OPTIMIZATION APPROACH TO JOINT CELL, CHANNEL AND POWER ALLOCATION IN WIRELESS COMMUNICATION NETWORKS

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ABSTRACT
In multicell wireless networks the resource allocation task includes the selection of the serving cell and the allocation of channels and transmission powers. While all of these tasks have been studied in the past, all three jointly are seldom addressed. In this paper we formulate the joint cell, channel and power allocation problem as an optimization task, whose purpose is to maximize the total user throughput. The joint problem is decomposed into separate subproblems of cell, channel and power allocation. We propose heuristic and optimization based algorithms to solve each of these tasks and present numerical results that give new and valuable insights into a sum throughput optimal joint resource allocation strategy.

1. INTRODUCTION
In multicell wireless networks, Mobile Stations (MS) need to be assigned to a serving base station such that the MSs enjoy continuous service coverage. This task is referred to as the serving link (cell) selection and it has been studied ever since cellular telecommunication systems started to gain popularity [1]. Link selection can be optimized according to different objective functions, such as overall system throughput [2], individual Quality of Service (QoS) targets [3] or other suitable utility functions [4].

Once an MS is assigned to a cell, radio resources – most importantly frequency/time channels and transmission powers – need to be allocated. Due to their relevance and complexity, channel assignment and power allocation have a vast literature, including classical papers from the late eighties (for a survey see [5]) to more recent research results [6, 7, 8, 9]. The authors of [8] propose a hybrid of centralized and distributed algorithms for subcarrier (i.e., channel) assignment to maximize the overall throughput. This scheme has been extended to include power allocation by [6]. Paper [7] studies three adaptive schemes for subcarrier (channel) allocation by means of cross-layer techniques for the purpose of throughput enhancement. Recently, it has been widely recognized that joint allocation of various radio resources has a clear potential over techniques that deal with a single resource, see for example [9]. However, cell selection is out of the scope of these papers.

Therefore, in this paper we examine joint cell (link), channel and power allocation for the purpose of getting insight into the gains when these three tasks are dealt with jointly. We focus on centralized algorithms for two reasons. First, centralized algorithms provide an insight into the potential gains of addressing the resource allocation tasks jointly and often serve as a starting point to distributed algorithm development [10]. Secondly, recent technology advancements indicate that centralized architectures and algorithms across multiple access points, including base band processing and radio resource management functions, may be an attractive technical solution in future wireless networks [11]. For cell selection and channel allocation a centralized entity may cover multiple cells, as proposed in [8] in the form of the "Radio Network Controller (RNC) algorithm”, while the power control problem advantageously can be implemented by means of distributed approaches.

We study these algorithms in a downlink Orthogonal Frequency-Division Multiple Access (OFDMA) context by means of a dynamic system level simulator called RUdimen- tary Network Emulator (RUNE) [12]. Even though the uplink is not considered in this paper, corresponding algorithms can easily be reformulated to this setting.

The rest of the paper is structured as follows. Section 2 describes our system model. In Section 3 we formulate the maximum throughput resource allocation optimization problem. Section 4 proposes solution approaches for link, channel and for power allocation. Section 5 contains the numerical results. Finally, Section 6 highlights our conclusions.

2. SYSTEM MODEL
We consider a multicell downlink wireless network with a Base Station (BS) in the center of each cell, where each BS maintains the coverage area of its associated cell. The set of BSs and the set of cells are both denoted by \( \mathcal{B} \). In the coverage area of the multicell network, there is the set of mobile stations, denoted by \( \mathcal{M} \). We say that there is a communication link between an MS and its serving BS. The link allocation describes the cell selection, and is denoted by the variable \( y \) within this paper. Furthermore, we assume that the radio resources that are used in each cell (for example subcarriers, time slots or codes) are orthogonal channels such that...
there is no intracell interference. We denote the set of channels with \( \mathcal{C} \). In general, a link may comprise multiple channels. The channel allocation variable is denoted by \( x \). The assumption on negligible intracell interference is valid for virtually all major modern telecommunication standards, including OFDMA or Orthogonal Code-Division Multiple Access (OCDMA) schemes and is often used in the literature of multicell models, see for example [13]. Finally, we allow for a complete reuse of all channels in each cell. Whenever a cell allocation is known, the set \( \mathcal{M}_b \) contains the MSs of cell \( b \in \mathcal{B} \) (i.e., BS \( b \)'s cell). Given a channel allocation, the set \( \mathcal{C}_b \) contains the allocated channels within the cell \( b \in \mathcal{B} \).

For ease of presentation, in this paper we assume that the basic radio resource is the transmission bandwidth \( W \) that is a known constant. This bandwidth is allocated in terms of frequency channels for the communication links and it is reused in each cell. The most important variables and constants of this (rather general) system are summarized by Table 1.

