CARRIER FREQUENCY OFFSET ESTIMATION IN TIME-SELECTIVE RAYLEIGH FLAT-FADING CHANNELS

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ABSTRACT

This paper focuses on carrier frequency offset (CFO) estimation in the presence of time-selective Rayleigh fading (i.e., Gaussian multiplicative noise) channel. The time-variant fading is modeled by considering the Jakes' and the first order autoregressive AR(1) correlation models. A high signal-to-noise-ratio maximum likelihood (ML) estimators based on the AR(1) correlation model and for slow-fading channels are derived when the channel statistics are unknown. The main objective is to reduce algorithm complexity to a *single-dimensional search* on the CFO parameter alone. Closed-form expressions of the Cramér-Rao lower bound (CRB) for the CFO parameter alone are derived for fast-fading and slow-fading channels. Approximate analytical expressions for the CRB are derived for low and high SNR that enable the derivation of a number of properties that describe the bound's dependence on key parameters such as SNR, channel correlation. Finally, simulation results illustrate the performance of the estimators and confirm the validity of the theoretical analysis.

***Index Terms*—** CFO estimation, ML estimator, Cramér Rao bound, Time-varying fading channel, Jakes' channel model, AR1 channel model.

1. INTRODUCTION

Carrier frequency offset (CFO) estimation in the presence of multiplicative noise is important for synchronization of communications signals in the context of time-selective fading channels (e.g., [1, 2, 3]). The time-selective fading is involved particularly in mobile wireless communications, which causes Doppler spread [4]. On the other hand, the instability of oscillators causes the CFO. These frequency bias degrade the receiver performance greatly, thus it needs to track and compensate these frequency bias in coherent communications. Multiplicative noise models are also appropriate in array processing when the source signal is spatially distributed [5], as well as for backscattered acoustic signals [6, 7] and SAR imagery systems.

Various techniques have been proposed for carrier frequency recovery (see e.g., [8, 9]). Among these, the class of data-aided techniques (see e.g., [10, 11, 12]), which use a training sequence for frequency offset estimation, become popular because they can attain good performance with a short training sequence. The data-aided techniques have been developed for additive white Gaussian noise (AWGN) channels (see e.g., [12, 13]). However, the additive noise model is not sufficiently representative in most practical applications. As an example, consider a digital communication system with burst transmission using coherent signal detection over a radio link where the transmitted signal is corrupted by a time-varying multiplicative distortion. In such conditions, frequency estimators based on additive Gaussian models do not perform well, and a low-accuracy carrier frequency estimation for the fading channel will cause serious performance degradation.

Many data-aided CFO estimators have been proposed in the presence of time-varying multiplicative noise distortions [2, 3, 14, 15, 16], which are mainly classified as correlation-based or periodogram-based estimators. The most popular one, the so-called pulse-pair (PP) estimator originally proposed by Benham et. al. in an application to Doppler spectrum estimation [17]. It has been shown that the correlation-based methods show very good performance and it achieve the Cramer-Rao bound (CRB) for the frequency estimate [2, 3]) when the phase unwrapping problem is solved. However, the correlation-based estimators (e.g., [2, 3, 14]) need to know the channel parameters (e.g., Doppler bandwidth of the channel), and not take into account the statistical properties of the time-varying fading channel. Note that if channel statistics are unknown to the estimator then a parametrization of the time-varying channel is required.

It is well-known that the derivation of the CFO ML estimator over an unknown time-selective Rayleigh fading (i.e., Gaussian multiplicative noise) channel is even more complex since it requires a multidimensional optimization procedure. In this paper we develop two high-SNR approximate CFO ML estimators for time-selective Rayleigh fading and slow-fading channels with unknown channel statistics. The fast-fading ML estimator is derived based on a simplified AR(1) correlation model. The main objective is to reduce estimators complexity to a single-dimensional search on the CFO parameter after deriving closed-form expressions of the ML estimates for the unknown channel and signal parameters. Exact and approximate closed-form expressions for the CRB on CFO estimation are derived for fast-fading and slow-fading channels. In addition, attention is focused on data-aided (DA) synchronization, wherein the transmitted symbol sequence is known a priori.

