SEQUENTIAL REMOTE SOURCE CODING IN WIRELESS ACOUSTIC SENSOR NETWORKS

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ABSTRACT
In this paper, we consider source coding in wireless acoustic sensor networks (WASNs). A WASN is a network of sensors, where each sensor is equipped with a microphone and thereby forming a distributed microphone array. In addition to a microphone, the sensors communicate via radio channels. Thus, a sensor observes an acoustic noisy measurement in addition to receiving a coded correlated noisy measurement from a neighboring sensor node. Given a set of sensors, the problem is to efficiently encode the noisy observations and/or re-encode the received coded noisy observations. Furthermore, one seeks a routing strategy that e.g., minimizes the sum rate subject to some distortion constraints. Towards that end, we define a new source coding concept related to WASNs which we denote sequential remote source coding, and present an achievable rate region in the quadratic Gaussian case.

1. INTRODUCTION
Wireless sensor networks (WSNs) and microphone arrays have both (separately) received increasing attention. Their combination, which is denoted wireless acoustic sensor networks (WASNs), forms a challenging field. In the following, we provide a brief survey of results related to WASNs and source coding within WASNs.

1.1. Wireless Sensor Networks
Wireless sensors are low power measurement devices, which are equipped with a radio transmitter and receiver, and in addition they might have a digital signal processing unit and some memory. Due to the low power constraints, the sensors cannot individually transmit their measurements over great distances, and it is therefore necessary to form sensor networks containing a number of wireless sensors. A routing protocol for WASNs describes the path and methodology used when conveying data via several sensors to a base station.

1.2. Microphone Arrays
A microphone array consists of a set of spatially distributed microphones. The main idea is to be able to achieve diversity, i.e., obtain several spatio-temporal versions of a noisy sound field. This is useful for e.g., source localization and tracking, as well as speech and audio enhancement, speaker separation, speaker identification etc.

It was recently shown that the underlying sound field in any spatial position may be accurately predicted by knowing the sound field in a certain number of positions [1].

1.3. Wireless Acoustic Sensor Networks
In a WASN each sensor node is equipped with (at least) one microphone to capture acoustic signals. In addition, a sensor node is equipped with a radio receiver and radio transmitter. Thus, a node may simultaneously receive acoustic information while receiving or transmitting via radio communication. We refer to [2] for a survey of applications of WASNs.

1.4. Source Coding in Wireless Sensor Networks
The measurements obtained by a wireless sensor are usually noisy due to the fact that the acquisition must be of finite accuracy and the fact that the signal is often corrupted by noise prior to being observed. Thus, instead of coding the source X one has to encode the observed noisy source, say X + N. This problem is well known in the information theoretic community and is usually referred to as remote source coding [3–5].

Let X and Y be two correlated sources, where X is known only at the encoder and Y is known only at the decoder. If the joint statistics of (X, Y) are known at both the encoder and the decoder, then it is possible to leverage on results from distributed source coding, and thereby encode X at a rate R, which is sandwiched by RX(D) ≥ R ≥ RX|Y(D), where RX(D) denotes Shannon’s rate-distortion function (RDF) and RX|Y(D) denotes Shannon’s conditional RDF [6–8]. Interestingly, it can be shown that in the quadratic Gaussian case, one can achieve R = RX|Y(D) [7].

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There might also be loudspeakers on the sensors, which could be used for acoustic communications. However, we do not explore this idea here.
The case where sensor node $i$ observes $Y_i = X + N_i$, $i = 1, \ldots, M$, and knows the joint statistics of $(Y_1, \ldots, Y_M)$, but does not know the actual realizations of $Y_j$, $j \neq i$, is usually referred to as the CEO problem [9, 10]. In this problem, the observations are separately encoded and transmitted to a common decoder and has been treated in several works related to wireless sensor networks, cf. [11, 12].

In [13], source coding was considered jointly with routing and power allocation for WSNs and the interaction of routing and source coding in multi-hop networks was addressed in [14,15]. Energy-efficient source coding in WSNs was considered in [16, 17].

Recently, ideas from compressed sensing have been applied to speech and audio in [18] and to WSNs in [19, 20]. There has also been some recent works regarding source coding especially for WASNs. In particular, the multi-terminal rate-distortion function for a Gaussian acoustic field was found in [21], and in [22] it was shown that oversampling and A/D conversion is rate beneficial in WASNs.

