BUILT IN PERFORMANCE EVALUATION FOR AN ADAPTIVE NOTCH FILTER

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ABSTRACT

The problem of estimating instantaneous frequency of a nonstationary complex sinusoid (cisoid) buried in wideband noise is considered. The proposed approach extends adaptive notch filtering algorithm with a nontrivial performance assessment mechanism which can be used to optimize frequency tracking performance of the adaptive filter. Simulation results confirm that the proposed extension allows one to improve accuracy of frequency estimates considerably, especially in nonstationary conditions.

Index Terms— frequency estimation, adaptive notch filtering

1. INTRODUCTION

Tracking of instantaneous frequency of nonstationary narrowband signals is often accomplished using adaptive notch filters (ANFs). Classical ANFs are based on constrained poles and zeros [1] or lattice [2] designs. Recent contributions to the field include, among others, a complex filter by Regalia [3] and its modification proposed in [4]. These algorithms have some desirable properties, like unbiased estimation and fast convergence.

Most adaptation laws of ANFs include so-called adaptation gains, usually adjusted so as to optimize *signal*, rather than *frequency*, tracking performance of the filter. For many applications, such as removing power interferences from EEG signals [5] or tracking harmonic currents [6], this is an understandable optimization goal.

However, there exist applications where instantaneous frequency is the quantity of primary interest. For instance, rotational speed of a combustion engine can be estimated by means of tracking fundamental frequency of acoustic noise or vibration signal generated by the engine. In such a case signal tracking performance of ANF is a secondary issue.

Unfortunately the two above mentioned goals (signal and frequency tracking) are conflicting ones – ANFs which exhibits optimal signal tracking performance usually underperform in terms of frequency tracking and vice versa. Furthermore, it is not trivial to evaluate frequency tracking performance of an ANF during its operation. While the quality of signal estimates can be easily quantified using prediction errors yielded by the filter, such an approach fails in case of frequency tracking. This makes optimization of frequency tracking performance of ANFs a tricky task. The situation becomes even more challenging when the filter is expected to operate in nonstationary conditions, e.g. under time-varying signal to noise ratio.

In the paper we show how the accuracy of frequency estimates yielded by an ANF can be evaluated on-line, without any prior knowledge of the true frequency trajectory. Gaining upon this foundation, we later propose a novel parallel frequency tracking scheme which is capable of adjusting its characteristics to unknown, and possibly time-varying, conditions.

The paper is organized as follows: Section 2 presents problem formulation and ANF algorithm which will be extended later. Section 3 introduces the proposed way of evaluating frequency tracking performance of the ANF. Section 4 presents simulation results. Section 5 concludes.

2. PROBLEM FORMULATION

Consider the problem of estimating instantaneous frequency of a complex-valued signal

$$s(t) = a(t)e^{j\sum_{\tau=1}^{t}\omega(\tau)} \tag{1}$$

using noisy measurements

$$y(t) = s(t) + v(t) \ , \tag{2}$$

where $t = 0, 1, \ldots$ denotes dimensionless, discrete time, $a(t)$ is a slowly time-varying complex “amplitude”, $\omega(t)$ denotes instantaneous frequency and $v(t)$ is a wideband measurement noise. Note that, since $a(0)$ is a complex quantity, it may as well incorporate the initial phase of $s(t)$. This is the reason behind the absence of initial phase in (1).

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The starting point of our discussion is ANF algorithm proposed in [7]. It takes the form

$$\begin{aligned}
\widehat{f}(t) &= e^{j[\widehat{\omega}(t-1)+\widehat{\alpha}(t-1)]}\widehat{f}(t-1) \\
\varepsilon(t) &= y(t) - \widehat{a}(t-1)\widehat{f}(t) \\
\widehat{a}(t) &= \widehat{a}(t-1) + \mu \widehat{f}^*(t)\varepsilon(t) \\
\widehat{\alpha}(t) &= \widehat{\alpha}(t-1) + \gamma_\alpha \delta(t) \\
\widehat{\omega}(t) &= \widehat{\omega}(t-1) + \widehat{\alpha}(t-1) + \gamma_\omega \delta(t) \\
\delta(t) &= \mathrm{Im}\left[\frac{\varepsilon(t)}{\widehat{a}(t-1)\widehat{f}(t)}\right] \\
\widehat{s}(t) &= \widehat{f}(t)\widehat{a}(t)\ , \qquad (3)
\end{aligned}$$

