

SHIP LOCALIZATION USING AIS SIGNALS RECEIVED BY SATELLITES

*Raoul Prévost^{1,2}, Martial Coulon¹, Philippe Paimblanc², Julia LeMaitre³,
Jean-Pierre Millerioux³ and Jean-Yves Tournet¹*

¹ University of Toulouse, INP-ENSEEIH/IRIT, 2 rue Charles Camichel, BP 7122, 31071 Toulouse cedex 7, France

² TésA, 14-16 Port Saint-Étienne, 31000 Toulouse, France ³ CNES, 18 Avenue Edouard Belin, 31400 Toulouse, France

{raoul.prevast, philippe.paimblanc}@tesa.prd.fr, {martial.coulon, jean-yves.tournet}@enseeiht.fr,
{julia.lemaitre, jean-pierre.millerioux}@cnes.fr

ABSTRACT

This paper addresses the problem of ship localization by using the messages received by satellites and transmitted by the automatic identification system (AIS). In particular, one considers the localization of ships that do not transmit their actual position in AIS signals. The proposed localization method is based on the least squares algorithm and uses the differences of times of arrival and the carrier frequencies of the messages received by satellite. A modification of this algorithm is proposed to take into account the displacement model of the ships as additional measurements. This modification shows a significant localization improvement.

Index Terms— AIS, satellite, localization, least squares, TDOA, FOA.

1. INTRODUCTION

This paper addresses the problem of ship localization by using the messages received by satellites and transmitted by the automatic identification system (AIS) [1]. AIS is a self-organized TDMA access system, whose objective is to avoid collision of large vessels. Although this system was not originally designed for satellite transmissions, it has been found possible to demodulate the AIS signals received by satellites in order to monitor maritime traffic. Some systems are already operational [2] and new receivers are being studied to enhance the detection rate of AIS signals [3, 4]. Reception of AIS signals by satellites can be used to locate most of the ships in the world thanks to the GPS position of the transmitter included in the data. However, some ships do not provide their actual position in the AIS messages they transmit. This can be due for instance to the absence of GPS receiver in the vessel or to the malfunctioning of these GPS receivers. Receiving messages by satellites can provide an estimated position of these vessels by the use of times of arrivals (TOA) and frequency shifts due to the Doppler effect [5–8].

The method proposed in this paper is an evolution of the usual strategy [9, 10] based on the technique of the non-linear

least squares (NLS) [11] exploiting a Marquardt parameter [12]. This approach minimizes a criterion equal to the sum of squared differences between the measurements and a non-linear function of the parameters of interest corresponding to a given transmitter location [13–15]. An originality of the proposed method is the consideration of a displacement model for the ship. The standard NLS criterion is augmented by an additional cost function depending on the distance between the tested parameters and those predicted by the displacement model.

The paper is organized as follows. Section 2 presents the system model considered in this paper. The proposed localization strategy is presented in Section 3. The displacement model and its integration to the localization algorithm are introduced in Section 4. Section 5 presents some simulation results obtained from a realistic simulator. Conclusions are finally reported in Section 6.

2. SYSTEM MODEL

2.1. Ship and satellite displacement models

The model used for ship movements is a random walk with a preferred direction. This model considers straight lines of constant speeds chosen uniformly between 0 and 25 knots during random time intervals distributed uniformly between 2 and 9 hours. The angles of rotation between the traveled segments are randomly chosen uniformly between -90 and 90 degrees.

The system constellation is composed of 5 low Earth orbit satellites at 800 km. The orbits of these satellites are heliosynchronous and satellites are in line of sight when the elevation is greater than 5° . The number of received AIS signals during one passage varies from 1 to 5. Fig. 1 illustrates an example of satellite trajectories considered in this study.