The paper is organized as follows. Sec. 2 describes the signal model, the Jakes' and AR1 correlation models and poses the estimation problem. Sec. 4 presents high-SNR approximate ML CFO estimator derived for fast fading channel and high-SNR approximate slow fading ML estimator is presented in Section 5. In Sec. 6, exact and approximate closed-form expressions for the CRB on CFO estimation are derived for fast and slow fading fading channels. Finally, the numerical results are analyzed in Sec. 7 and conclusions are presented in Sec. 8.

2. MODELING AND PROBLEM FORMULATION

Consider the transmission of a linearly modulated signal over flat Rayleigh fading channel. We assume Nyquist shaping and ideal sample timing so that the inter-symbol interference at each symbol spaced sampling instance can be ignored. In the presence of frequency offset, the signals at the output of the matched filter can be modeled as a complex signal as follows:

$$y_k = a_k h_k e^{j2\pi k f_0} + n_k, \;\; k = n_0, \ldots, n_0+N-1 \quad (2.1)$$

where $\{a_k\}$ is a sequence of a known symbols with $|a_k|^2 = 1$. The deterministic unknown parameter f_0 represents the carrier frequency offset normalized to the symbol rate. N is the total number of received samples in the observation interval. The processes h_k and n_k are the samples of the fading gains and additive noise process, respectively. The noise samples are assumed to be independent and identically distributed (IID) circular complex Gaussian with zero-mean and variance σ_n^2. Rayleigh fading is assumed and the fading process is normalized so that h_k is zero-mean circular complex Gaussian with unknown variance σ_h^2 and correlation function given by:

$$R_h^J(m) \stackrel{\text{def}}{=} \mathrm{E}(h_n h_{n-m}^*) = \sigma_h^2 J_0(2\pi f_d T m),$$

where $J_0(.)$ is the first kind 0th-order Bessel function, T is the symbol period and f_d denotes the maximum Doppler shift. This is frequently referred to as the Jakes' model [4]. The signal-to-noise ratio (SNR) is defined as $\rho \stackrel{\text{def}}{=} \frac{\sigma_h^2}{\sigma_n^2}$.

It is, however, not feasible to directly apply the Jakesḿodel in our computation for it leads to intractable solutions. Alternatively, an AR(1) process can often be used to approximate the Jakes' model with satisfactory accuracy (e.g. [1, 18]). Particularly, in this paper, an AR(1) process is adopted, i.e.,

$$h_k = \gamma h_{k-1} + \sqrt{1-\gamma^2} e_k, \quad (2.2)$$

here $e_k \sim \mathcal{N}(0, \sigma_h^2)$ is the additive driving noise and where $\gamma \stackrel{\text{def}}{=} J_0(2\pi f_d T)$ is assumed to be unknown. The fading amplitude at time n is constrained to follow a sequence from a known initial state, say h_0:

$$h_n = \gamma^n h_0 + \sqrt{1-\gamma^2} \sum_{k=0}^{n-1} \gamma^k e_{n-k}. \quad (2.3)$$

The correlation over m signalling intervals is given by

$$R_h^{\mathrm{AR}}(m) = \mathrm{E}(h_n h_{n+m}^*) = \sigma_h^2 \gamma^{|m|},$$

and it depends on the mobility environment (and on the symbol time T) at hand. Consequently, the covariance matrix for the AR1 channel model can be written as

$$\mathbf{R}_h^{\mathrm{AR}} = \sigma_h^2 \begin{pmatrix} 1 & \gamma & \gamma^2 & \ldots & \gamma^{N-1} \\ \gamma & 1 & \gamma & \ldots & \gamma^{N-2} \\ \gamma^2 & \gamma & 1 & \ldots & \gamma^{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{N-1} & \gamma^{N-2} & \gamma^{N-3} & \ddots & 1 \end{pmatrix} \quad (2.4)$$

We note that for $\gamma = 0$ the channel becomes an uncorrelated fading process, and for $\gamma = 1$, the channel is simply a realization of a single random variable (slow fading).