where $*$ denotes complex conjugation, the quantities $\widehat{a}(t)$, $\widehat{\omega}(t)$, and $\widehat{\alpha}(t)$ are the estimates of the signal's complex amplitude, instantaneous frequency and instantaneous frequency rate [$\alpha(t-1) = \omega(t) - \omega(t-1)$], respectively. The parameters $\mu > 0$, $\gamma_\omega > 0$, $\gamma_\alpha > 0$, $\gamma_\alpha \ll \gamma_\omega \ll \mu$, are scenario-dependent small adaptation gains determining the rates of amplitude adaptation, frequency adaptation and frequency rate adaptation, respectively. Finally, $\widehat{s}(t)$ denotes the filtered estimate of the narrowband signal $s(t)$.

In spite of its simplicity, the gradient frequency tracking algorithm (3) has very good statistical properties. As shown in [7], under the following assumptions:

(A1) The instantaneous frequency drifts according to the 2-nd order random walk (also called integrated random walk)

$$\begin{aligned}
\omega(t) &= \omega(t-1) + \alpha(t-1) \\
\alpha(t) &= \alpha(t-1) + w(t)\ , \qquad (4)
\end{aligned}$$

where $\{w(t)\}$ forms a stationary zero-mean white noise sequence,
(A2) The sequence $\{w(t)\}$ is Gaussian distributed, $w \sim \mathcal{N}(0, \sigma_w^2)$,
(A3) The sequence $\{v(t)\}$, independent of $\{w(t)\}$, is a zero-mean complex Gaussian white noise, $v \sim \mathcal{CN}(0, \sigma_v^2)$,
(A4) The magnitude of the narrowband signal is constant, $|s(t)| \equiv a_0$,

the algorithm (3) can be made statistically efficient, i.e. it can reach so-called Posterior Cramér-Rao bounds[1] which limit mean-squared frequency and frequency rate tracking errors. Although closed-form expressions for the optimal values of gains μ, γ_ω, γ_α do not exist, it was shown that they depend only on the following 'normalized' measure of signal nonstationarity [7]

$$\kappa = \frac{a_0^2 \sigma_w^2}{\sigma_v^2} \qquad (5)$$

which combines signal to noise ratio and frequency variability.

[1]Classical Cramér-Rao Bound applies to systems with unknown deterministic parameters. Posterior Cramér-Rao Bound applies to systems with random parameters, such as (1)-(2).

In practical situations the above parameter is unknown and possibly time-varying. Therefore, adaptation gains of the filter must be hand-tuned using some cost criteria. Unfortunately, when one is primarily interested in frequency tracking, determining the optimal values of the adaptation gains is difficult because it is unclear how one could measure frequency tracking performance.

A hand-on approach could rely on minimization of mean-squared prediction errors $\epsilon(t)$ yielded by the filter. This would actually correspond to the optimization of signal tracking properties of the filter [8]. However, the settings which minimize signal tracking errors are different from those which minimize frequency tracking errors [7].

To demonstrate this discrepancy between signal and frequency tracking properties of the ANF it is sufficient to perform a simple simulation experiment. For the purpose of such demonstration we used a nonstationary complex sinusoid with instantaneous frequency and amplitude governed by

$$\begin{aligned}
\omega(t) &= 0.2 + 0.1\cos(2\pi t/2000) \\
a(t) &= 3 + \sin(2\pi t/500)\ , \qquad (6)
\end{aligned}$$

where $t \in [0, 5000]$. The variance of the wideband noise was $\sigma_v^2 = 0.01$.