**Table 1.** Definition of the constants and variables.

<table>
<thead>
<tr>
<th>Constants</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>transmission frequency bandwidth</td>
</tr>
<tr>
<td>( g_{b m} )</td>
<td>path gain between BS ( b \in \mathcal{B} ) and MS ( m \in \mathcal{M} ), without channel variations</td>
</tr>
<tr>
<td>( g_{b m k} )</td>
<td>path gain between ( b \in \mathcal{B} ) and ( m \in \mathcal{M} ) on ( k \in \mathcal{C} )</td>
</tr>
<tr>
<td>( \sigma^2_m )</td>
<td>thermal noise at the receiving MS ( m \in \mathcal{M} )</td>
</tr>
<tr>
<td>( P_{b \text{max}} )</td>
<td>maximum transmit power of BS ( b \in \mathcal{B} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{b m} )</td>
<td>( \begin{cases} 1, &amp; \text{if } m \in \mathcal{M} \text{ belongs to cell } b \in \mathcal{B} \ 0, &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>( x_{b m k} )</td>
<td>( \begin{cases} 1, &amp; \text{if BS } b \in \mathcal{B} \text{ and MS } m \in \mathcal{M} \text{ is allowed} \ 1, &amp; \text{to communicate on channel } k \in \mathcal{C} \ 0, &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>( p_{b k} )</td>
<td>power that transmitter ( b \in \mathcal{B} ) uses on ( k \in \mathcal{C} )</td>
</tr>
<tr>
<td>( \eta_{b m k} )</td>
<td>throughput between ( b \in \mathcal{B} ) and ( m \in \mathcal{M} ) on ( k \in \mathcal{C} )</td>
</tr>
<tr>
<td>( R_m )</td>
<td>received throughput (rate) of MS ( m \in \mathcal{M} )</td>
</tr>
</tbody>
</table>

The interference that mobile \( m \) receives on channel \( k \), given that it belongs to cell \( b \), is given by

\[
I_{b m k} = \sum_{b \in \{B \setminus b\}} g_{b m k} p_{b k}, \quad b \in \mathcal{B}, m \in \mathcal{M}, k \in \mathcal{C},
\]

while the signal-to-interference-and-noise-ratio (SINR) between BS \( b \) and MS \( m \) on channel \( k \) is given by

\[
\gamma_{b m k} = \frac{g_{b m k} p_{b k}}{\sigma^2_m + I_{b m k}}, \quad b \in \mathcal{B}, m \in \mathcal{M}, k \in \mathcal{C}.
\]

The throughput between \( b \) and \( m \) on channel \( k \) is given by

\[
\eta_{b m k} = \frac{W}{|\mathcal{C}|} \log_2(1 + \gamma_{b m k}), \quad b \in \mathcal{B}, m \in \mathcal{M}, k \in \mathcal{C}.
\]

The total throughput (rate) of mobile user \( m \) is given by

\[
R_m = \sum_{b \in \mathcal{B}, k \in \mathcal{C}} x_{b m k} \eta_{b m k}, \quad m \in \mathcal{M}.
\]

Furthermore, to avoid modeling the details of scheduling algorithms, we assume that \(|\mathcal{M}| \leq |\mathcal{B}||\mathcal{C}|\).

### 3. Problem Formulation

Given the model setup, we introduce a joint cell, channel and power allocation optimization problem, which is formulated as

\[
\begin{align*}
& \text{maximize} & & R \\
& \text{subject to} & & R = \sum_{m \in \mathcal{M}} R_m, \\
& & & \sum_{b \in \mathcal{B}} y_{b m} = 1, \quad m \in \mathcal{M}, \\
& & & \sum_{m \in \mathcal{M}} x_{b m k} \leq 1, \quad b \in \mathcal{B}, k \in \mathcal{C}, \\
& & & x_{b m k} \leq y_{b m}, \quad b \in \mathcal{B}, m \in \mathcal{M}, k \in \mathcal{C}, \\
& & & 0 \leq y_{b m} \leq 1, \quad b \in \mathcal{B}, m \in \mathcal{M}, \\
& & & x_{b m k} \in \{0, 1\}, \quad b \in \mathcal{B}, m \in \mathcal{M}, k \in \mathcal{C}, \\
& & & \sum_{k \in \mathcal{C}} p_{b k} \leq P_{b \text{max}}, \quad b \in \mathcal{B}, \\
& & & p_{b k} \geq 0, \quad b \in \mathcal{B}, k \in \mathcal{C}.
\end{align*}
\]

where \( R_m \) is given by (4). Constraint (5c) ensures that all MSs are connected to exactly one BS, while constraint (5d) ensures that each BS is allowed to communicate with at most one MS per channel. Inequality (5e) ensures that channel \( k \) only can be allocated where MS \( m \) belongs to the same cell as BS \( b \). The constraints (5f) and (5g) define the variables \( y \) and \( x \), which in combination with (5e) ensure that \( y \) takes binary values whenever a channel is allocated within the cell. Finally constraints (5h) and (5i) ensure that the total power of each transmitter is at most that BSs’ maximum, and that each transmission power is non-negative. As a restricted max-sum problem is known to be NP-hard, see [14], heuristic approaches will be applied in the following section.

