ROBUST CODING AND MODULATION FOR BODY-AREA NETWORKS

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ABSTRACT

We consider channel modeling for transmission over body-area networks. Due to the difficulty in assessing an accurate statistical model valid for multiple scenarios, we advocate a technique that favors robustness. Our calculations, which follow in the footsteps of the results in [9], and generalize them, allow us to determine the variation of a performance metric when the nominal statistical distribution of fading is replaced by the worst distribution within a given Kullback–Leibler divergence from it. The sensitivity of the performance metric to the divergence from the nominal distribution can be used as an indication of the model robustness. This concept is applied by evaluating error probability for binary uncoded modulation and outage probability—the first parameter is useful to assess system performance with no error-control coding, while the second reflects the performance when a near-optimal code is used.

Index Terms— Body-area network, Channel model, Channel uncertainty, Coding and modulation.

1. INTRODUCTION AND MOTIVATION OF THIS WORK

Body-area networks (BANs), which use the human body to support communication using low-power wireless sensor network technology, of late have been attracting a considerable interest [8]. Now, modeling the transmission channel to allow reliable communication through a BAN is a challenging problem. First, communication takes place on different types of links, depending on the body parts to which transmit and receive antennas are attached, e.g., trunk-to-trunk, trunk-to-head, trunk-to-hand [1], on where the hardware is located (body-to-body, off-body, on-body, and in-body links [6]), and on antenna type and orientation, body size, location, and posture [7]. In addition, as observed in [6], propagation in on-body links "may be a combination of surface wave, creeping wave, diffracted waves, scattered waves, and free space propagation, depending on the antenna positions and the body postures" (see also [8] and references therein). Refs. [5, 11] describe analyses and measurements leading to a lognormal-distribution model for slow fading, and a Rice-distribution model for fast fading in single-link communication. The use of multiple-input, multiple-output (MIMO) systems in BANs has been first advocated in [12], and MIMO BAN channel models were discussed in [6] and [7].

Even if adaptive techniques are used to adjust modulation and coding to the changing environment, reliable mathematical models for the transmission channel are called for, which are difficult to obtain because of the variations of the environment in which the transmission is taking place. In this paper, we argue that the design of modulation and coding schemes in BANs should be based on their *robustness* to uncertainties of channel model. Specifically, we study how performance varies as the channel model runs through an *uncertainty set* surrounding the nominal one. We do this, following in the footsteps of [9], by examining the system performance as a function of the distance between the nominal distribution of channel statistics and the worst distribution in the uncertainty set. Based on this concept, the robustness of system design to channel modeling can be assessed.

To measure the distance of a probability measure $\mathcal{P}$ from a reference measure $\mathcal{P}_0$, we use the Kullback–Leibler (K–L) divergence $D(\mathcal{P} \,\|\, \mathcal{P}_0)$ [4, p. 18 ff.].[1] The solution of an optimization problem allows one to determine the worst distribution within a given K–L divergence from the nominal distribution, and assess the system performance when the former is used in lieu of the latter (see Fig. 1). A numerical example of K–L divergence is provided in Fig. 2, which shows the values of the divergence between Rayleigh density and Rice density [2, pp. 27-29] with parameter K (as usual, K denotes the ratio between the signal power in the dominant component and the scattered power).

The mathematical problem of evaluating a performance metric vs. the K–L divergence between the nominal and the worst distribution is described and solved in next section.

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[1] The K–L divergence is always positive, except when $\mathcal{P} = \mathcal{P}_0$. Yet, it is technically not a distance—it is not symmetric, as generally $D(\mathcal{P} \,\|\, \mathcal{P}_0) \neq D(\mathcal{P}_0 \,\|\, \mathcal{P})$. Thus, it can be interpreted as a "directed distance" between two probability models. A discussion of this point can be found in [9, Section IV].