Fig. 1 shows the steady-state (the first 1000 samples of the output was discarded to guarantee that steady-state conditions were reached) mean-squared signal tracking errors $\Delta s(t) = s(t) - \widehat{s}(t)$, frequency tracking errors $\Delta\omega(t) = \omega(t) - \widehat{\omega}(t)$ and prediction errors $\epsilon(t)$ for different settings of the filter. To reduce the number of degrees of freedom, the gains μ, γ_ω, γ_α were set according to the following rule of thumb, suggested in [7],

$$\gamma_\omega = \mu^2/2 \qquad \gamma_\alpha = \mu^3/8\ . \qquad (7)$$

It is clear from the results that signal prediction errors are not a good measure of frequency tracking performance and a different quantity must be used for this purpose.

3. PROPOSED WAY OF ASSESSING FREQUENCY TRACKING ERRORS

Prior to proposing a meaningful way of evaluating frequency tracking errors, one should gain some insight into operation of the filter (3). Let $|s(t)|^2 = a_0^2 \equiv \text{const}$, and $e(t) = -\mathrm{Im}\ [v(t)s^*(t)/a_0^2]$. Note that, when (A3) holds, $\{e(t)\}$ is a zero mean Gaussian (real-valued) white noise with variance $\sigma_e^2 = \sigma_v^2/2a_0^2$.

Using the approximating linear filter technique, introduced in [9] for the purpose of analyzing tracking properties of ANF algorithms, it can be shown that, under good tracking conditions, i.e. when $\Delta\omega(t) \approx 0$, the following relationship holds [7]

$$\Delta\omega(t) = H_1(q^{-1})e(t) + H_2(q^{-1})w(t)\ ,$$

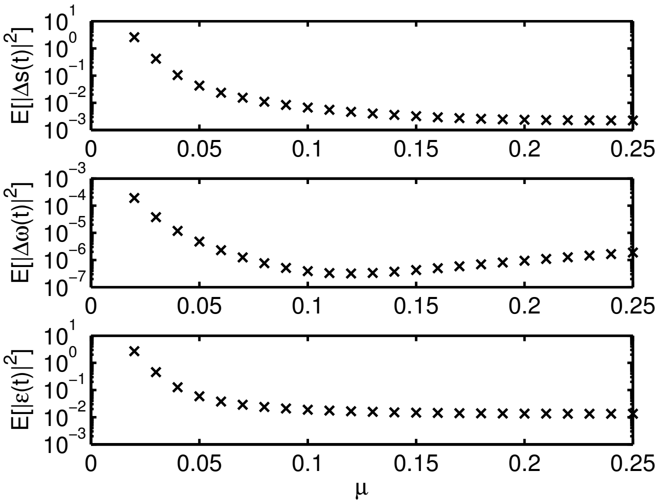

Fig. 1. Comparison of the steady state mean squared signal tracking errors $\Delta s(t) = s(t) - \widehat{s}(t)$, frequency tracking errors $\Delta\omega(t) = \omega(t) - \widehat{\omega}(t)$ and prediction errors $\epsilon(t)$ for different settings of the filter.

where

$$H_1(q^{-1}) = \frac{(1-q^{-1})[\gamma_\omega + (\gamma_\alpha - \gamma_\omega)q^{-1}]}{D(q^{-1})}$$
$$H_2(q^{-1}) = \frac{q^{-1}[1-\gamma_\omega - (1-\mu)q^{-1}]}{D(q^{-1})}$$
$$D(q^{-1}) = 1 + d_1q^{-1} + d_2q^{-2} + d_3q^{-3} \tag{8}$$
$$d_1 = \mu + \gamma_\omega + \gamma_\alpha - 3$$
$$d_2 = 3 - 2\mu - \gamma_\omega$$
$$d_3 = \mu - 1 \ .$$

The filters $H_1(q^{-1})$, $H_2(q^{-1})$ are asymptotically stable if the following (sufficient) conditions hold: $0 < \mu < 1$, $0 < \gamma_\omega < 1$, $0 < \gamma_\alpha < 1$ and $\mu(\gamma_\omega + \gamma_\alpha) > \gamma_\alpha$.

