NONLINEARITY CORRECTION ALGORITHM FOR WIDEBAND FMCW RADARS

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ABSTRACT

This paper presents a novel nonlinearity correction algorithm for wideband frequency modulated continuous wave (FMCW) radars based on high-order ambiguity functions (HAF) and time resampling. By emphasizing the polynomial-phase nature of the FMCW signal, it is shown that the HAF is an excellent tool for estimating the sweep nonlinearity polynomial coefficients. The estimated coefficients are used to build a correction function which is applied to the beat signal by time resampling . The nonlinearity correction algorithm is tested by simulation and validated on real data sets acquired with an X-band FMCW radar.

Index Terms— Frequency Modulated Continuous Wave (FMCW), Nonlinearity Correction, High-Order Ambiguity Function (HAF), Time Resampling.

1. INTRODUCTION

The frequency modulated continuous wave (FMCW) radar principle is currently used in applications such as radio-altimeters, navigation systems [1] or in sensors for inhomogeneity identification on transmission lines [2]. A classical problem of this type of radar is that the voltage controlled oscillator (VCO) adds a certain degree of nonlinearity which leads to a deteriorated resolution by spreading the target energy through different frequencies [3]. This problem is usually solved either by hardware [4] or software [5, 6, 7] approaches. The simplest hardware correction is the use of a predistorted VCO control voltage to have a linear frequency modulation output. However, this approach does not work when the external conditions (e.g. temperature, supply voltage) change. Software solutions typically use an additional path in the transceiver with a known propagation delay for estimating the nonlinearities. The estimation is done either by breaking the beat signal into several sub-bands and computing the frequency peaks [7] or by extracting the phase information from the analytical signal [5, 6].

This paper proposes a novel nonlinearity correction algorithm for large bandwidth nonlinearities. The method is de-signed for nonlinearities that can be described by a polynomial expression which leads to a polynomial-phase FMCW signal. The coefficients of the polynomial-phase signal (PPS) are estimated using the high-order ambiguity function (HAF) [8] on a reference response which can be either a delay line or a high reflectivity target whose propagation delay should be roughly known. Afterwards, with the estimated coefficients the nonlinearity correction function is built and applied through a time resampling procedure. The correction algorithm is tested by simulation and validated on real data from an X-band FMCW radar.

The algorithm presented in this paper differs from previous works in two ways. On one hand, a typical nonlinearity estimation method (used for example in Vossiek's work [5]) based on the determination of the instantaneous phase of a precision radar reference path is valid only for a single component response, while the HAF-based estimation can extract the nonlinearity coefficients from a multi-component response if there is one highly reflective target relative to other scatterers. On the other hand, the correction method proposed in [6] by Meta et al. needs to up-sample the beat signal in order to satisfy the Nyquist condition for the nonlinearity bandwidth and consequently this method is not well suited for large bandwidth nonlinearities (up to gigahertz). In the algorithm proposed in this paper the processing is applied to the beat frequency signal, so the bandwidth of the nonlinear term from the transmitted signal doesn't impose the sampling rate.

The rest of this paper is organized as follows. Section 2 presents the nonlinearity correction algorithm. The nonlinear FMCW signal model is introduced first. Then, the estimation method of the FMCW signal coefficients is described. The time resampling based correction procedure is exposed in the last part of the second section. Simulation results demonstrate the efficiency of the algorithm in Section 3. Section 4 shows results of the developed nonlinearity correction algorithm applied to real data collected with a FMCW radar demonstrator system. Finally, the conclusions are stated in Section 5.