2.2. Measurements exploited for localization

To localize vessels by using their AIS signals, we consider two types of measurements: the time differences of arrival (TDOA) (as the satellites have generally no precise time base)

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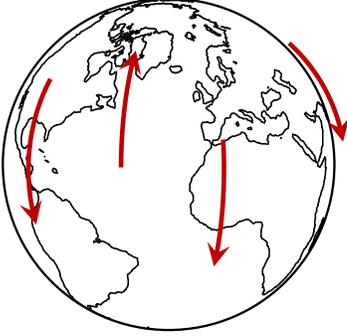


Fig. 1. Satellite trajectories.

and the frequencies of arrival (FOA). TOA measurements are affected by an additive Gaussian noise constituted by independent samples with zero mean and standard deviation $\sigma_{mt} = 60 \mu s$. Note that the noise variance of the TDOA measurements is twice that of the TOA measurements (because the noise samples associated with the TOAs are independent).

FOA measurements are the product of the emission frequency of the vessel and the Doppler coefficient, plus a Gaussian measurement noise with zero mean and standard deviation $\sigma_{mf} = 20$ Hz. Considering that the vessel speed is negligible compared to that of a given satellite, the frequency measured by this satellite can be written

$$f_r = f_e \left(1 - \frac{\bar{v}}{c}\right) + n \quad (1)$$

where \bar{v} is the satellite radial velocity, c is the signal propagation speed and n is the measurement noise. Note that the ship emission frequency f_e is a random variable representing the imprecision of modulators embedded in the vessels. We assume that f_e is constant during a passage (visibility period of a satellite) and varies according to a Gaussian distribution with zero mean and standard deviation $\sigma_{df} = 50$ Hz from a passage to another.

2.3. Coordinate system

The satellite positions in space are defined in the classical coordinate system Earth-centered, Earth-fixed (ECEF). The point $(X, Y, Z) = (0, 0, 0)$ is the Earth center of mass, the z-axis is pointing towards the north, the x-axis intersects the surface of the Earth at 0° longitude and 0° latitude and the y-axis is directed eastward.

Since ships are usually at sea level, only their longitude and latitude are estimated. The ellipsoid considered to convert the longitude – latitude coordinates to ECEF coordinates is that defined by WGS-84. Thus the ECEF coordinate conversion of a point \mathbf{P} is defined as follows

$$\begin{aligned} P_X &= R_e \cos P_\lambda \cos P_\phi \\ P_Y &= R_e \sin P_\lambda \cos P_\phi \\ P_Z &= R_p \sin P_\phi \end{aligned} \quad (2)$$

where (P_λ, P_ϕ) are the longitude and latitude coordinates of \mathbf{P} and (P_X, P_Y, P_Z) are the ECEF coordinates of \mathbf{P} . R_e and R_p are the equatorial and polar radii of the Earth defined by WGS-84, i.e., $R_e = 6378.136$ km and $R_p = 6356.752$ km. We use the notation \mathbf{P}_{XYZ} to represent the position vector of P in the ECEF coordinate system and the notation $\mathbf{P}_{\lambda\phi}$ for the position vector in longitude and latitude.

3. LOCALIZATION USING NON-LINEAR LEAST SQUARES

3.1. Principles

The measurement equation used for the proposed NLS method can be written

$$\mathbf{y} = h(\mathbf{x}; \boldsymbol{\theta}) + \mathbf{n} \quad (3)$$

where $\mathbf{y} = [\mathbf{y}_1, \dots, \mathbf{y}_N]^\top$ is the measurement vector, N is the number of measurements, $h(\mathbf{x}; \boldsymbol{\theta})$ is a non-linear function (that will be made explicit later) relating the measurements to the known pre-image $\mathbf{x} = [\mathbf{x}_1, \dots, \mathbf{x}_N]^\top$ (where \mathbf{x}_i can be the known coordinates of a satellite and its speed), $\boldsymbol{\theta}$ is the unknown parameter vector to be estimated and \mathbf{n} is the measurement noise. The NLS estimator associated with (3) minimizes a criterion based on the differences between the measurements and predictions (images obtained from the pre-images) with respect to $\boldsymbol{\theta}$. The NLS criterion to be minimized can be written as

$$C(\boldsymbol{\theta}) = [\mathbf{y} - h(\mathbf{x}; \boldsymbol{\theta})]^\top \boldsymbol{\Sigma}^{-1} [\mathbf{y} - h(\mathbf{x}; \boldsymbol{\theta})] \quad (4)$$

where $h(\mathbf{x}; \boldsymbol{\theta}) = [h_1(\mathbf{x}_1; \boldsymbol{\theta}), \dots, h_N(\mathbf{x}_N; \boldsymbol{\theta})]^\top$ and $\boldsymbol{\Sigma}$ is the noise covariance matrix.