Collecting the samples of the received signal to form a vector $\mathbf{y} \stackrel{\text{def}}{=} (y_{n_0}, \ldots, y_{n_0+N-1})^T$ yields the following model

$$\mathbf{y} = \mathbf{AFh} + \mathbf{n}, \quad (2.5)$$

where $\mathbf{A} \stackrel{\text{def}}{=} \mathrm{Diag}\,(a_{n_0}, \ldots, a_{n_0+N-1})$, $\mathbf{F} \stackrel{\text{def}}{=} \mathrm{Diag}\left(e^{j2\pi n_0 f_0}, \;\ldots, e^{j2\pi(n_0+N-1)f_0}\right)$, $\mathbf{h} \stackrel{\text{def}}{=} (h_{n_0}, \ldots, h_{n_0+N-1})^T$ and $\mathbf{n} \stackrel{\text{def}}{=} (n_{n_0}, \ldots, n_{n_0+N-1})^T$ is an $N \times 1$ noise vector with covariance matrix $\sigma_n^2\mathbf{I}$. Since the transmitted symbols a_k are known, $\mathbf{y}$ is a zero-mean complex Gaussian random vector, with correlation matrix given by

$$\mathbf{R}_y \stackrel{\text{def}}{=} \mathrm{E}(\mathbf{y}\mathbf{y}^H) = \mathbf{AFR}_h\mathbf{F}^H\mathbf{A}^H + \sigma_n^2\mathbf{I}, \quad (2.6)$$

where $\mathbf{R}_h \stackrel{\text{def}}{=} \mathrm{E}(\mathbf{h}\mathbf{h}^H)$ is the fading-channel correlation matrix. The probability density function (PDF) of $\mathbf{y}$ is given by:

$$p(\mathbf{y};\boldsymbol{\alpha}) = \frac{1}{\pi^N \det(\mathbf{R}_y)} e^{-\mathbf{y}^H\mathbf{R}_y^{-1}\mathbf{y}}, \quad (2.7)$$

where $\boldsymbol{\alpha} = (f_0, {\boldsymbol{\alpha}_n}^T)^T$ is an unknown parameter vector depending on parameter of interest, f_0, and a vector of nuisance parameters $\boldsymbol{\alpha}_n \stackrel{\text{def}}{=} (\gamma, \sigma_h^2, \sigma_n^2)^T$.

The estimation problem can now be formulated as follows: Given the received signal $\mathbf{y}$ whose pdf (2.7), estimate f_0 in the presence of a vector nuisance parameters $\boldsymbol{\alpha}_n$.

3. CFO ML ESTIMATORS

3.1. Fast fading

The direct maximization of the likelihood function (2.7) with respect to the unknown parameter $\boldsymbol{\alpha} \stackrel{\text{def}}{=} (f_0, \boldsymbol{\alpha}_n^T)^T$ is a difficult task. The straightforward approach for deriving the ML solution to the problem is to try to concentrate (2.7) (where (2.4) is used instead of the true channel correlation matrix in the case of fast-fading channel) with respect to the nuisance parameters, and to perform a search on the CFO parameter f_0 and the parameters which cannot be concentrated.

Using the fact that the diagonal matrices $\mathbf{A}$ and $\mathbf{F}$ are unitary matrices[1], the vector $\mathbf{z} \stackrel{\text{def}}{=} \mathbf{F}^H\mathbf{A}^H\mathbf{y}$ can be seen as a unitary transformation of $\mathbf{y}$. The concentrated log-likelihood function of $\mathbf{z}$ proved in [20] by applying the chain rule[2] can be expressed as (after dropping the constant term)

$$\begin{aligned} L(\boldsymbol{\alpha}_n;\mathbf{z}) =& \ln(\sigma_h^2+\sigma_n^2)+\frac{|z_0|^2}{\sigma_h^2+\sigma_n^2} \\ &+ (N-1)\ln\left(\sigma_h^2+\sigma_n^2+\frac{\gamma^2\sigma_h^4}{\sigma_h^2+\sigma_n^2}\right) \\ &+ \frac{1}{\sigma_h^2+\sigma_n^2-\frac{\gamma^2\sigma_h^4}{\sigma_h^2+\sigma_n^2}}\sum_{n=1}^{N-1}|z_n-z_{n-1}\frac{\gamma\sigma_h^2}{\sigma_h^2+\sigma_n^2}|^2. \end{aligned} \tag{3.8}$$