2. NONLINEARITY CORRECTION ALGORITHM

In [9] it is mentioned that the slope of the frequency-voltage characteristic for some VCOs may be reasonably approximated by a quadratic curve. However, a more general approach is to assume a polynomial frequency-voltage dependence. With this assumption, for a linear tuning voltage sweep, the transmitted analytical signal in a sweep period T_p can be written as:

$$s_T(t) = \exp\left[j2\pi\left(f_0 t + \frac{1}{2}\alpha_0 t^2 + \sum_{k=2}^{K}\frac{\beta_k}{k+1}t^{k+1}\right)\right],$$
(1)

where f_0 is the initial frequency, α_0 is the linear chirp rate in the origin and β_k with $k = \overline{2, K}$ are the nonlinearity coefficients. In the following, it is considered that the reflected signal comes from N different targets. The signal received from these targets can be expressed as a sum of delayed and attenuated versions of the transmitted signal with a certain complex amplitude:

$$s_R(t) = \sum_{i=1}^{N} A_i s_T(t - \tau_i),$$
(2)

where τ_i and A_i are the propagation delay and amplitude of the signal received from target i.

By mixing the transmitted and received signals, the beat signal is written as:

$$s_b(t) = \sum_{i=1}^{N} A_i s_T(t) s_T^*(t - \tau_i).$$
(3)

Due to the fact that the propagation delay τ is typically a few orders of magnitude smaller than the sweep period, the higher-order terms in τ can be neglected. In consequence the beat signal becomes a sum of polynomial-phase signals:

$$s_b(t) = \sum_{i=1}^{N} s_b(t, \tau_i),$$
(4)

where

$$s_b(t, \tau_i) = A_i \exp\left[j2\pi\left(f_0 + \alpha_0 t + \sum_{k=2}^{K}\beta_k t^k\right)\tau_i\right].$$
(5)

If the range profile is computed as the Fourier transform of the multi-component PPS signal in (4), the energy of each target would be spread and the resolution deteriorated.

The algorithm proposed in this paper aims to eliminate this effect by turning the multi-component PPS into a sum of N complex sinusoids. In this way each target appears as a *sinc* function in the range profile. The correction algorithm

consists of two steps: an estimation of the FMCW signal coefficients (linear chirp rate α_0 and nonlinearity coefficients β_k) using the high-order ambiguity function and a correction of the beat signal by time resampling.

2.1. Estimation of the FMCW signal coefficients

The estimation is based on the presence of a reference target response (with amplitude A_{ref} and propagation delay τ_{ref}) in the FMCW signal. This particular PPS component can be extracted by bandpass filtering the beat signal around the beat frequency corresponding to τ_{ref} (which means selecting a certain range interval centered on the reference target). The filtered signal can be written as:

$$s_{b,f}(t) = s_{b,f}(t, \tau_{ref}) + \sum_{m=1}^{M} s_{b,f}(t, \tau_m),$$
(6)

where M is the number of significant PPS components located near the reference response in the filter's pass band which cannot be eliminated. In the estimation, is considered that the reference target is highly reflective relative to the remaining M components. Although the filtered signal has other components besides the reference response, the FMCW signal coefficients can be estimated using the high-order ambiguity function due to its ability to deal with multiple component PPS's if the highest order phase coefficients of the components are not the same [10, 11] (as happens for the FMCW signal because each component is linked to a target with a certain propagation delay).

The estimation procedure starts from the high-order instantaneous moment (HIM), which can be defined for a signal $s(t)$ as [8]:

$$\text{HIM}_k[s(t); \tau] = \prod_{i=0}^{k-1}\left[s^{(*i)}(t - i\tau)\right]^{\binom{k-1}{i}},$$
(7)

where k is the HIM order, τ is the lag and $s^{(*i)}$ is an operator defined as:

$$s^{(*i)}(t) = \begin{cases} s(t) & \text{if } i \text{ is even,} \\ s^*(t) & \text{if } i \text{ is odd,} \end{cases}$$
(8)

where i is the number of conjugate operator "*" applications. The high-order ambiguity function (HAF) is defined as the Fourier transform of the HIM.