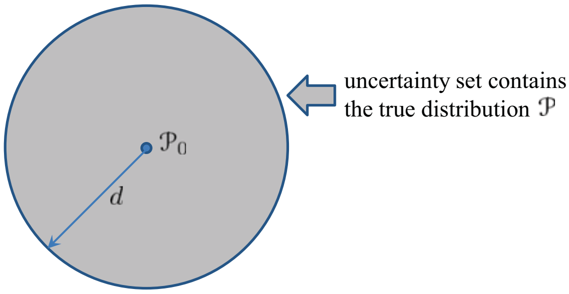

Fig. 1. *Illustration of the search for the worst distribution within an uncertainty set of distributions $\mathcal{P}$ having a K–L divergence $\leq d$ from the nominal distribution $\mathcal{P}_0$.*

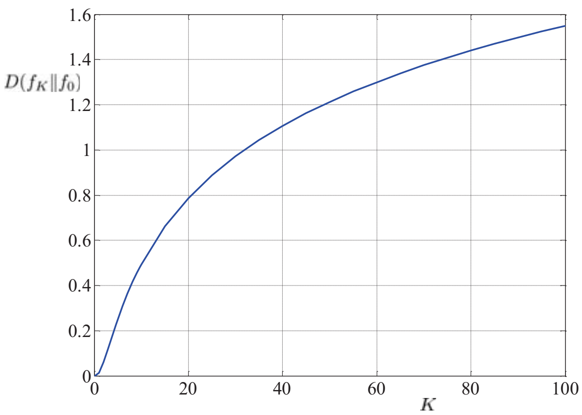

Fig. 2. *K–L divergence $D(f_K \| f_0)$ between the Rayleigh density f_0 and the Rice density with parameter K, denoted f_K.*

This method is applied in Section 3 to the evaluation of two performance metrics, viz., error probability for binary uncoded modulation and of outage probability—the first parameter being useful to assess system performance with no error-control coding, the second reflecting the performance when a near-optimal code is used.

2. THE OPTIMIZATION PROBLEM

A nominal probability measure $\mathcal{P}_0$ is assumed for a channel random parameter X. Assume that the *actual* probability measure $\mathcal{P}$ has a K–L divergence $D(\mathcal{P} \| \mathcal{P}_0)$ from $\mathcal{P}_0$ less than or equal to a given value d. If the system performance is evaluated as the expected value of a known function h of the random variable X, following [9] (see also [13] and references therein) we consider the solution of the optimization problem

$$(P') \qquad \max_{\mathcal{P}} \mathbb{E}\, h \qquad \text{s.t.} \quad D(\mathcal{P} \| \mathcal{P}_0) \leq d$$

i.e., the search for the distribution whose K–L divergence is within d from the nominal distribution $\mathcal{P}_0$ and yields the maximum (i.e., worst) value of the cost function $\mathbb{E}\, h$.

Introducing the inaccurately modeled random variable X and the probability density functions f, f_0 corresponding to measures $\mathcal{P}$, $\mathcal{P}_0$, respectively, we may explicitly rewrite problem (P') in the equivalent form

$$\begin{aligned}(P) \qquad & \max_f \int h(x) f(x)\, dx \\ & \text{s.t.} \quad \int \log \frac{f(x)}{f_0(x)} f(x)\, dx \leq d \\ & \qquad\;\; \int f(x)\, dx = 1\end{aligned}$$

(Condition $f(x) \geq 0$ should be added unless automatically satisfied by the solution of (P).)

Since the objective of (P) is linear and the constraint is convex, then (P) is a convex optimization problem [9, p. 6834]. The Lagrangian of (P) is given by

$$\begin{aligned} L = & \int h(x) f(x)\, dx \qquad (1) \\ & - \nu^{-1} \left(\int \log \frac{f(x)}{f_0(x)} f(x)\, dx - d \right) - \mu \left(\int f(x)\, dx - 1 \right) \end{aligned}$$

Taking the functional derivative of L with respect to f, the Karush–Kuhn–Tucker (KKT) conditions can be written in the form

$$h(x) - \nu^{-1} \left(\log \frac{f(x)}{f_0(x)} + 1 \right) - \mu = 0 \qquad (2a)$$

$$\int f(x)\, dx = 0 \qquad (2b)$$

$$\nu^{-1} \left(\int \log \frac{f(x)}{f_0(x)} f(x)\, dx - d \right) = 0 \qquad (2c)$$

$$\nu^{-1} \geq 0 \qquad (2d)$$

For $\nu > 0$ the maximum is achieved at the boundary (see (2c)). In this case, (2a) and (2b) yield the form of the optimizing $f(x)$:

$$f^\star(x) = \frac{e^{\nu^\star h(x)} f_0(x)}{\xi(\nu^\star)} \qquad (3)$$

where

$$\xi(\nu) \triangleq \int e^{\nu h(x)} f_0(x)\, dx, \qquad (4)$$

$\nu^\star$ is the solution of

$$\nu \frac{\xi'(\nu)}{\xi(\nu)} - \log \xi(\nu) = d, \qquad (5)$$

and

$$\xi'(\nu) \triangleq \frac{d\xi(\nu)}{d\nu} = \int h(x) e^{\nu h(x)} f_0(x)\, dx \tag{6}$$

