WIDER BANDWIDTHS FOR IMPROVED ANGLE AND VELOCITY RESOLUTION

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ABSTRACT

Angle and velocity are two primary radar parameters where resolution improvements are mostly needed, for detection of targets in complex environments (e.g. multiple targets, severe and wide-spectrum clutter). In this paper, techniques for improving resolution in velocity and angle through widening of the instantaneous bandwidth are examined, and the possibility of complementing such solutions with high resolution processing is emphasized, with two illustrating examples: wideband MTI (Moving Target Indicator) radar, and multiple transmissions system.

***Index Terms*—** Radar waveforms, Digital Beam Forming, Space-time coding, Space-time adaptive processing, Waveform diversity, MIMO.

1. INTRODUCTION

Most radars divide their observation space along 3 or 4 coordinates: angle (1 or 2 coordinates), range, and velocity, and it is well-known that resolution in angle depends on the spatial extent of the antenna, resolution in range depends on the instantaneous bandwidth, and resolution in velocity depends on the coherent integration time.

So, improving angular resolution makes the radar bulky, and improving velocity resolution makes it slow, whereas improving range resolution makes it …costly (essentially for the receiver channels).

Interestingly though, what radar systems are really lacking is mainly:

1. angular resolution, since at long ranges the angular separation, expressed in meters, is at least 10 times poorer than the range resolution; Angular resolution is also needed for strong targets rejection when looking for small targets;
2. velocity resolution, for improving slow targets detection against clutter and other targets.

So, the two parameters where improvement would be most appreciated – angle and velocity – also have the most severe consequences on the physical characteristics of the system – bulkiness and reaction time.

That is why very often, improving range resolution (through bandwidth widening) is actually the only way to mitigate poor resolution in the other domains, separating the targets and reducing the clutter level for the most difficult situations – especially for situations where multiple slow targets and heavy clutter are encountered, such as ground or sea surveillance.

In this paper, we will examine how this track can be followed one step further: actually improving angular and velocity resolution – and rejection of interfering signals – through bandwidth widening. Two concepts will be analyzed: wideband non-ambiguous waveforms, and multiple simultaneous transmissions (or coherent MIMO).

2. WIDEBAND MTI

2.1 Principle

An essential limitation for standard radars using bursts of periodic pulses[1] comes from pulsed radar range-Doppler ambiguity relation, which states that the ambiguous speed V_a and the ambiguous range D_a are related by:

$D_a \, . \, V_a = \lambda \, . \, c \, / \, 4$ (λ being the wavelength)

This relation means that many ambiguities, either in range or speed (or both), have to be dealt with, which in turn implies the transmission of successive pulse trains with different repetition frequencies, requiring more time to be spent on target for ambiguity and blind speeds removal.

An alternative solution is obtained by increasing the range resolution, (or the instantaneous bandwidth) so that the moving target range variation (rangewalk) during the pulse train becomes non-negligible compared with the range resolution – which is equivalent to stating that the Doppler shift is varying across the whole bandwidth (compared with the Doppler resolution), and can not be considered as a global frequency shift any more – : such radars may use bursts of wideband pulses, with low Pulse Repetition Frequency (no range ambiguities) such that the rangewalk phenomena during the whole burst is significant enough (compared with the range resolution) to remove the velocity ambiguity. It then becomes possible to detect the target and measure range and speed with only one coherent pulse burst.

The ambiguity function, and the clutter rejection capability of such radars have been analyzed in simulations

[1] The use of a non-periodic pulse train, sometimes advocated for mitigating this ambiguity, has been shown [1] to bring much more severe drawbacks, especially when multiple second trace echoes are received from strong and distant clutter echoes.

[1] and experimental results have demonstrated the feasability of the approach [2].

The condition for wideband situation is written, if P is the number of pulses in the pulse train, $T_r=1/F_r$ the repetition period, V_a the standard ambiguous velocity ($V_a = \lambda/2T_r$), ΔF the instantaneous bandwidth, and δR the range resolution [δR = c/(2ΔF)]:

$$P\,V_a\,T_r >> \delta R \Leftrightarrow (\lambda_0/2)P >> \delta R \Leftrightarrow P >> \frac{F_0}{\Delta F}$$

where F_0=c/λ is the central carrier frequency. For example, a burst of 50 pulses at 10 GHz carrier frequency and 1 kHz repetition frequency (V_a = 15 m.s^{-1}) with 500 MHz bandwidth (δR = 0.30 m) would be a possible candidate for non-ambiguous MTI detection, since the migration is then 15. 50.10^{-3} = 0.75m, significantly larger than the 0.30m range cell.