If we assume a PPS model for the analyzed signal, i.e.:

$$s_{\text{PPS}}(t) = A \exp\left[j2\pi\sum_{m=0}^{k} a_m t^m\right],$$
(9)

the essential property of the HIM is that, the kth order HIM is reduced to a harmonic with amplitude $A^{2^{k-1}}$, frequency $\tilde{f}_k$ and phase $\tilde{\Phi}_k$:

$$\mathrm{HIM}_k[s_{\mathrm{PPS}}(t);\tau] = A^{2^{k-1}}\exp\left[j\left(2\pi\tilde{f}_k t + \tilde{\Phi}_k\right)\right],\quad (10)$$

where

$$\tilde{f}_k = k!\tau^{k-1}a_k. \qquad (11)$$

So the HAF of this HIM should have a spectral peak at the frequency $\tilde{f}_k$. Based on this result, an algorithm that estimates sequentially the coefficients a_k was proposed in [12]. Starting with the highest order coefficient, at each step, the spectral peak is determined, and an estimation value $\hat{a}_k$ of a_k is computed from (11). With this value, the phase term of the higher order is removed:

$$s_{\mathrm{PPS}}{}^{(k-1)}(t) = s_{\mathrm{PPS}}{}^{(k)}(t)\exp\left(-j2\pi\hat{a}_k t^k\right) \qquad (12)$$

and the procedure repeats iteratively. A classical problem of this nonlinear method is the propagation of the approximation error from one higher order to the lower ones, but in the case of typical frequency-voltage VCO characteristics this effect is not critical because an approximation order of only 3 or 4 is required. Still, if a higher order is necessary a warped-based polynomial order reduction as described in [13] could be employed.

After applying this iterative algorithm to the FMCW reference signal and obtaining the polynomial phase coefficients, the linear chirp rate and the nonlinearity coefficients can be computed as:

$$\begin{aligned}\alpha_0 &= \frac{\hat{a}_1}{\tau_{ref}}\\ \beta_k &= \frac{\hat{a}_k}{\tau_{ref}}, k = \overline{2,K}.\end{aligned} \qquad (13)$$

2.2. Time Resampling

The frequency-voltage characteristic of a VCO is bijective which means that for a linear voltage sweep the resulting polynomial phases of the beat signal components are bijective functions for $t \in [0, T_p]$. Therefore, the beat signal in (4) can be rewritten as:

$$s_b(t) = \sum_{i=1}^{N} A_i \exp\left\{j2\pi\left[f_0 + \alpha_0\theta(t)\right]\tau_i\right\}, \qquad (14)$$

where

$$\theta(t) = t\left(1 + \sum_{k=2}^{K}\frac{\beta_k}{\alpha_0}t^{k-1}\right) \qquad (15)$$

is a bijective function of time t, which can be interpreted as a new time axis. Hence, if the time axis is changed to θ, the beat signal becomes a sum of N complex sinusoids, which

was the scope of the correction algorithm. Moreover, in the context of radar detection, the highly correlated clutter from a nonlinear range profile gets decorrelated in a range profile computed for the new time axis.

Notice that in the definition of θ the nonlinearity coefficients β_k are normalized to the linear chirp rate α_0 which means that the exact value of the reference propagation delay is not needed. Nevertheless, an approximate value is required for the estimation section in order to extract the reference response.

From the implementation point of view, the beat signal is a digital signal $s_b[n]$ uniformly sampled at the moments t_n, $n = \overline{0, N_s - 1}$ where N_s is the number of samples. However, the samples $s_b[n]$ of the beat signal related to the moments $\theta_n = \theta(t_n)$ of the θ time axis lead to a non-uniformly sampled signal. It can be shown that the average sampling interval of θ is:

$$\overline{\theta_S} = \frac{\overline{\alpha}}{\alpha_0}T_s, \qquad (16)$$

where $\overline{\alpha}$ is the mean chirp rate and T_s the uniform sampling interval. According to [14], for a nonuniformly sampled signal, the average sampling rate must respect the Nyquist condition. For $\overline{\alpha} > \alpha_0$, this condition can be fulfilled if the beat signal is oversampled (the chirp rate in the origin and the average chirp rate typically have the same order of magnitude, so an oversampling of at most 10 is enough). If the signal is alias-free it can be resampled with an interpolation procedure (e.g. with spline functions) in order to obtain a uniformly sampled signal in relation with the θ time axis. Afterwards, the range profile is computed by applying the discrete Fourier transform to the resampled signal.