The resulting minimum value of $\mathbb{E}\, h$, denoted $p_{\max}$, is given by

$$p_{\max} = \frac{\xi'(\nu^\star)}{\xi(\nu^\star)} \tag{7}$$

that is, $p_{\max}$ is the slope of the logarithmic derivative of $\xi(\nu)$ at $\nu = \nu^\star$.

Define $p_0 \triangleq \int h(x) f_0(x)\, dx$, and observe that, since $\xi(0) = 1$ and $\xi'(0) = p_0$, (5) has the solution $\nu^\star = 0$ for $d = 0$, which implies that $d = 0$ yields $p_{\max} = p_0$, as it should be. In addition, it can be proved that $p_{\max}(d)$ increases with d.

2.1. A special case

An important special case, examined in [9], occurs when $h(x)$ turns out to be the indicator function of an interval $\mathcal{I}$:

$$h(x) = \begin{cases} 1, & x \in \mathcal{I} \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

so that

$$\mathbb{E}\, h(X) = \mathbb{P}[X \in \mathcal{I}] \tag{9}$$

In this case, we have explicitly

$$\xi(\nu) = e^\nu \int_{\mathcal{I}} f_0(x)\, dx + \int_{\bar{\mathcal{I}}} f_0(x)\, dx \tag{10}$$

$$= p_0 e^\nu + (1 - p_0) \tag{11}$$

$$= 1 + p_0(e^\nu - 1) \tag{12}$$

where

$$p_0 \triangleq \int_{\mathcal{I}} f_0(x)\, dx, \tag{13}$$

and also

$$\eta(\nu) = p_0 e^\nu \tag{14}$$

Thus,

$$p_{\max} = \frac{p_0 e^{\nu^\star}}{1 + p_0(e^{\nu^\star} - 1)} \tag{15}$$

where $\nu^\star$ is the solution of

$$-\log\left(1 + p_0(e^\nu - 1)\right) + \frac{p_0 \nu e^\nu}{1 + p_0(e^\nu - 1)} = d \tag{16}$$

For $d = 0$ the solution is $\nu^\star = 0$, which yields $p_{\max} = p_0$. When $d = \log(1/p_0)$ we have $p_{\max} = 1$, and $\nu^\star \to \infty$.

Fig. 3 shows the behavior of $p_{\max}$ vs. the value of the K–L divergence d. As stressed in [9], in the special case examined in this section the nominal distribution enters $p_{\max}$ only via the nominal probability p_0.

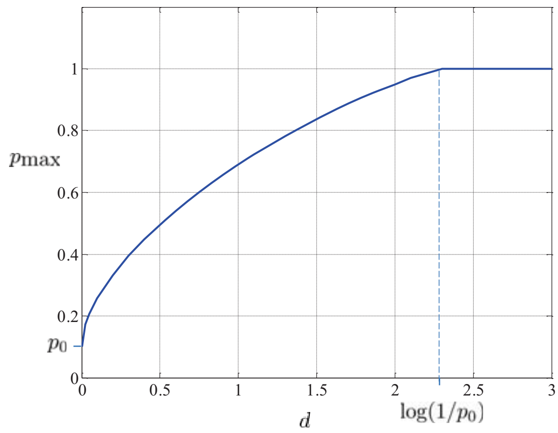

Fig. 3. $p_{\max}$ *vs.* d. *Here* $p_0 = 0.1$.