Employing the fact that [c.f. (4)]

$$\omega(t) = \frac{w(t-1)}{(1-q^{-1})^2}$$

it is straightforward to arrive at

$$\widehat{\omega}(t) = Q(q^{-1})[\omega(t) + (1-q^{-1})e(t)] \ , \tag{9}$$

where

$$Q(q^{-1}) = \frac{H_1(q^{-1})}{1-q^{-1}} = \frac{\gamma_\omega + (\gamma_\alpha - \gamma_\omega)q^{-1}}{D(q^{-1})} \ . \tag{10}$$

Observe that the estimates yielded by the notch filter may be treated as a result of processing the signal

$$u(t) = \omega(t) + (1-q^{-1})e(t) \tag{11}$$

with the filter $Q(q^{-1})$. It is therefore the signal $u(t)$, rather than $y(t)$, which plays the role of noisy 'measurements' in the frequency estimation problem.

Furthermore note that, even though $u(t)$ is not directly accessible, it may be recovered, whenever necessary, by means of inverse filtering frequency estimates $\widehat{\omega}(t)$

$$u(t) = \frac{1}{Q(q^{-1})}\widehat{\omega}(t) \ . \tag{12}$$

It follows from the above discussion that the quality of frequency estimates could be measured using a one-step predictor $\widehat{u}(t|t-1)$ of the signal $u(t)$. To make the proposed approach sound, one should design the predictor in such a way so as to 'share' its settings with that of the ANF. Such a predictor can be designed using Wiener approach.

It can be shown that the transfer function of the optimal (in the mean-square sense) Wiener predictor of the signal $u(t)$ takes the form (see [10] for details of the derivation)

$$X(q^{-1}) = \frac{N_P(q^{-1})}{N(q^{-1})} \ . \tag{13}$$

where $N_P(q^{-1})$, $\deg N_P(q^{-1}) = \deg N(q^{-1}) - 1$ is the solution of the following Diophantine equation

$$q\alpha A(q^{-1}) + N_P(q^{-1}) = qN(q^{-1}) \ .$$

and $N(q^{-1})$ is a stable transfer function such that

$$N(q^{-1})N(q) = \sigma_e^2 + A(q^{-1})A(q)C(q^{-1})C(q)\sigma_w^2$$
$$A(q^{-1}) = (1-q^{-1})^2 = 1 - 2q^{-1} + q^{-2}$$
$$C(q^{-1}) = (1-q^{-1}) \tag{14}$$

On the other hand, the transfer function $Y(q^{-1})$ of the optimal, in the mean-squared sense, estimator of $\omega(t)$ takes the form

$$Y(q^{-1}) = \frac{F(q^{-1})}{N(q^{-1})} \ . \tag{15}$$

Note that, under (A1)-(A4), for the optimal values of adaptation gains of the algorithm (3) it must hold that $Q(q^{-1}) = Y(q^{-1})$, i.e.

$$D(q^{-1}) = cN(q^{-1}) \ ,$$

where c is some constant. This also means that

$$X(q^{-1}) = \frac{D_P(q^{-1})}{D(q^{-1})} \ , \tag{16}$$

where

$$D_P(q^{-1}) = (2+d_1) + (d_2 - 1)q^{-1} + d_3q^{-2} \tag{17}$$

solves

$$q\beta A(q^{-1}) + D_P(q^{-1}) = qD(q^{-1}) \ .$$

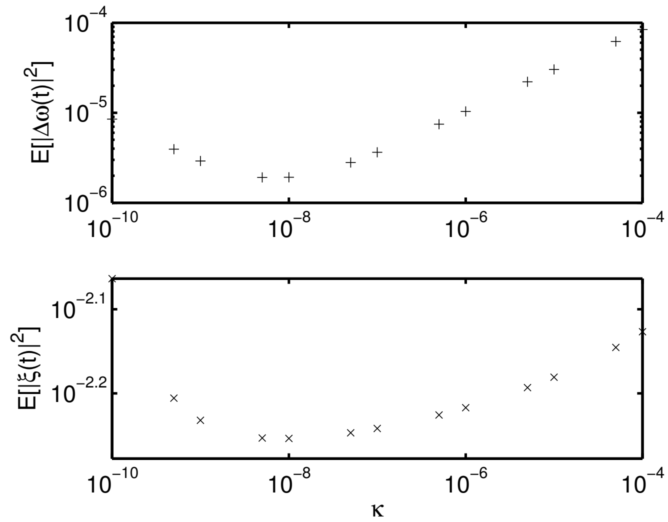

Fig. 2. Comparison of the mean-squared steady-state frequency tracking errors yielded by the adaptive notch filter with the mean-squared prediction errors yielded by the proposed predictor for frequency changes governed by the integrated random walk model.