Note that the maximization of (3.8) is significantly complicated by the presence of the nuisance parameters. However, since we are interested in a simpler estimation procedure, we resort to a high-SNR approximation. For high-SNR (σ_n^2 is very small), (3.8) can be simplified by assuming that $\gamma \neq 1$

$$\begin{aligned} L(\boldsymbol{\alpha}_n^{high};\mathbf{z}) =& \ln(\sigma_h^2)+\frac{|z_0|^2}{\sigma_h^2}+(N-1)\ln\left(\sigma_h^2(1+\gamma^2)\right) \\ &+ \frac{1}{\sigma_h^2(1-\gamma^2)}\sum_{n=1}^{N-1}|z_n-\gamma z_{n-1}|^2, \end{aligned} \tag{3.9}$$

where $\boldsymbol{\alpha}_n^{high} \stackrel{\text{def}}{=} (\sigma_h,\gamma)^T$ is the nuisance parameters vector after omitting σ_n from the nuisance parameters vector $\boldsymbol{\alpha}_n$.

The following result proved in [20], shows that it is possible to reduce the optimization problem, under a high SNR approximation, to a *single-parameter search* with respect to the CFO parameter f_0 only.

Result 1 *For high SNR environment and with $\gamma \neq 1$, the joint ML estimates that minimize the log-likelihood function* (3.9) *are given by the following:*
$\hat{f}_{0,ML}$ is obtained by the minimizing with respect to f_0

$$\begin{aligned} F_{fa}(f_0;\mathbf{z}) =& \ln(\hat{\sigma}_{h,ML}^2)+\frac{|z_0|^2}{\hat{\sigma}_{h,ML}^2} \\ &+ (N-1)\ln\left(\hat{\sigma}_{h,ML}^2(1+\hat{\gamma}_{ML}^2)\right) \\ &+ \frac{1}{\hat{\sigma}_{h,ML}^2(1-\hat{\gamma}_{ML}^2)}\sum_{n=1}^{N-1}|z_n-\hat{\gamma}_{ML}z_{n-1}|^2 \end{aligned} \tag{3.10}$$

where $\hat{\sigma}_{h,ML}^2$ and $\hat{\gamma}_{ML}$ are the ML estimates of the nuisance parameters given by

$$\hat{\gamma}_{ML} = -\frac{k_{2,z}(f_0)}{2k_{4,z}(f_0)}, \tag{3.11}$$

$$\begin{aligned} \hat{\sigma}_{h,ML}^2 =& \frac{1}{N}\left(k_{3,z}(f_0)+\frac{1}{1-\hat{\gamma}_{ML}^2}\left(-\hat{\gamma}_{ML}k_{2,z}(f_0)\right.\right. \\ &+ \left.\left.\hat{\gamma}_{ML}^2k_{1,z}(f_0)\right)\right), \end{aligned} \tag{3.12}$$

where the frequency-dependent coefficients $k_{l,z}(f_0)$, $l = 1,\ldots,4$, are given by $k_{1,z}(f_0) \stackrel{\text{def}}{=} \sum_{n=1}^{N-1}(|\mathbf{z}_n|^2+|z_{n-1}|^2)$, $k_{2,z}(f_0) \stackrel{\text{def}}{=} \sum_{n=1}^{N-1}(z_n^*z_{n-1}+z_{n-1}^*z_n)$, $k_{3,z}(f_0) \stackrel{\text{def}}{=} \sum_{n=0}^{N-1}|z_n|^2$ and $k_{4,z}(f_0) \stackrel{\text{def}}{=} k_{3,z}(f_0)-k_{1,z}(f_0)$.

The overall estimation procedure can be summarized as follows. For each value of f_0 in the search domain, the ML estimates of γ and σ_h^2 are given by (3.11) and (3.12), respectively. Substituting then the estimates of the nuisance parameters into (3.9) yields (3.10). The CFO estimate is that value which maximizes (3.10). Thus, for high SNR, the nuisance parameters are given in closed-form expressions that depend on the CFO parameter, reducing the search to a *single-parameter search* only.

Note that the ML approach only requires maximizing (3.10) with respect to a scalar f_0, which can be efficiently implemented using derivative-free uphill search methods such as the Nelder-Mead algorithm[3] [21].