The criterion minimization is achieved by applying the Gauss-Newton algorithm [11, 16] which is a modification of the Newton's method for a multidimensional parameter vector $\boldsymbol{\theta}$. It is an iterative algorithm that generates vectors $\boldsymbol{\theta}_j$ approaching the minimum of $C(\boldsymbol{\theta})$ by successive steps. The iterations considered in the Gauss-Newton algorithm are classically defined as

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + [\mathbf{H}_j^\top \boldsymbol{\Sigma}^{-1} \mathbf{H}_j]^{-1} \mathbf{H}_j^\top \boldsymbol{\Sigma}^{-1} [\mathbf{y} - h(\mathbf{x}; \boldsymbol{\theta}_j)] \quad (5)$$

where \mathbf{H}_j is the Jacobian matrix of h at the point $\boldsymbol{\theta}_j$. The initialization of the algorithm will be discussed in Section 3.5.

Parameters to be estimated for the AIS system are the position of the boat and its frequency. These parameters are included in the state vector $\boldsymbol{\theta} = [B_\lambda, B_\phi, f_e]^\top$ where (B_λ, B_ϕ) are the longitude and latitude coordinates of the boat. Note that the frequency f_e is not estimated when the number of AIS signals received during the passage is less than 3 (in this case, one uses the latest estimation of the frequency and the nominal AIS frequency $f_e^0 = 161.975$ MHz is used for the first estimation). Note that the position of a vessel can be estimated from the reception of two AIS signals during a passage.

3.2. Measurement predictions

The measurement prediction $h_i(\mathbf{x}_i; \boldsymbol{\theta})$ depends on the type of measurement \mathbf{y}_i . The functions h_i used for TDOA and FOA measurements are detailed in the next sections.

3.2.1. TDOA prediction

The TDOA prediction associated with two satellite positions S_1 and S_2 and a boat position B can be computed using the following TOA expressions

$$\begin{aligned} t_1 &= \frac{\|\mathbf{S}_{1XYZ} - \mathbf{B}_{XYZ}\|}{c} + t_0 + b_1, \\ t_2 &= \frac{\|\mathbf{S}_{2XYZ} - \mathbf{B}_{XYZ}\|}{c} + t_0 + b_2 \end{aligned} \quad (6)$$

where $\|\cdot\|$ is the standard ℓ^2 norm ($\|x\|^2 = x^\top x$ and \cdot^\top denotes transposition), t_0 is the satellite clock offset, and b_1 and b_2 are the measurement noises. The difference between the two TOAs in (6) is

$$\begin{aligned} \Delta t &= t_1 - t_2 \\ &= h_i(\mathbf{x}_i; \boldsymbol{\theta}) + b_{12} \end{aligned} \quad (7)$$

with $b_{12} = b_1 - b_2$ and

$$h_i(\mathbf{x}_i; \boldsymbol{\theta}) = \frac{\|\mathbf{S}_{1XYZ} - \mathbf{B}_{XYZ}\| - \|\mathbf{S}_{2XYZ} - \mathbf{B}_{XYZ}\|}{c}. \quad (8)$$

The ECEF coordinates of B are obtained from the longitude and latitude coordinates contained in $\boldsymbol{\theta}$ by using (2).