The nonlinearity correction algorithm is summarized in the block diagram from Fig. 1.
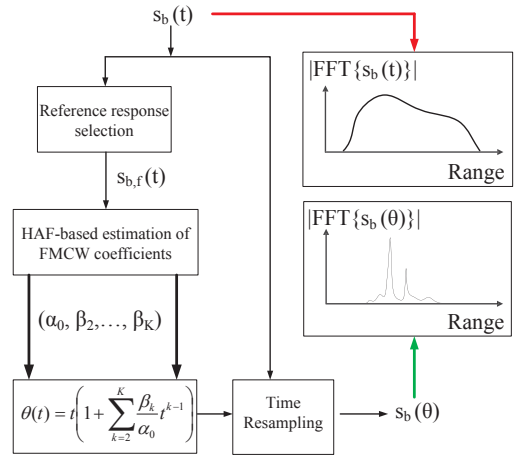

Fig. 1: Wideband nonlinearity correction algorithm based on high-order ambiguity functions and time resampling.

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3. SIMULATION RESULTS

The range profiles of a FMCW radar based on an X-band VCO with 15% linearity (according to the linearity definition given in [15]) were simulated. The chirp bandwidth was $4GHz$, the sweep rate $50Hz$ and the sampling frequency $1MHz$. The responses of six stationary targets with different amplitudes were considered. The reference target was the one located at 50m. The nonlinear and corrected range profiles are shown in Fig. 2. In the nonlinear range profile the targets can't be distinguished due to the overlapping of the frequency spread responses of each target. The correction algorithm enhances the $-3dB$ resolution up to the theoretical limit (around $4.9cm$ for a Hamming window), although in the filtered signal used for estimating the FMCW coefficients there are four other targets close to the reference response. This result is in keeping with the capability of the HAF estimation method to extract the maximum amplitude PPS component if the ratio to the other components is above a certain threshold (around $10dB$). Besides the dramatically enhanced resolution, the correction algorithm improves the peak level of each target which leads to an increase of the signal to noise ratio with over $20dB$.

4. REAL DATA VALIDATION

The nonlinearity correction algorithm was tested for a FMCW radar based on a RFVC1800 X-band VCO. The calibration curve of the VCO was measured prior to the data acquisition. The range profile obtained with a predistorted command signal based on the measured calibration curve was considered as reference. A few data sets were collected using as targets two delay lines with air-equivalent lengths of $30cm$ and respectively $240cm$. The longer delay line was used as the reference response for the correction algorithm. The chirp bandwidth was $4GHz$ (centered on $10GHz$) and the sweep interval $100ms$. Fig. 3 shows three range profiles: the profile resulted after applying the nonlinearity correction algorithm to the linear sweep data, the one corresponding to a predistorted sweep and the nonlinear range profile obtained for the linear voltage sweep. The profile obtained with the correction algorithm is very similar to the one for the predistorted sweep, but a clear advantage of the algorithm-based correction is that the FMCW signal coefficients are computed for each sweep and can include various frequency drifts (due to temperature, frequency pushing, etc.) that can occur between data acquisitions.