Remark 1 *It is well known that $\gamma = J_0(2\pi f_dT) \approx 1-\frac{1}{4}.(2\pi f_dT)^2$ for $f_dT \ll 1$. Therefore, using* (3.11)*, the approximate ML estimate of the maximum Doppler shift, f_d, can be expressed as*

$$\hat{f}_{d,ML} = \frac{1}{\pi T}\sqrt{1-\hat{\gamma}_{ML}}.$$

3.2. Slow fading

In the sequel, the results for the fast fading channel will be compared with those of the slow fading (i.e., with the fading process remaining constant throughout the observation window). We prove in [20] that the negative log-likelihood function for slow fading channel can be expressed as

$$\begin{aligned} L_{sl}(\boldsymbol{\alpha}^{sl};\tilde{\mathbf{z}}) =& (N-1)\ln(\sigma_n^2)+\ln(N\sigma_h^2+\sigma_n^2) \\ &+ \frac{1}{\sigma_n^2}\sum_{n=0}^{N}|\tilde{\mathbf{z}}_n|^2-\frac{\sigma_h^2}{(N\sigma_h^2+\sigma_n^2)\sigma_n^2}\left|\sum_{n=0}^{N}\tilde{\mathbf{z}}_n\right|^2. \end{aligned} \tag{3.13}$$

with $\tilde{\mathbf{z}} \stackrel{\text{def}}{=} (\tilde{\mathbf{z}}_n,\ldots,\tilde{\mathbf{z}}_{N-1})^T$ where $\tilde{\mathbf{z}}_n \stackrel{\text{def}}{=} a_n^*e^{-j2\pi nf_0}y_n$ and where $y_n = ha_ne^{j2\pi nf_0}+n_n$ with $h \sim \mathcal{N}(0,\sigma_h^2)$. $\boldsymbol{\alpha}^{sl} \stackrel{\text{def}}{=} (f_0,{\boldsymbol{\alpha}^{sl}_n}^T)^T$ and where $\boldsymbol{\alpha}_n^{sl} \stackrel{\text{def}}{=} (\sigma_h^2,\sigma_n^2)^T$ is an unknown parameter vector of nuisance parameters. We prove in [20], the following result.

Result 2 *For high SNR environment, the CFO ML estimate $\hat{f}_{0,ML}^{sl}$ that minimize the log-likelihood function* (3.13) *is obtained by minimizing with respect to f_0 only the following function*

$$F_{sl}(f_0;\tilde{\mathbf{z}}) = \left|\frac{1}{N}\sum_{n=0}^{N}\tilde{\mathbf{z}}_n\right|^2. \tag{3.14}$$

4. CRB EXPRESSIONS

The following result is proved in [20] summarized by the following result.

[1]The matrices $\mathbf{A}$ and $\mathbf{F}$ satisfying $\mathbf{A}\mathbf{A}^H = \mathbf{A}^H\mathbf{A} = \mathbf{I}$ and $\mathbf{F}^H\mathbf{F} = \mathbf{F}\mathbf{F}^H = \mathbf{I}$.

[2]The AR(1) process allows to use the chain rule, and has only a limited memory of its own history for which $p(z_k|z_0,\ldots,z_{k-1};\boldsymbol{\alpha}) = p(z_k|z_{k-1};\boldsymbol{\alpha})$.

[3]The Nelder-Mead algorithm has already been incorporated in the function "fminsearch" in MATLAB®.

Result 3 *The DA CRB for CFO parameter over time-selective fast fading channel is given by:*

$$\mathrm{CRB}^{\mathrm{DA}}(f_o) = \frac{1}{8\pi^2\rho^2}\frac{1}{\mathrm{Tr}((\mathbf{R}'_h\bar{\mathbf{R}}_h^{-1}\mathbf{N}_0)(\mathbf{N}_0\mathbf{R}'_h - \mathbf{R}'_h\mathbf{N}_0))},$$

where $\mathbf{R}'_h \stackrel{\mathrm{def}}{=} \frac{1}{\sigma_h^2}\mathbf{R}_h$, $\mathbf{N}_0 \stackrel{\mathrm{def}}{=} \mathrm{Diag}(n_0,\ldots,n_0+N-1)$ and $\bar{\mathbf{R}}_h \stackrel{\mathrm{def}}{=} \rho\mathbf{R}'_h + \mathbf{I}$.