3.2.2. FOA prediction

The FOA prediction requires the knowledge of the radial velocity of the satellite relative to the ship. It is given by

$$\bar{v} = \frac{\mathbf{V}_{XYZ}^\top (\mathbf{S}_{XYZ} - \mathbf{B}_{XYZ})}{\|\mathbf{S}_{XYZ} - \mathbf{B}_{XYZ}\|} \quad (9)$$

where \mathbf{V}_{XYZ} is the velocity vector of the satellite in the ECEF reference. By integrating the radial velocity in (1), the FOA prediction can be written as

$$h_i(\mathbf{x}_i; \boldsymbol{\theta}) = f_c \left[1 - \frac{\mathbf{V}_{XYZ}^\top (\mathbf{S}_{XYZ} - \mathbf{B}_{XYZ})}{c \|\mathbf{S}_{XYZ} - \mathbf{B}_{XYZ}\|} \right]. \quad (10)$$

3.3. Jacobian matrix of h

The computation of the state vector $\boldsymbol{\theta}_{j+1}$ at the $(j+1)$ th iteration of the Gauss-Newton algorithm requires to compute the Jacobian matrix \mathbf{H} of $h(\mathbf{x}; \boldsymbol{\theta})$ as a function of $\boldsymbol{\theta}$ and evaluate it at the current point $\boldsymbol{\theta}_j = [B_{\lambda,j}, B_{\phi,j}, f_{e,j}]^\top$. This matrix is defined as follows

$$\mathbf{H}_j = \begin{bmatrix} \frac{\partial h_1}{\partial B_\lambda}(\mathbf{x}_1; \boldsymbol{\theta}_j) & \frac{\partial h_1}{\partial B_\phi}(\mathbf{x}_1; \boldsymbol{\theta}_j) & \frac{\partial h_1}{\partial f_e}(\mathbf{x}_1; \boldsymbol{\theta}_j) \\ \frac{\partial h_2}{\partial B_\lambda}(\mathbf{x}_2; \boldsymbol{\theta}_j) & \frac{\partial h_2}{\partial B_\phi}(\mathbf{x}_2; \boldsymbol{\theta}_j) & \frac{\partial h_2}{\partial f_e}(\mathbf{x}_2; \boldsymbol{\theta}_j) \\ \vdots & \vdots & \vdots \\ \frac{\partial h_N}{\partial B_\lambda}(\mathbf{x}_N; \boldsymbol{\theta}_j) & \frac{\partial h_N}{\partial B_\phi}(\mathbf{x}_N; \boldsymbol{\theta}_j) & \frac{\partial h_N}{\partial f_e}(\mathbf{x}_N; \boldsymbol{\theta}_j) \end{bmatrix}. \quad (11)$$

Each row of the matrix \mathbf{H}_j contains the gradients of h for a TDOA or an FOA measurement.

3.4. Noise Covariance Matrix Σ

FOA measurements are altered by independent noise samples. Thus, the part of the matrix Σ corresponding to these measurements is diagonal and its diagonal elements are all equal to σ_{mf}^2 . TDOA measurements are the differences between time instants that are contaminated by independent noise samples of variance σ_{mt}^2 . The variance of these measurements is $2\sigma_{mt}^2$ and the covariance of TDOAs sharing a common arrival time is $-\sigma_{mt}^2$. This allows one to build the covariance matrix elements associated with TDOAs.

As an example, a passage corresponding to the reception of 4 AIS signals yields 4 FOA measurements and 3 TDOA measurements, hence

$$\mathbf{y} = [y_1^f, y_2^f, y_3^f, y_4^f, y_1^t - y_2^t, y_2^t - y_3^t, y_3^t - y_4^t]^\top \quad (12)$$

where y_i^f and y_i^t are the i th FOA and TOA measurements. The covariance matrix associated with these measurements is

$$\Sigma = \begin{bmatrix} \sigma_{mf}^2 \mathbf{I}_4 & \mathbf{0} \\ \mathbf{0} & \Sigma_{\text{TDOA}} \end{bmatrix} \quad (13)$$

with

$$\Sigma_{\text{TDOA}} = \begin{bmatrix} 2\sigma_{mt}^2 & -\sigma_{mt}^2 & 0 \\ -\sigma_{mt}^2 & 2\sigma_{mt}^2 & -\sigma_{mt}^2 \\ 0 & -\sigma_{mt}^2 & 2\sigma_{mt}^2 \end{bmatrix}. \quad (14)$$

3.5. Initialization of the state vector $\boldsymbol{\theta}$

The estimation of a ship position is iterative and thus requires to define an initial parameter vector $\boldsymbol{\theta}_0$. To mitigate the possible algorithm divergence and the incorrect convergence issues, multiple initializations are classically considered.

