Sigma}}_u^D \, (\underline{\underline{V}}_u^D)^H \tag{29}$$

(in which we only need $\underline{\underline{V}}_u^D$) and express $\underline{\underline{D}}_u$ and $\underline{\underline{\tilde{D}}}_u$ as

$$\begin{aligned} \underline{\underline{D}}_u &= \underline{\underline{V}}_u^D \, \underline{\underline{\Lambda}}_1 && (30)\\ \underline{\underline{\tilde{D}}}_u &= \underline{\underline{V}}_u^D \, \underline{\underline{\Lambda}}_2 && (31) \end{aligned}$$

so that $\underline{\underline{D}}_u$ and $\underline{\underline{\tilde{D}}}_u$ have the same left singular vectors and contribute optimally to minimize the inverse trace criterion for $\underline{\underline{H}}_u \, \underline{\underline{N}}_u$ in (27). Matrices $\underline{\underline{\Lambda}}_1$ and $\underline{\underline{\Lambda}}_2$ are the new (diagonal) unknowns.

[1]Note that $\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S$ is a "fat" matrix so that the product $\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S \, (\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S)^\dagger$ in (26) is always equal to an identity matrix

Based on these definitions, we can now review and solve the three criteria listed in the beginning of this section:

1) Using (31) and (26), Equation (19) becomes

$$\text{tr}\left((\underline{\underline{H}}_u^C \, \underline{\underline{B}}_u^S \, \underline{\underline{M}}_u)^H \, \underline{\underline{H}}_u^C \, \underline{\underline{B}}_u^S \, \underline{\underline{M}}_u\right) \tag{32}$$

$$= \text{tr}\left(\underline{\underline{\Lambda}}_2^2 \, \underline{\underline{\tilde{X}}}\right) = \frac{R - R_u}{U - 1} \sigma_n^2 \tag{33}$$

where $\underline{\underline{\tilde{X}}}$ is defined as follows:

$$\begin{aligned} \underline{\underline{X}} &= \underline{\underline{H}}_u^C \, \underline{\underline{B}}_u^S \, (\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S)^\dagger \, \underline{\underline{H}}_u \, \underline{\underline{B}}_u^N \, \underline{\underline{V}}_u^D && (34)\\ \underline{\underline{\tilde{X}}} &= \underline{\underline{X}}^H \, \underline{\underline{X}}. && (35) \end{aligned}$$

Additionally, since all $R - R_u$ diagonal terms inside the trace of (33) must have equal power, $\underline{\underline{\Lambda}}_2$ must be equal to:

$$\underline{\underline{\Lambda}}_2 = \frac{\sigma_n}{\sqrt{U-1}} \text{diag}(\underline{\underline{\tilde{X}}})^{-1/2}, \tag{36}$$

where $\text{diag}(\underline{\underline{\tilde{X}}})$ is a diagonal matrix having its diagonal equal to the main diagonal of $\underline{\underline{\tilde{X}}}$.

2) Using Lemma 1, (20) is minimized by $\underline{\underline{\Lambda}}_1$ and $\underline{\underline{\Lambda}}_2$ that satisfy

$$\underline{\underline{\Lambda}}_1 + \underline{\underline{\Lambda}}_2 = \frac{\alpha}{\sqrt{\text{tr}\left((\underline{\underline{\Sigma}}_u^D)^{-2/3}\right)}} (\underline{\underline{\Sigma}}_u^D)^{-1/3} \tag{37}$$

where we have used (25) with $\underline{\underline{\Sigma}}_A$ replaced by $\underline{\underline{\Sigma}}_u^D$ defined in (29). The scaling constant α must be introduced because the sum $(\underline{\underline{D}}_u + \underline{\underline{\tilde{D}}}_u)$ in (27) is not constrained to have a Frobenius norm equal to 1 as in Lemma 1.

3) Finally, the normalization of $\underline{\underline{N}}_u$ translates into

$$\text{tr}\left(\underline{\underline{N}}_u^H \, \underline{\underline{N}}_u\right) = \text{tr}\left(\underline{\underline{\Lambda}}_1^2\right) + \text{tr}\left(\underline{\underline{\Lambda}}_2^2 \, \underline{\underline{\tilde{Y}}}\right) = 1 \tag{38}$$

where $\underline{\underline{\tilde{Y}}}$ is defined from (26) as follows:

$$\begin{aligned} \underline{\underline{Y}} &= (\underline{\underline{H}}_u \, \underline{\underline{B}}_u^S)^\dagger \, \underline{\underline{H}}_u \, \underline{\underline{B}}_u^N \, \underline{\underline{V}}_u^D && (39)\\ \underline{\underline{\tilde{Y}}} &= \underline{\underline{Y}}^H \, \underline{\underline{Y}}. && (40) \end{aligned}$$

The three unknowns are $\underline{\underline{\Lambda}}_1$, $\underline{\underline{\Lambda}}_2$ and α. $\underline{\underline{\Lambda}}_2$ is given directly by (36). Since $\underline{\underline{\Sigma}}_u^D$ is known in (37), solving for $\underline{\underline{\Lambda}}_1$ and α from (37) and (38) is not shown here but is trivial (α is the only positive root of a second order polynomial). Inserting $\underline{\underline{\Lambda}}_1$ and $\underline{\underline{\Lambda}}_2$ in (30) and (31) and then $\underline{\underline{D}}_u$ and $\underline{\underline{\tilde{D}}}_u$ in (28) provides the MU precoder $\underline{\underline{N}}_u$.