1.5. Scope of Paper

In this paper, we are interested in coding the noisy measurements $Y_i = X + N_i$, $i = 1, \ldots, M$, obtained in a WASN having at least $M$ nodes. We assume that a given sensor is not able to directly communicate with the central server. In particular, we assume that the coded measurement from sensor node $i$ needs to be transmitted to sensor node $i + 1$. Thus, each sensor node receives a noisy coded version, say $\tilde{Y}_{i-1}$, of the sound source from a neighboring sensor node as well as its own noisy measurement $Y_i = X + N_i$. The problem at hand is therefore to determine an efficient coding strategy for sensor node $i$ to jointly encode the two correlated sources $(Y_i, \tilde{Y}_{i-1})$ subject to a distortion constraint $D_i$. Towards that end, we define the concept of sequential remote source coding, and present an achievable rate region in the quadratic Gaussian case. The rate region is tight in the case where no side information is available. We further show how to find a rate-efficient routing in a WASN by minimizing the sum rate over all nodes given a set of node distortion constraints.

2. GENERAL PROBLEM FORMULATION

Let $X_i \in \mathbb{R}$ be the $i$th source, where $i = 1, \ldots, M$. Since we have several sources broadcasting simultaneously as well as background noise, the received source $Y_j \in \mathbb{R}$ at sensor $j$ is given by:

$$Y_j = H_j^T \tilde{X} + N_j, \quad j = 1, \ldots, S,$$

$$= H_{j,i} X_i + \sum_{k \neq i} H_{j,k} X_k + N_j,$$

where $H_j \in \mathbb{R}^M$ denotes the acoustic channel mixing vector between the $M$ sources and sensor $j$, $H_{j,k}$ is the $k$th element of $H_j$, $N_i \in \mathbb{R}$ denotes the background noise at sensor $j$, and $\tilde{X} \in \mathbb{R}^M$ is the vector composed of the original $M$ independent sources. Each of the $S$ sensors receives a mixed and noisy version of all the $M$ sources, but we are here only interested in one of them, say $X_i$, and treat the remaining sources as noise.

Sensor $j$ needs to encode $Y_j$ and transmit the coded version, say $\hat{Y}_j$, to the gateway. The gateway may be located far away from the sensor node, and it is therefore necessary to use multiple sensor nodes as intermediate relays. However, in addition to act as a relay, the sensor node also has to transmit its own measurement. Thus, the sensors have to decide about a jointly optimal strategy for compressing relay signals and measurements.

3. RESULTS

We begin by introducing a new concept in remote source coding, which we denote sequential remote source coding. First, let $Y_i = X + N_i$ be a noisy observation of $X$ where $N_i$ is independent of $X$. Let $f_i$ be an encoding function. Let $g_i, i = 1, \ldots, M$, be the corresponding decoding functions satisfying $d(X, g_i(f_i)) \leq D_i$, where $d(\cdot, \cdot)$ is the distortion measure, which in this work is the mean squared error. Thus, even though we wish to encode $Y_i$, the distortion is w.r.t. $X$.

Definition 1 (Sequential Remote Source Coding). At node $i$, $Y_i$ is observed, the output of $f_{i-1}$ is received, and the problem is to form $f_i(Y_i, f_{i-1})$, which will be transmitted noiselessly to node $i + 1$.

Remark 1. Note that sequential remote source coding is related to successive refinement, where the main difference is that, at each refinement stage, we do not have access to the original source but a new noisy version of it. In addition, we also allow that $D_i < D_j$ for $j > i$, which does not make any sense in conventional successive refinement.

Definition 2 (Achievable Rates in Sequential Remote Source Coding). Let $\mathcal{D} = \{D_i\}_{i=1}^M$ denote an ordered set of $M$ distortions. A rate tuple $(R_1, \ldots, R_M)$ is said to be sequentially achievable with respect to the distortion tuple $\mathcal{D}$, if there exists a sequence of encoders $f_i, i = 1, \ldots, M$, such that

$$\log_2 |f_i| \leq R_i, \quad i = 1, \ldots, M,$$

where $|f_i|$ denotes the codebook size of $f_i$, and such that

$$d(g_i(f_i), X) \leq D_i, \quad i = 1, \ldots, M.$$

The following theorem provides a set $\mathcal{R} = \{R_i\}_{i=1}^M$ of sequentially achievable rates given the set $\mathcal{D} = \{D_i\}_{i=1}^M$ of distortions, where $(R_i, D_i)$ denotes the rate-distortion pair associated to node $i$.
Theorem 1. Let $X$ be a zero-mean memoryless scalar Gaussian process with variance $\sigma_X^2$ and let the noisy observations be given by $Y_i = X + N_i$, where $X$ is independent of $N_i$, $\forall i$ and $N_i, N_j, i \neq j$ are mutually independent Gaussian sources of variances $\sigma_N^2$. Then, a sequentially achievable rate $R_i$ for the $i$th node at distortion $\frac{\sigma_X^2 \sigma_i^2}{\sigma_X^2 + \sigma_i^2} < D_i \leq \sigma_X^2$ is:

$$R_i(D_i) = \frac{1}{2} \log_2 \left( \frac{\sigma_X^2}{D_i} \right) + \frac{1}{2} \log_2 \left( \frac{\sigma_X^2}{\sigma_X^2 + \sigma_i^2 - \frac{\sigma_X^2 \sigma_i^2}{D_i}} \right),$$

(3)

where $\sigma_i^2 = \sigma_N^2$, and for $i \geq 2$

$$\sigma_i^2 = \frac{1}{\left( \frac{1}{\sigma_{i-1}^2 + \phi_{i-1}} + \frac{1}{\sigma_N^2} \right)},$$

where

$$\phi_i = \left( \frac{\sigma_X^2 + \sigma_i^2}{\sigma_X^2} \right)^{1/2} \frac{1}{2R_i - 1}.$$

Proof. See Section 6.

Remark 2. The general case with correlated noises given by (2) is also considered in Section 6.

Remark 3. If at node $i$, the joint statistics of subsequent nodes are known and no distortion $D_i$ is enforced, it is clear that one can improve upon Theorem 1 by using conventional distributed source coding techniques (e.g., Wyner-Ziv binning). However, if distortion $D_i$ is enforced without the knowledge of additional observations (side information), then it can be shown that Theorem 1 is tight.

For a WASN having $M$ nodes, one might be interested in the distortion at each node, or perhaps only at a select few nodes, e.g., the end node. In the latter case, one aims at minimizing the sum-rate over all nodes, subject to the final distortion at the end node. We put these observations into a Lemma.

Lemma 1. An achievable sum-rate $R$ subject to a subset of distortions $\tilde{D}_i, i \in \ell \subseteq \{1, \ldots, M\}$ and a set of noisy nodes $\{Y_i = X + N_i\}, i = 1, \ldots, M$, is given by

$$R = \min_{\{D_i\}} \sum_{i=1}^M R_i(D_i), \text{ s.t. } \frac{\sigma_X^2 \sigma_i^2}{\sigma_X^2 + \sigma_i^2} < D_i \leq \tilde{D}_i, \ i \in \ell,$$

and $\frac{\sigma_X^2 \sigma_i^2}{\sigma_X^2 + \sigma_i^2} < D_i \leq \sigma_X^2, \ \forall i, \ \text{where } R_i(D_i) \text{ is given by (3).}$

Interestingly, Lemma 1 provides an optimal routing (in terms of an achievable sum-rate) in a WASN as well as the optimal bit distribution subject to a set of node distortion constraints.

4. EXAMPLES

In this section, we illuminate some subtle implications of Theorem 1 and Lemma 1.

Let $Y_1 = X + N_1$ and $Y_2 = X + N_2$, where $\sigma_X^2 = 1$ and let us consider the Gaussian case where we only are concerned about the distortion $D_2$ at sensor 2. In this case, we observe $Y_2$ and we could also receive a coded version of $Y_1$ from sensor 1.

4.1. Example 1

Let $\sigma_{X_1}^2 = 1/4$ and $\sigma_{X_2}^2 = 1/2$. Moreover, let $D_2 = 1/4$. Notice that without coding at sensor 2, the MMSE at sensor 2 is 1/3 and it is therefore not possible to achieve the desired distortion $D_2 = 1/4$ using only sensor 2. However, choosing $D_1 = 1/3$, leads to $R_1 = 1.29$ bits and $R_2 = 2$ bits, which gives $D_2 = 1/4$ as required.

4.2. Example 2

Let $D_2 = 1/3 + 0.00001$ and $\sigma_{X_2}^2 = 1/2$. Then, the desired distortion $D_2$ is achievable by coding only $Y_2$ at a rate $R_2 = 8.01$ bits. However, letting $\sigma_{X_2}^2 = \sigma_{X_1}^2$, then, by choosing e.g., $D_1 = 1/2$, the resulting rates are $R_1 = 1$ bit and $R_2 = 1.58$ bits, which leads to a significantly smaller sum-rate than when only coding $Y_2$.