As shown in Fig. 2, we propose to initialize the state vector $\boldsymbol{\theta}$ by using starting points $\boldsymbol{\theta}_0$ located in a grid centered on the average track of the satellite during the message receptions. This grid is composed of 25 elements placed every 10 degrees in longitude and latitude, i.e., the starting points are $(\lambda_0 + l, \Phi_0 + m)$ with $(l, m) \in \{-20^\circ, -10^\circ, 0^\circ, 10^\circ, 20^\circ\}^2$ and the central point (λ_0, Φ_0) is the average satellite ground track computed using multiple message receptions.

3.6. Ambiguity mitigation

The estimation of the ship position is performed at each satellite passage and generally provides two possible positions. These positions are located symmetrically with respect to the satellite trajectory. Indeed, the satellite trajectory during a passage is similar to a straight line and there is no information which allows one to determine whether the signals are received by the satellite from one side or from the other side of this line. To estimate a unique position, we propose to select the positions that minimize the total distance traveled by the ship. So we choose the path that minimizes the sum of the distances resulting from successive passages.

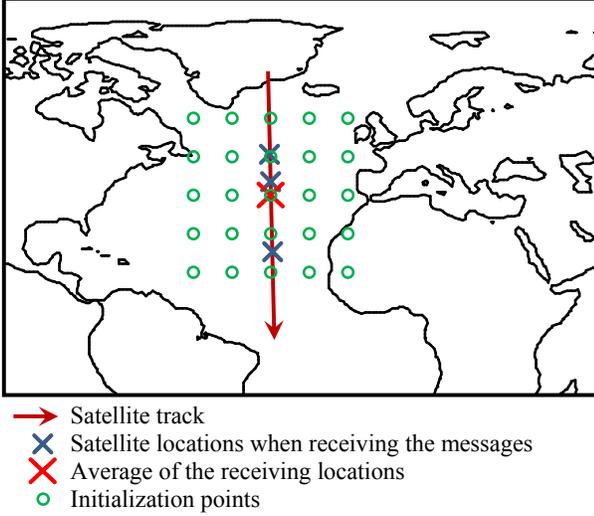


Fig. 2. Initialization points.

4. DISPLACEMENT MODEL

The position and the emission frequency of a ship during a satellite passage are highly dependent on those of the same ship during the previous passage. Less than one hour usually separates two satellite passages above a given ship. It is then possible to use the estimated parameters of the ship at a passage in order to improve the estimation of parameters in the next passage. It is also possible with this additional information to estimate the position of a ship after reception of a single message during a passage.

The proposed method consists of predicting the ship position and using this prediction as additional measurements that are combined to the TDOA and FOA measurements. The predicted location can be defined as the last estimated ship location (for a pure random walk model) or as an extrapolation of the ship trajectory build from on the latest estimated locations. The variance of the prediction error is included in the covariance matrix Σ of the state vector. This variance depends on the delay between the current passage and the previous one as well as on the variance of the previous position estimation. The ship displacement model considered in this paper is a random walk with a preferred direction. The predicted state can be expressed

$$\hat{\theta}_{k|k-1} = \hat{\theta}_{k-1} + \begin{bmatrix} \hat{v}_{k-1} \Delta t_k \\ 0 \end{bmatrix} \quad (15)$$

with

$$\hat{v}_k = \alpha \frac{B_{\lambda\phi k} - B_{\lambda\phi k-1}}{\Delta t_k} + (1 - \alpha) \hat{v}_{k-1} \quad (16)$$

where $B_{\lambda\phi k}$ and $B_{\lambda\phi k-1}$ are the estimated ship locations at the k th and $(k - 1)$ th satellite passages, Δt_k is the delay between the k th and $(k - 1)$ th satellite passages and \hat{v}_k is the average ship velocity in longitude and latitude computed

on the last part of the trajectory. Note that α is a weighting parameter that needs to be adjusted. The value used in this work ($\alpha = 0.3$) was obtained by cross validation. It is such that the speeds of the last 5 segments represent about 83 % of the computed average.