Note that this result is valid for any type of fading correlation (e.g., Jakes, AR) when the fading process is complex normal and of zero mean. We have also observed numerically that the CRB (4.15) does not depends on the time n_0 at which the first sample is taken.

In the special cases of slow fading[4] ($i.e.$, $\gamma = 1$ and $\mathbf{R}_h = \sigma_h^2\mathbf{1}\mathbf{1}^T$), result 3 can be extended to the following result.

Result 4 *The DA CRB for CFO over slow fading channel is given by*

$$\mathrm{CRB}^{\mathrm{Slow}}(f_0) = \mathrm{MCRB}(f_0)\frac{N\rho+1}{N\rho}\frac{1}{1-\alpha_{n_0}}, \quad (4.15)$$

where

$$\mathrm{MCRB}(f_0) \stackrel{\mathrm{def}}{=} \frac{1}{8\pi^2\rho\sum_{n=n_0}^{n_0+N-1} n^2},$$

is the modified CRB (MCRB) when the fading channel is assumed constant over the observation interval [19]. The coefficient α_{n_0} *is defined as* $\alpha_{n_0} \stackrel{\mathrm{def}}{=} \frac{(\sum_{n=n_0}^{n_0+N-1} n)^2}{\sum_{n=n_0}^{n_0+N-1} n^2}$.

It is clear that for sufficiently high SNR, the bound (4.15) is approximately inversely proportional to SNR (decreasing rapidly with SNR), and therefore tends to zero for asymptotically high SNR. Moreover, when the CRB is analyzed relative to the middle of the signal vector (i.e., $n_0 = -\frac{N-1}{2}$), α_{n_0} equal to zero, and then the CRB (4.15) becomes

$$\mathrm{CRB}^{\mathrm{slow}}(f_0) = \mathrm{MCRB}(f_0)\frac{N\rho+1}{N\rho}, \quad (4.16)$$

where

$$\mathrm{MCRB}(f_0) = \frac{6}{(2\pi)^2N(N^2-1)\rho}. \quad (4.17)$$

Using AR correlation model (where (2.4) is used instead of the true channel correlation matrix), we prove in [20] that (4.15) at high SNR can be simplified as

$$\mathrm{CRB}^{\mathrm{high}}(f_o) = \frac{1-\gamma^2}{\gamma^2}\frac{1}{8\pi^2(N-1)}. \quad (4.18)$$

5. SIMULATION RESULTS

This section presents numerical examples that illustrate the accuracy of the derived fast-fading and slow-fading ML estimators and compared with the derived fast-fading and slow-fading CRBs as a function of SNR and doppler-time product.

The channel is simulated according to AR1 correlation model [1, 4] with doppler-time product of f_dT. The symbols $\{a_n\}$ are assumed known at the receiver. In all simulations, we set $f_0 = 0.03$, $N = 100$ and 1000 Monte Carlo trials were run to obtain the empirical mean squares error (MSE) of the estimates.

Fig.1 plot the derived exact fast-fading and slow-fading CRBs on CFO estimation, the high-SNR CRB (4.18) and low-SNR approximates CRB calculated in [20], the MCRB calculated by (4.17), and the Monte Carlo results for the fast-fading and slow-fading ML estimators derived in results 1 and 2, respectively. It can be seen that the exact fast-fading and slow-fading CRBs provides an exact match to the Monte Carlo MSE results associated with the high-SNR ML estimators for both fast-fading and slow-fading channels. Note also, that although the slow-fading CFO ML estimate presented in result 2 for high SNR, the performance of this estimate reaches the slow-fading CRB for a large range of SNR up to -10 dB. This figure shows that for sufficiently high SNR, the SNR does not have a effect on the fast-fading bound on CFO estimation as shown the equation (4.18).