5. SIMULATION STUDY

Since audio is generally not Gaussian distributed, Theorem 1 provides only an approximation for real-world signals. To illustrate the usefulness of Theorem 1, we consider next a practical setup with a single loudspeaker and two sensors (microphones) and where we only exploit the spatial correlation of the observed signals. The sensors are distanced 16 cm apart, and the loudspeaker is oriented with a -13 degree angle towards the sensors. The source signal emitted from the loudspeaker is a trumpet signal, and the signal $X$ reaching sensor 1 contains delayed and reflected versions of the source. The mixed signal $X$ at sensor 1 will be used as the reference signal that needs to be coded and sent to the base station via sensor 2. Due to measurement noise, the measured signal is $Y_1 = X + N_1$, where $\sigma_{X_1} = 0.01$.

At the second sensor, we measure the acoustic signal $Y_2$, which can be interpreted as $Y_2 \approx X + N_2$, where the variance $\sigma_{X_2}^2$ might not be exactly known at the sensors. In this simulation, we find it to be $\sigma_{X_2}^2 = 1.23$. For comparison, we also measure the performance when the sensor wrongly assumes that $\sigma_{X_2}^2 = 0.5$ (see below). Thus, the minimum distortion due to linear estimation of $X$ given only $Y_2$ is $D_{Y_2}^{\text{min}} = 0.8631$. To achieve this minimum, one would require an infinite bitrate $R_2$ when coding $Y_2$. On the other hand, if we encode $Y_1$ at sensor 1 and send it to sensor 2, then the distortion $D_2$ can be significantly reduced below $D_{Y_2}^{\text{min}}$.
\( \Delta_1 = 1/10 \) denotes the step-size of the quantizer and \( \lfloor \cdot \rfloor \) denotes rounding to nearest integer. Then, at sensor 2, we sweep the step-size \( \Delta_2 \) of the quantizer in steps of 0.1 in the interval \([0.1; 1] \). For each value of \( \Delta_2 \), we form the sufficient statistics \( V_2 \) see (4), which is then quantized into \( U_2 = \lfloor \alpha_2 V_2 / \Delta_2 \rfloor \Delta_2 \) using rate \( R_2 \). The resulting simulated (theoretical) average rate \( R_1 \) at sensor 1 is 5.24 (5.24) bits/sample after entropy coding and the corresponding distortion is \( D_1 = 0.0107 \) (0.0106). The rate \( R_2 \) and distortion \( D_2 \) at sensor 2 is shown in Fig. 1. It can be seen that the simulations using real audio signals are close to the theoretically achievable performance based on Gaussian assumptions.

Node 2: We receive \( U_1 \) from Node 1 and observe \( Y_2 \), thus, we have two noisy observations of \( X \). Let \( \frac{1}{\alpha_1} U_1 = X + N_1 + \frac{1}{\alpha_1} Z_1 = X + N_1 + Z_1' \). Then we have a remote source coding problem with two noisy observations \( X + N_1 + Z_1' \) and \( X + N_2 \). A similar problem was treated in [23] and it was shown that rather than jointly encoding both sources it is enough to consider a single scalar source, say \( V_2 \), which provides a sufficient statistics of \( (Y_2, U_1) \) with respect to \( X \). First let \( \sigma_2^2 \) be given by

\[
\sigma_2^2 = \frac{1}{\frac{1}{\sigma_X^2} + \frac{1}{\sigma_{N_1}^2} + \frac{1}{\sigma_{Z_1'}^2}},
\]

where

\[
\sigma_{Z_1'}^2 = (\sigma_X^2 + \sigma_{N_1}^2) \frac{2^{-2R_1}}{1 - 2^{-2R_1}}.
\]

Then we form \( V_2 \) as the following linearly weighted sum of \( \frac{1}{\alpha_1} U_1 \) and \( Y_2 \):

\[
V_2 = \sigma_2^2 \left( \frac{\alpha_1}{\sigma_X^2 + \sigma_{N_1}^2} U_1 + \frac{Y_2}{\sigma_X^2 + \sigma_{N_1}^2} \right).
\]

(4)