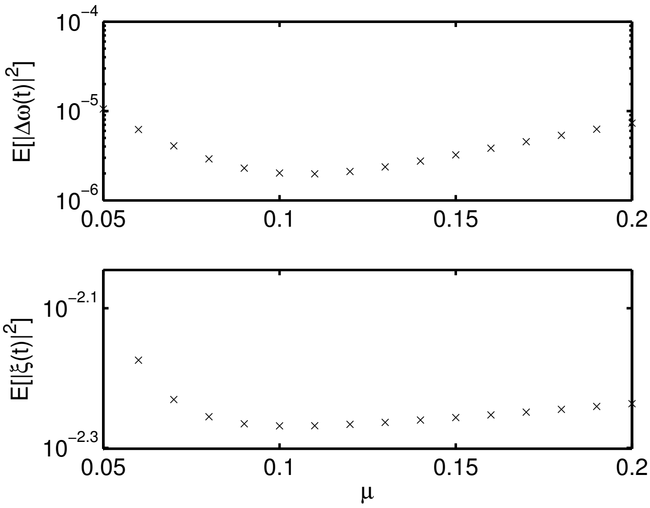

Fig. 3. Comparison of the mean-squared steady-state frequency tracking errors yielded by the adaptive notch filter with the mean-squared prediction errors yielded by the proposed predictor for sinusoidal frequency and amplitude changes.

Combining all the partial results, after some elementary calculations, one can arrive at the following expression for computing prediction errors of $u(t)$

$$\begin{aligned}\xi(t) &= u(t) - \widehat{u}(t|t-1) = [1 - q^{-1}X(q^{-1})]u(t) \\ &= \frac{[1 - q^{-1}X(q^{-1})]}{Q(q^{-1})}\widehat{\omega}(t) \\ &= \frac{1 - 2q^{-1} + q^{-2}}{\gamma_\omega + (\gamma_\alpha - \gamma_\omega)q^{-1}}\widehat{\omega}(t) \ . \qquad (18)\end{aligned}$$

4. SIMULATION RESULTS

4.1. Predictor verification

In order to check how well the performance of the proposed predictor matches frequency estimation performance of the ANF algorithm (3), two computer simulations were conducted.

In the first simulation, frequency changes were governed by the integrated random-walk model (A1)-(A4) with $\sigma_w^2 = 10^{-8}$, $|a_0|^2 = 100$ and $\sigma_v^2 = 1$, i.e. $\kappa = 10^{-8}$. Several ANFs of the form (3), with adaptation gains optimized for κ ranging from 10^{-10} to 10^{-4} (the optimal values of adaptation gains were found using numerical methods, see [7] for more details) were used for frequency estimation.

Fig. 2 shows the comparison of mean-squared frequency estimation errors yielded by the algorithm (3) for different settings with the corresponding mean-squared prediction errors yielded by (18). Note the agreement between shapes of both curves. In particular the minima of both curves coincide.

In the second simulation, the profiles of instantaneous frequency and amplitude were modified to more realistic ones

$$\begin{aligned}\omega(t) &= 0.2 + 0.1\cos(2\pi t/2000) \\ a(t) &= 3 + \sin(2\pi t/500) \ . \qquad (19)\end{aligned}$$

The gains of the filters were now set as follows: $\mu_k \in [0.05,\ 0.25]$, $\gamma_{\omega,k} = \mu_k^2/2$, $\gamma_{\alpha,k} = \mu_k^3/4$. The results of the experiment, shown in Fig. 3, confirm that the proposed predictor can be used to assess the performance of algorithm (3) filters, not only for the 2nd order random walk type changes, but also in a realistic setup.

4.2. Parallel tracker

Denote by $T(t) = [t - M - 1,\ t]$ the local evaluation window consisting of M samples. Combining (3), (12) and (16) one can propose the following parallel frequency tracking scheme.