If $x_{p,t}$ is the signal received from the p^{th} pulse, at t^{th} time sample, the quantity to be compared to the threshold, taking into account the range migration from pulse to pulse, for hypothesis t in delay (range) and V in velocity, is [1]:

$$T_{t,V} = \sum_{p=0}^{P-1} x_{p,\Gamma\left[t-p\frac{VT_r}{\delta R}\right]} e^{-2\pi j p \frac{F_0}{F_r}\frac{2V}{c}} \qquad (1)$$

with $\Gamma(u)$ the nearest integer from u.

This processing should also be followed by a non-coherent integration along t , for an assumed length of the targets of interest, in order to recover the totality of the energy scattered by the target (this post-integration is also providing the usual diversity gain, as explicited in [1]).

2.2 Velocity and angular resolution

For comparing two radars, one with standard range resolution, for instance 75m (corresponding to 2 MHz bandwidth), and another with high resolution, 30cm, obtained by an instantaneous bandwidth of 500 MHz, some basic assumptions should be made regarding the waveforms.

The two radars will be assumed to operate in X band, for medium/long range surveillance at 100 km. The narrow-band radar typically requires 5 successive bursts of 10ms each for blind speeds (every 15 m.s^{-1}) and velocity ambiguity removal, with repetition frequency 1 kHz (non-ambiguous in range). Within the same illumination time, the wideband radar previously sketched (F_r=1kHz, δR = 0.30 m) would use one coherent burst of 50ms, providing a theoretical velocity resolution of 0.3 m.s^{-1}.

With this wideband radar, <u>extended targets can be resolved in angle</u> [1], using monopulse measurements as shown in Figure 1. Since an angular measurement can be made in each range cell, if the SNR allows, then the histogram of ecartometry measurements on an extended target will provide the information that two targets are present and their respective angular locations; Schematically, the angular resolution comes down to the – much better – angular accuracy.

With these baseline parameters, the following properties can then be stated:

1. The velocity resolution is improved by a factor 5, due to the 5 times longer coherent integration time;
2. The benefit of frequency diversity, obtained for the narrowband radar by varying the carrier frequency from burst to burst, is also obtained (as explained in detail in [1]) in the wideband case by non-coherent integration along the impulse response of the target;
3. Angular resolution is significantly improved (a factor 5 might be a reasonable estimate), through analysis of angular measurements in the range cells covered by the impulse response of the target.

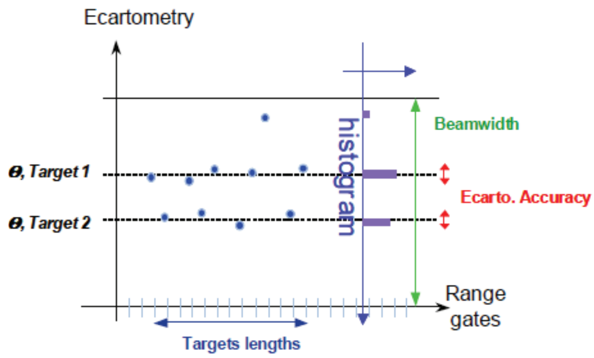

Figure 1: Wideband angular resolution

This brief overview shows that significant <u>improvements can be expected from widening the bandwidth, regarding both angular and velocity resolution and ambiguities</u>, without changing the physical dimensions of the antenna nor the illumination time. To these improvements should also be added, of course, the benefits of range resolution itself, for instance for target classification – a function for which high velocity resolution is also required.

2.3 Higher resolution: discussion

It must be emphasized that, unlike high resolution signal processing techniques, these gains in resolution are obtained without any assumption about the number of targets, the characteristics of clutter, or their nature (point scatterers vs distributed echoes, both for targets and clutter).

On top of these improvements, one can also imagine application of high resolution adaptive processing techniques to the range-velocity analysis in the wideband situation. However, improving the velocity resolution beyond 0.3 m.s^{-1}, as obtained at the beginning of Section 2.4, might prove to be an illusory gain, since non-stationarity of the target can be expected to become the limiting factor.

Though much work still has to be done regarding wideband radars – especially on clutter suppression and angular resolution – it is clear that this path towards high resolution radar is a very promising one, for small targets detection and classification in adverse environments. Further discussions on the advantages and drawbacks of such radars can be found in [1] – attention has been focused here on the less well-known properties in terms of velocity and angular resolution.