3. APPLICATIONS

3.1. Binary error probability

The general expression for the error probability of uncoded binary antipodal modulation with equally likely signals, under the assumption of ergodic fading with amplitude R, additive white Gaussian noise with power spectral density $N_0/2$, and perfect channel state information at the receiver, is

$$p = \mathbb{P}[\sqrt{R^2 \mathcal{E}} + n < 0] \tag{17}$$

$$= \mathbb{E}_R Q\left(R\sqrt{2\,\mathsf{snr}}\right) \tag{18}$$

where $\mathsf{snr} \triangleq \mathcal{E}/N_0$. With Rayleigh fading, we have

$$p_0 = \frac{1}{2}\left[1 - \sqrt{\frac{\mathsf{snr}}{1 + \mathsf{snr}}}\right] \tag{19}$$

The resulting variation of $p_{\max}$ with $d = D(f_K \| f_0)$ and f_K, f_0 as in Fig. 2 is illustrated in Fig. 4. Fig. 5 shows the evolution of $f^\star(x)$ with d for $\mathsf{snr} = 0$ dB.

3.2. Outage probability

On a nonergodic channel affected by fading with random gain R and additive white Gaussian noise, the *information outage probability*, i.e., the probability that the transmission rate ρ bits per channel use exceeds the instantaneous mutual information of the channel, is given by [2, Chapter 4]

$$p_{\text{out}} = \mathbb{P}[\log_2(1 + R^2\mathsf{snr}) < \rho] \tag{20}$$

With a nonergodic channel, this is the information-theoretical rate limit which cannot be exceeded by the word error probability of any coding scheme, and hence can be utilized for estimating the error probability of coded systems [3, 10]. From (20) we obtain

$$p_{\text{out}} = \mathbb{P}[R \in \mathcal{I}] \tag{21}$$

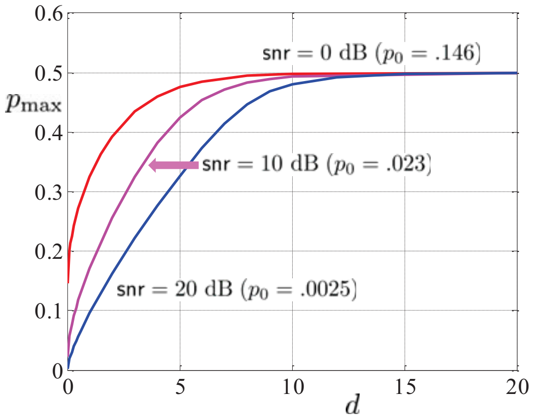

Fig. 4. *p_{out} vs. d for binary error probability with* $\mathsf{snr}=0$ *dB,* $\mathsf{snr}=10$ *dB, and* $\mathsf{snr}=20$ *dB.*

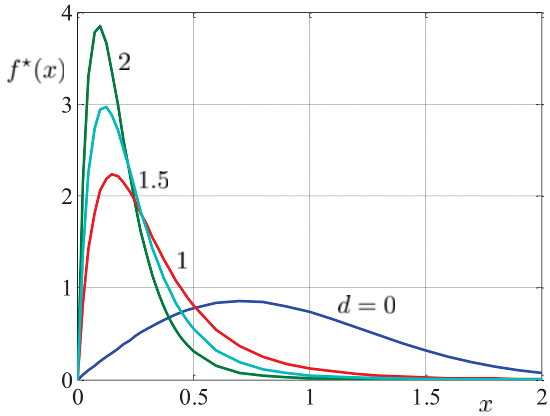

Fig. 5. *"Worst" probability density functions $f^\star(x)$ as in Fig. 4 with* $\mathsf{snr}=0$ *dB for various values of d.*

where

$$\mathcal{I} = \left(0, \sqrt{(2^\rho - 1)/\mathsf{snr}}\right)$$

In the special case of Rayleigh-distributed fading, i.e., a probability density function of R given by

$$f_0(r) = 2re^{-r^2}, \qquad r \geq 0$$

we obtain

$$p_{\text{out},0} = 1 - \exp[-(2^\rho - 1)/\mathsf{snr}]$$

We can evaluate the robustness of outage probability to fading model uncertainty using the previous theory, and in particular Section 2.1 and Fig. 3.

4. CONCLUSIONS

Efficient channel modeling for transmission in body area networks is a challenging task due to the difficulty in assessing an accurate statistical model valid for multiple scenarios. We advocate a scheme that favors robustness in terms of coding and modulation. To study robustness, we determine the variation of a performance metric when the nominal statistical distribution of fading is replaced by the worst distribution having a given Kullback–Leibler divergence from it. This concept is applied to two performance metrics, viz., error probability and outage probability.

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