The variance of \( V_2 \) is \( \sigma_{V_2}^2 = \sigma_X^2 + \sigma_{Z_1'}^2 \). Since \( X, Y_2, \) and \( V_2 \) are jointly Gaussian, it can be shown that \( V_2 \) is a sufficient statistics for \( X \), so that \( X \leftrightarrow V_2 \leftrightarrow (U_1, Y_2) \). It was shown in [23], that the remote RDF of a source is the same as the remote RDF for a sufficient statistics of the noisy source. Thus, we will consider coding of \( V_2 \). Towards that end, we form \( \mathbb{E}[X|V_2] = \alpha_2 V_2 \) and encode it into \( U_2 = \alpha_2 V_2 + Z_2 \) using rate \( R_2 \). The Shannon DRF for the source \( \alpha_2 V_2 \) is given by

\[
D_{\alpha_2 V_2} (R_2) = \text{var}(\alpha_2 V_2) 2^{-2R_2} = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_{Z_2}^2} 2^{-2R_2}.
\]

(5)

The MSE due to estimating \( \alpha_2 V_2 \) from \( \alpha_2 V_2 + Z_2 \) is

\[
\text{var}(\alpha_2 V_2) \sigma_{Z_2}^2 / (\text{var}(\alpha_2 V_2) + \sigma_{Z_2}^2),
\]

which when equalized to (5), makes it possible find \( \sigma_{Z_2}^2 \) as a function of \( R_2 \), that is \( \sigma_{Z_2}^2 = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_{Z_2}^2} 2^{-2R_2} \).

Node 3: We receive \( U_2 \) from Node 2 and observe \( Y_2 \). We now form \( V_3 \) such that \( X \leftrightarrow V_3 \leftrightarrow (U_2, Y_3) \). To do this, we first let \( \sigma_3^2 \) be given by

\[
\sigma_3^2 = \frac{1}{\frac{1}{\sigma_X^2 + \sigma_{Z_2}^2} + \frac{1}{\sigma_{N_2}^2}}
\]

and then form \( V_3 \) as \( V_3 = \sigma_3^2 \left( \frac{U_2}{\sigma_X^2 + \sigma_{Z_2}^2} + \frac{Y_3}{\sigma_{N_2}^2} \right) \). The variance of \( V_3 \) is given by \( \sigma_{V_3}^2 = \sigma_X^2 + \sigma_3^2 \). Note: In general, we receive \( U_{i-1} \) and observe \( Y_i \). We now form \( V_i \) such that \( X \leftrightarrow V_i \leftrightarrow (U_{i-1}, Y_i) \). The variance of \( V_i \) is given by \( V_i = \sigma_X^2 + \sigma_{N_i}^2 \) and for \( i > 1 \) we have \( \sigma_i^2 = \sigma_X^2 + \sigma_i^2 \), where \( \sigma_i^2 = \frac{1}{\frac{1}{\sigma_X^2 + \sigma_{N_i}^2} + \frac{1}{\sigma_{Z_i}^2}} \).
where $\sigma^2_i = \sigma^2_{N_i}$ and for $i > 1$, we have

$$
\sigma^2_{N_i} = (\sigma^2_X + \sigma^2_i)^{-2} \frac{2^{2H_2} - 1}{2 - 2^{2H_2}}.
$$

We finally note that in the more general case with correlated noises described by (2), we are interested in obtaining a representation of, say $X_i$. At sensor $j$ we receive $Y_j = H_j X_i + \sum_{k \neq j} H_{j,k} X_k + N_j$, and at sensor $\ell$ we receive $Y_\ell = H_\ell X_i + \sum_{k \neq \ell} H_{\ell,k} X_k + N_\ell$. Here we would form the new variable $\hat{Y} = \frac{1}{\alpha_{i+j} + \alpha_{i+\ell}} (\frac{\alpha_{i+j}}{\alpha_{i+j}} Y_j + \frac{\alpha_{i+\ell}}{\alpha_{i+\ell}} Y_\ell) = X_i + \phi$, where $\phi$ is independent of $X_i$, and where the $\alpha$'s denote the MMSE estimators of $X_i$ given $\frac{1}{\alpha_{i+j}} Y_j, \frac{1}{\alpha_{i+\ell}} Y_\ell$. Then, it can be shown that $\hat{Y}$ is a sufficient statistics of $(Y_j, Y_\ell)$ wrt. $X_i$. \hfill $\square$

7. CONCLUSIONS AND DISCUSSION

We have provided an overview of source coding in WASNs. Moreover, we presented new results for the case where a sensor receives only a single coded observation from a neighboring sensor in addition to its own observation. We showed how to efficiently jointly encode the received and observed sources in the quadratic Gaussian case. It should be noted, that it is straightforward to extend these results to the case where a sensor may receive several coded observations. Future work involves considering vector sources, colored sources, and the use of noise-shaped coding, not affecting the transfer function of the source [24].

8. REFERENCES