The measurement vector and the predicted measurements taking into account the prediction are defined as

$$y_e = \begin{bmatrix} y \\ \hat{\theta}_{k|k-1} \end{bmatrix}, \quad h_e(x; \theta) = \begin{bmatrix} h(x; \theta) \\ \theta \end{bmatrix}. \quad (17)$$

The measurement covariance matrix is

$$\Sigma_e = \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \Sigma_p \end{bmatrix} \quad (18)$$

where Σ_p is the covariance matrix of the prediction vector $\hat{\theta}_{k|k-1}$ given by $\Sigma_p = F P F^T + Q$, where F is the Jacobian matrix of the prediction function (15) (here, we have used $F = I_3$ since this function is a translation), P is the covariance matrix of the state vector at the previous instant (used for prediction), and Q is the covariance matrix of the prediction error

$$Q = \begin{bmatrix} \sigma_\lambda^2 & 0 & 0 \\ 0 & \sigma_\phi^2 & 0 \\ 0 & 0 & \sigma_f^2 \end{bmatrix} \quad (19)$$

with σ_λ^2 , σ_ϕ^2 and σ_f^2 the variances associated with the prediction of each parameter. These variances depend on the maximum speed of the ships v_{\max} (in degrees per second) via the following relations

$$\sigma_\lambda^2 = \frac{1}{6} \left(\frac{\Delta t_k 360 v_{\max}}{2\pi R_e \sin \hat{B}_{\phi k}} \right)^2, \quad (20)$$

$$\sigma_\phi^2 = \frac{1}{6} \left(\frac{\Delta t_k 360 v_{\max}}{2\pi R_p} \right)^2, \quad (21)$$

$$\sigma_f^2 = \sigma_{df}^2 \quad (22)$$

where $\hat{B}_{\phi k}$ is the predicted latitude of the ship.

The cost function (4) taking into account the displacement model is defined as

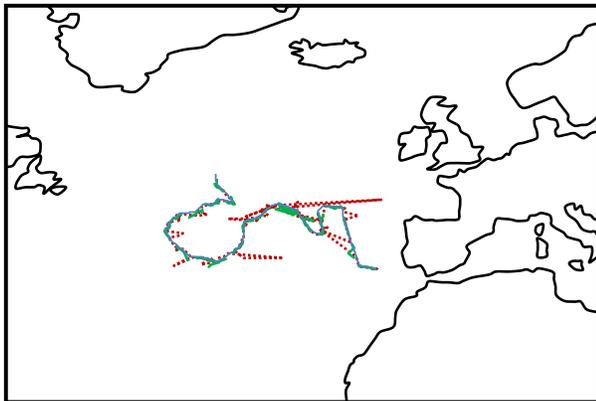
$$C_e(\theta) = [y_e - h_e(x; \theta)]^T \Sigma_e^{-1} [y_e - h_e(x; \theta)] \quad (23)$$

and can be decomposed into the sum of the original cost function and a cost function associated with the model

$$C_e(\theta) = [y - h(x; \theta)]^T \Sigma^{-1} [y - h(x; \theta)] + [\hat{\theta}_{k|k-1} - \theta]^T \Sigma_p^{-1} [\hat{\theta}_{k|k-1} - \theta]. \quad (24)$$

Note that this decomposition is possible because Σ_e is a block diagonal matrix.

The cost function (24) is minimized by using the Gauss-Newton algorithm initialized by an LS estimator. The algorithm is reinitialized to such a position each time it diverges.



— Actual ship trajectory
 Estimated trajectory without displacement model
 - - - Estimated trajectory with displacement model

Fig. 3. Examples of estimated trajectory with and without the displacement model.

5. SIMULATIONS

Fig. 3 shows examples of estimated ship trajectories with and without the displacement model for a duration of 10 days. The use of the displacement model significantly reduces the number of large amplitude errors which improves the mean and variance of the positioning error.