### 4. Solution Approaches

We resort to heuristic algorithms that are based on the decomposition of the problems to the separate tasks of finding link \( y \), channel \( x \) and power \( p \) allocation. Given an initial feasible allocation, the channel and the power allocation is updated repeatedly until no further improvement is obtained.


4.1. Link allocation (cell allocation)

In the link allocation it is decided which base station each mobile user will communicate with, i.e., regular cell allocation. We focus on link allocation algorithms which ensure that the number of MSs at any BS does not exceed the total number of channels, i.e., $|M_b| \leq |C|$ for each $b \in B$. Given this link allocation condition, it will be possible to assign at least one channel to each MS in the channel allocation. Two different link allocation approaches are presented below.

4.1.1. Link allocation Greedy (LaG)

The Link allocation Greedy (LaG) approach iteratively assigns mobile users to a cell according to the largest available path gain. Eventually any MS will have the highest path gain, and it then gets assigned to its desired base station (among the BSs that still provide at least one channel), see Algorithm 1.

Algorithm 1 Link allocation Greedy (LaG)

Let $y_{ij} \leftarrow 0, \bar{g}_{ij} \leftarrow g_{ij}, i \in B, j \in M$

for $\Delta = 1$ to $|M|$ do

$(b, m) \leftarrow \arg \max_{i \in B, j \in M} \bar{g}_{ij}$, breaking ties arbitrarily

Let $y_{bm} \leftarrow 1$ and $\bar{g}_{im} \leftarrow -1, i \in B, \{\text{removes MS } m\}$

if $\sum_{j \in M} y_{bj} = |C|$ then

$\bar{g}_{bj} \leftarrow -1, j \in M, \{\text{removes BS } b\}$

end if

end for

4.1.2. Link allocation All (LaA)

The Link allocation All (LaA) approach performs an exhaustive search over all feasible cell allocations, i.e., the remaining channel and power allocation problem is solved for each feasible link allocation, returning the best solution in terms of objective value.

4.2. Channel allocation

For a given $y$ and $p$ the Channel allocation Greedy (CaG) is performed within each cell separately, according to the approach proposed in [15]. Within each cell $b \in B$, every channel is allocated to the MS $m \in M_b$ that increases its throughput the most. The highest total throughput, given the cell and power allocation, is attained by this approach in downlink.

4.3. Power allocation

The power allocation assigns the powers $p$ given a feasible allocation of links $y$ and channels $x$.

4.3.1. Power allocation Evenly (PaE)

The Power allocation Evenly (PaE) approach assigns the powers of each BS $b \in B$ evenly over its allocated channels $C_b$. 

4.3.2. Power allocation Optimization (PaO)

The Power allocation Optimization (PaO) heuristic applies a nonlinear optimization solver on the optimization problem (5), given $y$ and $x$, using the power allocation obtained by PaE as initial starting point.

4.4. The overall allocation

Here we present the overall approaches used within this paper to solve the optimization problem (5).

4.4.1. The allocation Update (TaU)

The allocation Update (TaU) algorithm strives to obtain a good solution to (5), by using LaG or LaA, then CaG and finally either PaG or PaO allocation approaches described above. Thereafter, the channel and power allocations are alternately updated until no further improvement, in terms of objective value, is obtained.

4.4.2. Link Power grid Allocation (LPgA)

The Link Power grid Allocation (LPgA) approach performs an extensive search over all feasible cell allocations $y$, and a grid of power allocations as starting points when solving the remaining problem. The channel allocation is given by CaG, while each cell $b$ either uses no power, $P_{b}^{\max}$ power one of its channels, $P_{b}^{\max}/2$ power on two of its channels, or $P_{b}^{\max}/3$ power on each of its three channels.

5. NUMERICAL RESULTS

In this section we describe the considered simulation environment, and present our numerical results.

5.1. Simulation environment

Our simulation environment has been MATLAB, where the RUdimensionary Network Emulator (RUNE) was used to simulate realistic cellular systems. A detailed description of RUNE is available in [12]. The nonlinear programming problems have been solved using SNOPT [16]. The computations were run under 64-bit Linux on a single Intel Xeon 3 GHz processor core with hyperthreading disabled and with 32 GB of memory.