Regarding the field of application of high resolution techniques, one question remains open: can the required bandwidth for obtaining these benefits in angle and velocity be minimized, through the use of high resolution techniques? In lack of experimental results, we will leave this question open…

3. SPACE-TIME CODING (COHERENT MIMO)

For radar systems using multiple simultaneous transmissions, bandwidth widening will be shown to be a solution for improving Doppler and angle resolution.

3.1. Principle

Standard digital beamforming is a procedure where wide angular sector instantaneous coverage is obtained with a wide beam illumination on transmit (transmission through one subarray), and directive beams are formed on receive through coherent summations of signals received on different subarrays, in parallel for each direction.

Digital beamforming generally does not essentially change the power budget, compared to standard focused exploration, since the lower gain on transmit (due to wider illumination) is traded against a longer integration time (made possible by the simultaneous observation of different directions). In fact, the main benefit provided by digital beamforming is an improved velocity resolution, especially useful for identification purposes, or for detection of slow targets.

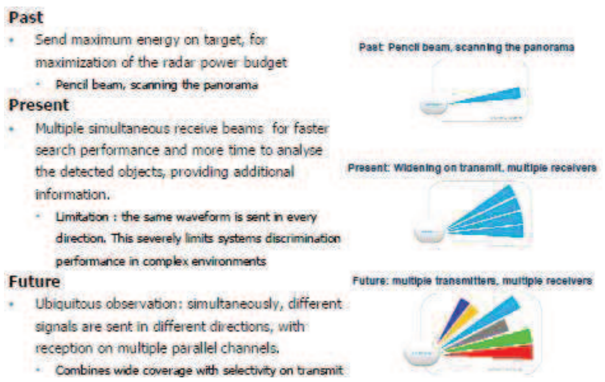

Figure 2: Radar exploration of space.

However, this velocity resolution comes at a cost: the non-directive beam on transmit, which induces a poorer rejection of echoes coming from adjacent directions. For airborne applications, a severe limitation arises from the clutter spreading in Doppler, due to the wider beam on transmit: this leads to a poor minimum detectable velocity, and to a poor clutter rejection, since only half the dBs are obtained, compared to focused beam illumination.

In order to recover this angular separation on transmit (which was basic to standard focused beam techniques), it is necessary to code the transmitted signals (space-time coding), such that the signals transmitted in the different directions be separable on receive (Figure 2).

3.2. Space-time coding

An example of such coding is well-known: it consists of using a dispersive antenna, which effectively sends different frequencies in the different directions. The different frequencies are then separated on receive, and digital beamforming is performed as usual for each transmitted beam/frequency.

A space-time coding which produces the same effect in a more flexible way uses a circulating signal, which can be any complex waveform (e.g. code or chirp) having good auto-correlation properties – which is normally obtained with a large time-bandwidth product. $s(t)$ is "circulating" along the array if the n-th channel signal $s^n(t)$ is (Fig. 3):

$$s^n(t) = s(t-(n-1)\Delta t), \qquad (2)$$

where n=1, 2, ..N, Δt is a 1-sample time shift, $\Delta t = 1/\Delta F$, where ΔF is the bandwidth of the circulating signal $s(t)$. For signals with a large time-bandwidth product, the relative time shift is very small compared to the pulse duration T_p, since $\Delta t = T_p / \Delta F\, T_p = 1 / \Delta F$.

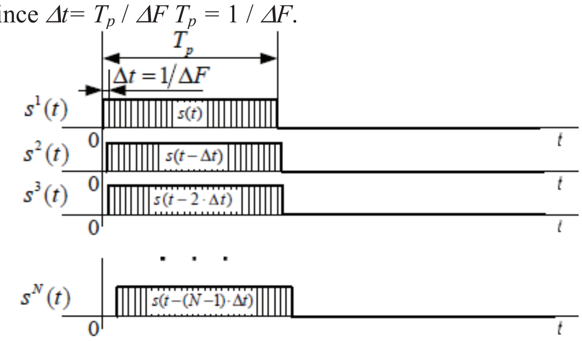

Figure 3: Space-time circulating signals.

3.3. Transmit ambiguity function

In order to make a fair comparison between space-time coded systems and standard ones [4,5], it is necessary to examine their ambiguity functions (only systems with similar ambiguity functions should be directly compared).