Simulations have been conducted to evaluate the estimation performance by using signals received from 300 vessels for a period of 60 days. The number of satellite passages during this simulation is 433081, with an average duration of 9.8 minutes and the average time separating them is approximately 1 hour. The results are presented in Table 1. The optimal LS method is the method which systematically selects the solution closest to the actual position of the vessel in case of ambiguity. It is used as a reference for the evaluation of the ambiguity mitigation strategy (Optimal LS in Table 1). As can be observed, the proposed strategy used to mitigate ambiguities (Estimated LS in Table 1) has a performance close to the one obtained with the optimal LS method (that cannot be applied in practice since it requires to know the ship location).

Taking into account the displacement model (DME in Table 1) reduces the average localization error by a factor of 2.5 (the average positioning error drops from about 50 km to 20 km). The standard deviation of the error is also significantly reduced by a factor of 3.5 which provides better confidence in the estimated position.

6. CONCLUSIONS

This paper presented a ship localization method based on time differences of arrival and frequencies of arrival for AIS signals received by satellites. The proposed method was based on the least squares algorithm combined with a technique avoiding positioning ambiguity. A ship displacement model

Table 1. Estimation performance.

Estimator	Average error	Standard deviation
Optimal LS	49.0 km	177.3 km
Estimated LS	49.6 km	180.2 km
DME	19.5 km	52.9 km

was introduced to improve the localization performance. Future work will focus on investigating localization methods based on Kalman filtering in order to reduce the computational complexity.

7. REFERENCES

- [1] Recommendation ITU-R M.1371, "Technical characteristics for a universal automatic identification system using time division multiple access in the VHF maritime mobile band." ITU, 2001.
- [2] "exactEarth," <http://www.exactearth.com>, 2010.
- [3] N. Bouny, J. LeMaitre, and J.-P. Millerieux, "Results of measurement campaign for characterisation of AIS transmitters," in *Proc. Adv. Sat. Mul. Sys. Conf.*, vol. 12, Sept. 2012, pp. 258–265.
- [4] R. Prévost, M. Coulon, D. Bonacci, J. LeMaitre, J.-P. Millerieux, and J.-Y. Tourneret, "Joint phase-recovery and demodulation-decoding of AIS signals received by satellite," in *Proc. IEEE Int. Conf. Acoust., Speech, and Signal Process. (ICASSP)*, Vancouver, Canada, May 2013.
- [5] J. Mason, "Emitter location accuracy using TDOA and differential Doppler," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-18, no. 2, pp. 214–218, March 1982.
- [6] B. Friedlander, "A passive localization algorithm and its accuracy," *IEEE Journal of Oceanic Engineering*, vol. OE-12, no. 1, pp. 234–245, Jan. 1987.
- [7] K. C. Ho and Y. T. Chan, "Geolocation of a known object from TDOA and FDOA measurements," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 3, pp. 770–783, July 1997.
- [8] K. C. Ho and W. Xu, "An accurate algebraic solution for moving source location using TDOA and FDOA measurements," *IEEE Trans. Signal Process.*, vol. 52, no. 9, pp. 2453–2463, Sept. 2004.
- [9] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-20, no. 2, pp. 183–198, March 1984.
- [10] X.-J. Tao, C.-R. Zou, and Z.-Y. He, "Passive target tracking using maximum likelihood estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, no. 4, pp. 1348–1354, Oct. 1996.
- [11] C. T. Kelley, *Iterative methods for optimization*. Society for Industrial Mathematics, 1999, vol. 18.
- [12] D. W. Marquardt, "An algorithm for least-squares estimation of nonlinear parameters," *Journal of the society for Industrial and Applied Mathematics*, vol. 11, no. 2, pp. 431–441, 1963.
- [13] R. Schmidt, "Least Squares range difference location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, no. 1, pp. 234–242, Jan. 1996.
- [14] T. Pattison and S. I. Chou, "Sensitivity analysis of dual-satellite geolocation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 36, no. 1, pp. 56–71, Jan. 2000.
- [15] J. Mason, "Algebraic two-satellite TOA/FOA position solution on an ellipsoidal Earth," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 40, no. 3, pp. 1087–1092, July 2004.
- [16] Å. Björck, *Numerical methods for least squares problems*. Society for Industrial Mathematics, 1996, vol. 51.