We consider a geographical location consisting of a number of hexagons. In the center of each hexagon a BS with the same fixed bandwidth is located. In total we consider three BSs and eight mobile users with a fixed bandwidth that is divided into three orthogonal channels. The MSs are randomly distributed within the hexagons, all using the uniform distribution. Given this data, the program RUNE considers path loss and fading effects to generate the channel gain between each BS and MS in the network. The main parameters of this
system together with parameters relevant for generating the path gain matrix are described in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell radius</td>
<td>500 m</td>
</tr>
<tr>
<td>Number of sectors per site</td>
<td>1</td>
</tr>
<tr>
<td>Number of clusters per system</td>
<td>1</td>
</tr>
<tr>
<td>Maximum power of base station</td>
<td>40 dB</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Chunk bandwidth</td>
<td>0.2 MHz</td>
</tr>
<tr>
<td>Gain at 1 meter distance</td>
<td>-28 dBm</td>
</tr>
<tr>
<td>Thermal noise</td>
<td>-103 dBm</td>
</tr>
<tr>
<td>Distance depended path gain coefficient</td>
<td>3.5</td>
</tr>
<tr>
<td>Standard deviation for the log-normal fading</td>
<td>6 dB</td>
</tr>
<tr>
<td>Log-normal correlation downlink</td>
<td>0.5</td>
</tr>
<tr>
<td>Correlation distance</td>
<td>110 m</td>
</tr>
<tr>
<td>Fast fading</td>
<td>Rayleigh</td>
</tr>
</tbody>
</table>

Within the simulations we consider the same thermal noise at each receiver, \( \sigma = \sigma_m, \ m \in \mathcal{M} \), and the same maximum power \( p^\text{max}_b = P^\text{max}_b, \ b \in \mathcal{B} \) at each transmitter. We consider 30 different test-problems that are solved for each combination of cell, channel and power allocations.

5.2. Numerical results

The simulation results are presented in Figures 1-2 where the performance of the different allocation approaches are compared.

Figure 1 shows that the approaches have fairly similar behavior in terms of performance. The optimization power allocation approach outperforms the greedy approach slightly.

In Figure 2 the differences are highlighted by subtracting the total throughput of LaG-PaG from the others. The extensive searches (LaA and LPgA) outperforms the greedy approach. There is also additional throughput that is attained by using PaO instead of the PaE approach. However, these earnings come at a high computational cost, see Table 3.

<table>
<thead>
<tr>
<th></th>
<th>( \text{time}, n_{\text{iter}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaG</td>
<td>0.0118 [s], 2.00</td>
</tr>
<tr>
<td>LaA</td>
<td>9.07 [s], 1680 \times 2.00</td>
</tr>
<tr>
<td>LPgA</td>
<td>14.7 [min], 1.85 \times 10^8</td>
</tr>
<tr>
<td>PaE</td>
<td></td>
</tr>
<tr>
<td>PaO</td>
<td>0.567 [s], 2.33</td>
</tr>
<tr>
<td></td>
<td>24.3 [min], 1680 \times 2.40</td>
</tr>
<tr>
<td></td>
<td>3.92 [days], 1.85 \times 10^8</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

In this work, we considered the joint serving cell (link), channel and power allocation problem in cellular systems that use orthogonal channel assignments within the cells. The complexity of this rather general problem motivated to decompose it to separate link, channel and power allocation problems. To these tasks we proposed low complexity algorithms together with a more advanced power allocation optimization approach. An allocation update procedure iteratively reallocates the channel and power resources to improve the system performance. Further, two extensive search approaches were applied (LaA and LPgA). The LaA approach performs an ex-
haustive search over all feasible cell allocations, while LPgA considers all cell allocations in combination with a set of initial power allocations.

We implemented the algorithms in a realistic system simulator called RUNE and generated numerical results for the downlink. The system performance was studied in terms of the total system throughput. One of the main results is that low complexity heuristics based on a heuristic resource allocation of the link, channel and power resources (LaG, CaG and PaE respectively) perform surprisingly well in comparison with the more expensive extensive search and optimization based approaches.

From the two proposed link allocation approaches, LaG and LaA, it is obvious that the best cell allocation seldom is obtained by the LaG. The greedy channel allocation CaG solves its problem to global optimality. The difference between the two power allocation methods (PaE and PaO) is notable, where the more advanced PaO attains higher total throughput. The allocation update (TaU) approach protrudes in terms of its few number of iterations. The average number of iterations for the different approaches varies between 2 and 2.40, where the last iteration did not improve the objective value. Hence, in comparison with merely solving the link, channel and power allocation once, the updates of TaU that actually improves the objective value comes at a fairly high additional computational time.

Although the proposed low complexity heuristics perform well in comparison to the approaches with higher computational complexity, there still exists available earnings to strive for in order to maximize the overall system performance. To develop distributed algorithms based on the insights of this paper is left for future work.

Acknowledgments

The authors are grateful to Andrea Goldsmith, Johan Håstad and Mikael Prytz for many valuable suggestions.

7. REFERENCES