The signal transmitted in a given direction θ_0 is the sum of all transmitted signals, from antennas positions $\vec{x}(n)$, with appropriate phase shifts for this direction defined by the wave vector $\vec{k}(\theta_0)$:

$$s_T(t,\theta_0) = \sum_{n=1}^{N} e^{j\,\vec{k}(\theta_0)\,\vec{x}(n)} \,.\, s_T^n(t) \qquad (3)$$

The signal received by a target in this direction θ_0, at range cτ_0/2, is written:

$$s_R(t,\theta_0) = \sum_{n=1}^{N} e^{j\,\vec{k}(\theta_0)\,\vec{x}(n)} \,.\, s_T^n\left(t - \frac{\tau_0}{2}\right) \qquad (4)$$

We assume here that there is no Doppler effect during the duration of one pulse (Doppler effect will then be processed, as usual, from pulse to pulse).

The signal received by one antenna element at position $\vec{x}(r)$ is written $s(t,\theta_0)$:

$$s(t,\theta_0) = A_0\; e^{j\varphi_0}\; e^{j\vec{k}(\theta_0)\,\vec{x}(r)} \sum_{n=1}^{N} e^{j\,\vec{k}(\theta_0)\,\vec{x}(n)} \,.\, s_T^n(t-\tau_0) \quad (5)$$

The received signal is processed, as usual, through matched filtering for every possible position (τ,θ) of an expected target, thus providing, to within an unsignificant complex coefficient, the output function $\chi_{\theta_0}(\tau,\theta)$:

$$\chi_{\theta_0}(\tau,\theta) = \sum_{\substack{n=1\\m=1}}^{N} e^{j\left(\vec{k}(\theta_0)\,\vec{x}(n) - \vec{k}(\theta)\,\vec{x}(m)\right)} \,.\int s_T^n(t)\left(s_T^m(t+\tau)\right)^* \,dt \qquad (6)$$

The ambiguity function $\left|\chi_{\theta_0}(\tau,\theta)\right|^2$ is thus a 3-dimensional function, giving for each aiming direction θ_0 the delay-angle ambiguity.

For complete analysis, it would also be necessary to take into account the beamforming on receive – coherent sumation on the different receiving channels –, and the coherent integration (Doppler filtering) from pulse to pulse. However, the gains and properties of those two operations are well-known, and the added clutter rejection and target separation they provide is unaffected by the transmit beamforming analyzed here.

For analysis of the "transmit" ambiguity function $\left|\chi_{\theta_0}(\tau,\theta)\right|^2$, it is useful to consider two bi-dimensional cuts of this ambiguity function, expressed in the following figures as functions of cosines and range variables:

- $\left|\chi_{\theta_0}(0,\theta)\right|^2 = D(\theta,\theta_0)$ which is the angular transmit diagram (at the exact range of the target), as a function of the angular aiming direction θ_0,
- and $\left|\chi_{\theta_0}(\tau,\theta)\right|^2$, which is the delay (range) - angle ambiguity function, for boresight aiming direction – Ideally, this delay-angle ambiguity function should also be analyzed for each possible aiming direction θ_0: such results are not presented here, but simulations have shown that the variation with angle θ_0 is not very significant.

More details and illustrations about the use of ambiguity functions for space-time coding systems are provided in [5].

3.4. Example: time-shifted chirps

The antenna array is made of N=8 elementary omnidirectional antennas, spaced λ/2 from each other, the carrier frequency is 10 GHz, and the time-bandwidth product $\Delta F\ T_p$ is 256, with 100µs pulse length.

Figure 4 examines the case of a circulating chirp, with the following characteristics:

Pulse duration T_p = 100 µs

$\Delta F\ T_p$ product = 257

Time shift Δt between adjacent chirps:

$\Delta t = T_p / \Delta F\ T_p = 0.4$ µs

These chirps are an example of a "circulating code" [3] – the same chirp, with time origin shifted from 0.4 µs, from antenna to antenna.

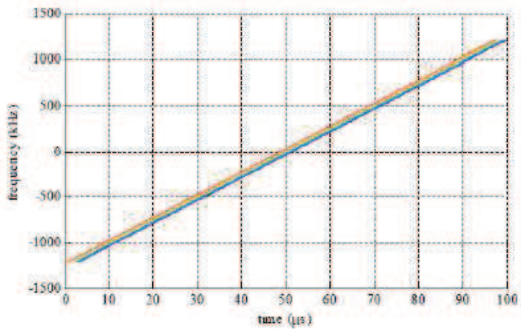

Figure 4: Frequency-time representation of multiple circulating chirps;

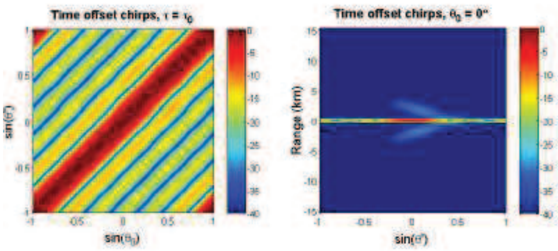

Figure 5: Multiple chirps (circulating code): Cuts of the 3-D ambiguity function at τ=0 and at θ_0=0 (angle-angle and angle-range cuts).