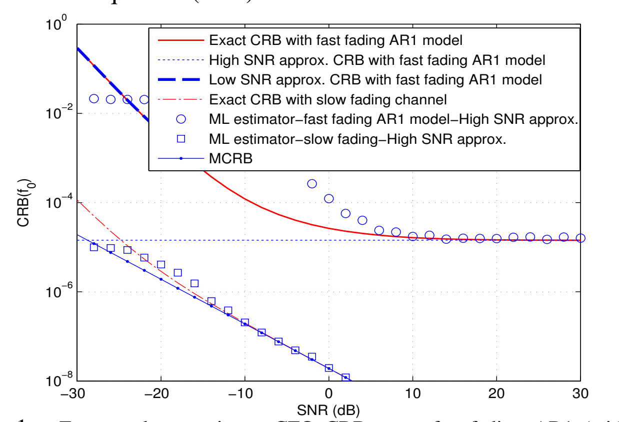

Fig.1 Exact and approximate CFO CRBs over fast-fading AR1 (with $f_dT = 0.1$) and slow-fading channels, MCRB, and estimated MSEs $\mathrm{E}(\hat{f}_{0,ML} - f_0)^2$ given by the high-SNR approximate ML estimators, versus f_dT for SNR$= 15dB$.

We also see from this figure, the validity of the high and low SNR fast-fading theoretical approximations of the CRBs which are very close to the true CRB, and that the MCRB and slow-fading CRB on CFO estimation are identical except for low SNR.

Fig.2 shows the behavior of the CRBs versus the doppler-time product f_dT with $SNR = 15$ dB. We observe the fast-fading CRB of the AR1 CFO estimation increases as the time-Doppler product increases. To understand why, recall that the AR1 high SNR approximation of the CRB given by (4.18) depends on the term $\gamma^2/(1-\gamma^2)$ which, for $\gamma \in [0, 1[$, is a monotonically increasing function of γ. The time-doppler product has a significant effect on the bound. We also observe that the fast-fading CRB on CFO estimation remains identical to the slow-fading CRB on CFO estimation up to $f_dT = 0.0001$ (where the channel is slowly time-varying). Note also that the MSE results associated with the high-SNR fast-fading ML estimator reaches the fast-fading CRB except when f_dT decreases. This is because the fast-fading AR1 ML estimator was derived under the assumption that $\gamma \neq 1$ ($\gamma = 1$ implies $f_dT = 0$, the slow-fading case). Thus, the estimator

[4] $\mathbf{1}$ is the all-one $(N \times 1)$ vector.

will not produce a valid result as γ approaches 1. We also see from this figure that the high SNR approximation of the fast-fading CRB (4.18) is very close to the exact fast-fading CRB except when the channels becomes slowly time-varying.

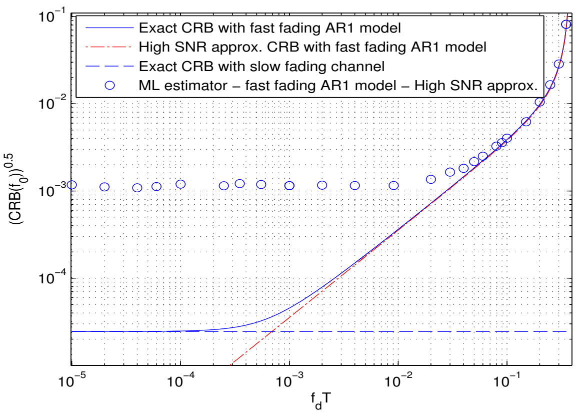

Fig.2 Exact and High-SNR approximate CFO CRBs over fast-fading AR1 and slow-fading channels, and estimated MSE $\mathrm{E}(f_{0,\hat{ML}} - f_0)^2$ given by the high-SNR approximate ML estimator versus f_dT with SNR$= 15dB$.

6. CONCLUSION

This paper considers the estimation of the carrier frequency offset in the presence of time-varying Rayleigh flat fading channels. We have derived fast-fading and slow-fading ML estimators for estimating CFO parameter with unknown channel parameters. The fast-fading ML approach was based on a mismatched AR(1) channel-correlation model upon which a high SNR CFO estimator is derived. The fast-fading ML estimator was compressed into a single-parameter search over the CFO parameter alone. A closed-form expression of the DA CRB for CFO parameter alone are derived for fast and slow fading channels. Approximate analytical expressions for the CRB of the CFO parameter over low and high SNR are derived. Some properties that highlight how the bound depends on key parameters such as SNR and time-Doppler product are presented. The performance of the fast-fading and slow-fading CFO estimators are analyzed through simulations and compared with the CRB.

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