In order to validate the HAF-based nonlinearity estimation method for a multi-component response, a data set was acquired for a scene containing three main scatterers: one highly reflective metal disc and two vertical metal bars. The range profiles obtained in this case are shown in Fig. 4. The nonlinearity coefficients are computed on the $1.2 - 5.2m$ range interval taking as reference target the metal disc. While
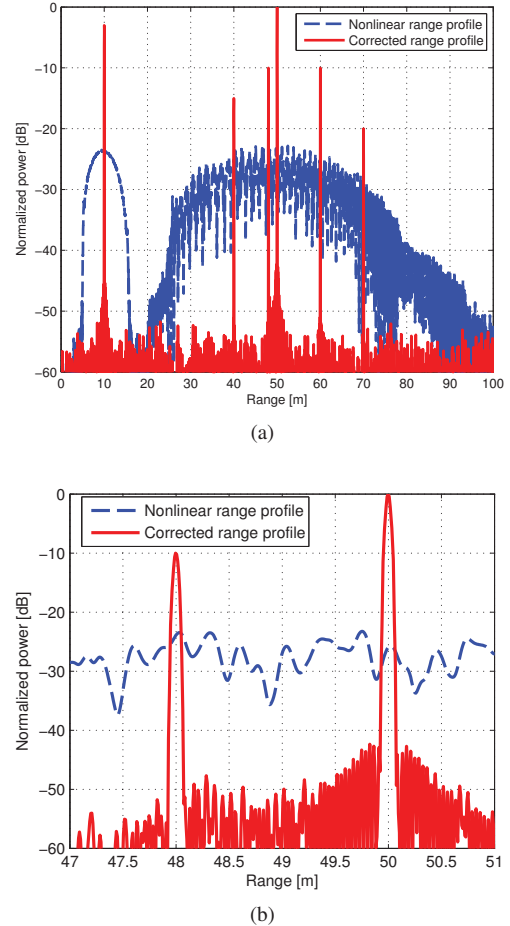

(a)

(b)

Fig. 2: Range profiles simulation results: (a) 0-100m range, (b) detailed 47-51m range. In the nonlinear range profile five targets are completely indistinguishable. The corrected range profile leads to a resolution near the theoretical limit and improves the signal to noise ratio.

on the initial nonlinear range profile obtained for the linear voltage sweep appears only a large continuous target, on the corrected profile the three targets are clearly highlighted. Notice that the power reflected by the metal disc is more than 10 dB higher in comparison to the other scatterers which is in agreement with the HAF method applicability threshold.

5. CONCLUSION

A novel wideband nonlinearity correction algorithm for FMCW radars is proposed. The algorithm estimates the nonlinearity coefficients using the HAF-based method on the beat signal corresponding to a delay line or a highly reflective target. Afterwards, a correction function is built and applied to the beat signal by time resampling. The algorithm was tested by simulations and validated on real data from an X-band FMCW radar.
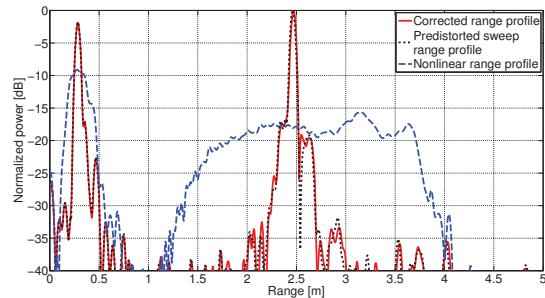
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Fig. 3: Experimental range profiles for two delay lines. The range profile obtained with the nonlinearity correction algorithm is compared with the predistorted sweep range profile and with the nonlinear range profile corresponding to the linear voltage sweep.
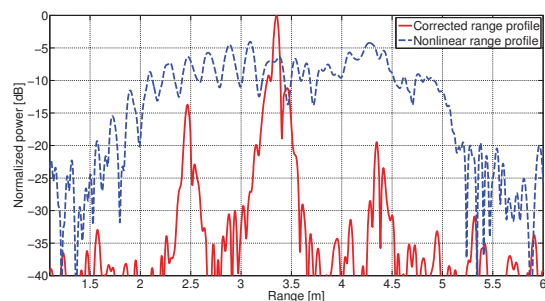

Fig. 4: Experimental range profiles for a scene containing two metal bars and one highly reflective metal disc. On the corrected range profile the three targets are clearly separated, while in the nonlinear range profile the scatterers can't be distinguished.

6. ACKNOWLEDGMENT

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