The obtained ambiguity function (Fig. 5) has two very nice properties:

1. A very stable level as a function of aiming direction θ_0, meaning that the energy has been evenly distributed across the whole angular domain – as in the case of focused beam systems;
2. A very clean range-angle ambiguity function (rejection higher than 40 dB over most of the domain). Similar properties, analog to the case of

standard focused beam systems, are obtained for many different types of circulating codes.

But the ambiguity function also shows the price paid for this clean ambiguity function: the range resolution (thickness of the horizontal line in the angle-range cut) is degraded by a factor equal to N, the number of antenna elements – just because of the dispersivity previously described: in each direction, only the fraction $1/N$ of the bandwidth is sent, thus degrading the range resolution by a factor N, as illustrated on Figure 6 for a 4 elements array.

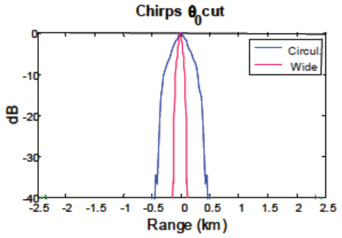

Figure 6: Multiple chirps (circulating code): Cut of the 3-D ambiguity function at θ=0 and θ_0=0 (range profile) for a widened beam with no coding, and a circulating code, with 4 elements antenna.

Comparing this multiple transmission radar with a more standard radar using a wide beam on transmit – for which the ambiguity function would have the same general characteristics, but with a range resolution N times better – , it appears that space-time coding has allowed to trade the angular resolution on transmit against range resolution.

In other words, for this space-time coding, increasing the bandwidth on transmit by a factor N is a way to obtain the full angular resolution (and associated rejection of strong echoes) on transmit, while keeping the original range resolution, in comparison with the wide beam standard illumination (no angular resolution on transmit). Since the final angular resolution is the geometric mean of the transmit and receive resolutions (product of the diagrams, for gaussian beams assumption), the final angular resolution is improved by a factor $\sqrt{2}$.

Coming back to the very basic focused beam system described on the top of Figure 2, we can also compare this radar with the wide-beam space-time coded radar just described: it then appears that, globally, we have improved the Doppler resolution of the radar (and slightly the angular resolution), and kept all other characteristics (angular rejection, range resolution and rejection) equal, by widening the bandwidth of the same factor. For each pencil beam radar with active antenna, it is possible to improve the Doppler resolution at the cost of widening the bandwidth – *ceteris paribus*. This very simple relation, though quite unsettling at first for any radar expert, opens ways for future modes dedicated to slow targets detection and analysis.

Moreover, as in the previous analysis of wideband radars, it must be emphasized that, contrary to high resolution processing techniques, these gains in resolution are obtained without any assumption about the number of targets, the levels or spectra of clutter, or their nature (point scatterers vs distributed echoes, both for targets and clutter).

3.5. Higher resolution: discussion

The previous discussion has shown that widening the bandwidth is a way for improving the Doppler resolution, or the angular resolution (depending on the terms of comparison), even in the presence of multiple interfering echoes.

This is clearly not a reason for waiving the benefits that can be provided by high resolution processing techniques: these benefits also apply to the signals collected after space-time processing, since space-time processing just comes down to a coherent summation of received signals. This statement is true for circulating chirps – more detailed analysis might be required for other types of coding, such as phase coding along the antenna, for which stationarity of the signal in time or space is not satisfied.

For instance, STAP processing could also be applied, after circulating codes on a linear airborne array, for improving detection of air or ground targets from airborne platforms.

4. CONCLUSION

The gains in angle and velocity resolution that can be obtained through bandwidth widening have been exhibited in two typical situations: wideband radar signals for unambiguous detection, and wide beam observations with space-time coding. The complementarity of such solutions with high resolution processing techniques has been emphasized, showing that modern signal processing is part of the global optimization of radar systems.

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