A bank of K adaptive notch filters of the form (3) with gains μ_k, $\gamma_{\omega,k}$, $\gamma_{\alpha,k}$, $k = 1,\ 2,\ \ldots$ provides (independent) partial estimates $\widehat{\omega}_k(t)$, $k = 1,\ 2,\ \ldots, K$. The quality of all partial estimates is evaluated using (18) and the best performing filter is selected using the following mechanism

$$k_*(t) = \arg\min_{k=1,2,\ldots,K} \left[\sum_{i=t-M+1}^{t} |\xi_k(i)|^2 \right] ,$$

At any time instant, the output of the parallel estimator corresponds to the locally best estimate

$$\widehat{\omega}(t) = \widehat{\omega}_{k_*(t)}(t) \ .$$

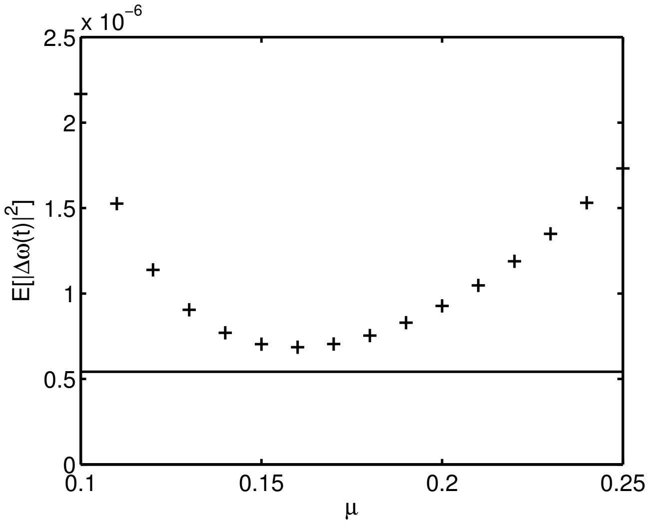

Fig. 4. Comparison of the mean-squared frequency tracking errors of the individual filters (pluses) and the parallel scheme (line) for $M = 50$.

Performance of the proposed parallel scheme was evaluated using the following two-mode signal

$$\omega(t) = \begin{cases} 0.3 & \text{for t} < 2000 \\ 0.3 + 0.1\cos(2\pi t/1000) & \text{for t} \geq 2000 \end{cases}$$
$$a(t) = 3 + \sin(2\pi t/500)\ . \tag{20}$$

Note that the above signal incorporates two sources of nonstationarity: magnitude variation and frequency variation. The variance of wideband measurement noise in the experiment was $\sigma_v^2 = 0.01$.

A total of sixteen ANFs was used to provide partial estimates. The gains of preliminary estimators was set as $\mu_k = 0.05 + 0.01(k-1)$, $\gamma_{\omega,k} = \mu_k^2$, $\gamma_{\alpha,k} = \mu_k^3/8$, $k = 1,\ 2,\ \ldots,\ 16$.

Fig. 4 compares mean-squared frequency tracking errors of individual filters in the bank with the final estimate for $M = 50$. Note that the accuracy of the parallel scheme exceeds performance of all the ANFs making up the bank. This confirms that the parallel scheme is capable of adjusting its properties to time-varying conditions.

The experiment was repeated with noise variance increased by 10 and 20 dB, respectively. In both cases the parallel solution yielded smaller ($2.47 \cdot 10^{-6}$ and $9.06 \cdot 10^{-6}$, respectively) mean squared errors than the best tuned filter from the bank ($2.91 \cdot 10^{-6}$ and $10.32 \cdot 10^{-6}$, respectively).

5. CONCLUSIONS

The problem of estimating instantaneous frequency of a nonstationary complex sinusoid (cisoid) buried in wideband noise was considered. We proposed a nontrivial extension of the adaptive notch filtering algorithm which enables one to evaluate its frequency tracking performance without any prior knowledge of true frequency values. The new assessment mechanism was successfully employed in a parallel frequency tracker, which was shown to cope well with unknown and time-varying conditions.

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