Very importantly, we have achieved the MU separation with $\underline{\underline{N}}_u$ but we have in the same process also optimized $\underline{\underline{N}}_u$ such that the resulting matrix $\underline{\underline{H}}_u \underline{\underline{N}}_u$ will have a better performance when applying the SDM precoding on top.

B. SDM Precoding

Now that the MU precoder $\underline{\underline{N}}_u$ is known, calculating the SDM precoder is straightforward using our inverse trace criterion (10) and (13). The channel of user u, including the MU precoder for user u is $\underline{\underline{H}}_u \, \underline{\underline{N}}_u$. Defining the SVD of $\underline{\underline{H}}_u \, \underline{\underline{N}}_u$ as

$$\underline{\underline{H}}_u \, \underline{\underline{N}}_u = \underline{\underline{U}}_u^R \, \underline{\underline{\Sigma}}_u^R \, (\underline{\underline{V}}_u^R)^H, \tag{41}$$

we use $\underline{\underline{\Sigma}}_u^R$ and $\underline{\underline{V}}_u^R$ to calculate directly the SDM precoder from (10) and (13), yielding $\underline{\underline{E}}_u$ and $\underline{\underline{\Sigma}}_{tu}$. The full precoder $\underline{\underline{F}}_u$, including the MU and SDM precoders, is then

$$\underline{\underline{F}}_u = \underline{\underline{N}}_u \, \underline{\underline{V}}_u^R \, \underline{\underline{\Sigma}}_{tu} = \underline{\underline{N}}_u \, \underline{\underline{E}}_u \tag{42}$$

and the post-filter $\underline{\underline{G}}_u$ is calculated with (14) (the post-filter $\underline{\underline{G}}_u$ is not needed at the TX side).

V. Simulation Results

To illustrate the performance of the proposed MU-MIMO scheme, we consider a multi-user MIMO situation where a BS, equipped with 9 or 10 antennas, is communicating with three 3-antenna UTs. Hence, this set-up has 3x3=9 simultaneous symbol streams in parallel. The system is fully loaded when the AP has 9 antennas. In the second case, when the AP has 10 antennas, there is one available degree of freedom for increased diversity. Each input bit streams at the AP consists of 12000 bits and is 16-QAM modulated, resulting in 3000 symbols per stream. 2000 channel realizations were generated. The entries of the channel matrix $\underline{\underline{H}}$ are zero mean i.i.d. gaussian random variables with variance 1 and are generated independently for each channel realization. So the MIMO channel is Rayleigh fading. The total transmit power per symbol period across all antennas is normalized to 1 (this is guaranteed by the design described in Section IV).

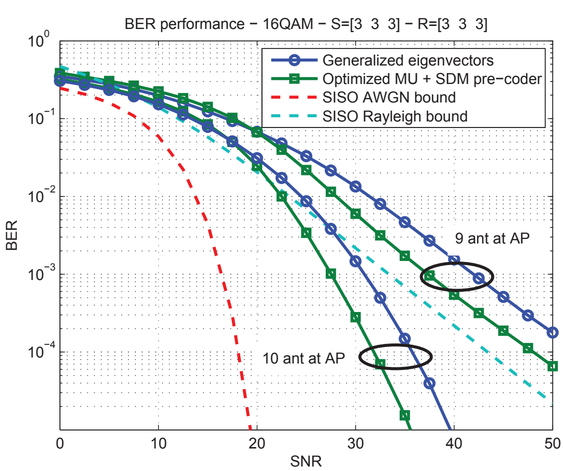

Fig. 2. Performance comparison of MU-MIMO designs. The generalized eigenvector design is with circular markers, our design with square markers. Simulation conditions are nine or ten TX antennas, three 3-antenna terminals, 16-QAM, uncoded, Rayleigh channels.

Fig. 2 shows the performance of the proposed system for two designs. The first design is based on [7], using the generalized eigenvectors as the MU precoder and is shown with circular markers. The second design is based on our optimized joint MU-SDM precoder and is shown with square markers. The better performance of our design is clearly visible both at full system load (9 antennas at the AP) and at partial system load (10 antennas at the AP).

VI. Conclusion

We have proposed a linear precoding scheme for downlink MU-MIMO that jointly optimizes the MU separation (SDMA) and the SDM precoding. We used a criterion that allows to perform this optimization for each user independently so that no iterations are necessary. The resulting precoder has been shown by simulation to outperform the precoder based on the generalized eigenvectors because the latter does not optimize the SDM precoding on top.

The key idea for this design was to identify analytically - and for each user separately - the contributions of the precoder vectors to both the MUI and MSI and, then, to minimize jointly those contributions.

This scheme is very attractive for downlink MU-MIMO (or MIMO-SDMA). Extension to frequency selective channels is straightforward with multi-carrier techniques such as OFDM since our scheme can be applied per sub-carrier. As such, this precoding scheme is suitable for emerging standards such as LTE-Advanced and IEEE 802.11ac.

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