QUICK SEARCH FOR RARE SPECTRUM OPPORTUNITIES

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ABSTRACT

In cognitive radio networks the secondary users are allowed to seek and exploit the under-utilized segments of the frequency spectrum. Driven by the ongoing growth of data networks in scale and traffic demands, the frequency spectrum will be populated by frequent opportunistic access by the cognitive users. Hence, the spectrum opportunities become rare and scattered across the entire spectrum. Due to their transient nature, such rare spectrum opportunities should be identified quickly. This paper develops a data-adaptive search algorithm for identifying such rare spectrum opportunities quickly and reliably.

Index Terms— Detection, quick, rare, search, spectrum sensing.

1. INTRODUCTION

Current statistics about the spectrum occupancy patterns indicate that a considerable fraction of the frequency spectrum is under-utilized. This observation has promoted the notion of cognitive communication, which envisions granting spectrum access to unlicensed users when the licensed users under-utilize the spectrum. Under such envisioned scenarios many unlicensed users compete for the same spectrum resources and the under-utilized segments of the spectrum, which we hereinafter call spectrum holes, will not be as abundant as they otherwise should be. Reduced availability of the spectrum holes becomes even more severe as the networks grow in size and in terms of level of data traffic that they are expected to sustain. Spectrum opportunities, as a result, become *rare* and will be scattered throughout the entire frequency spectrum.

Besides rarity, the occupancy statuses of the spectrum holes also vary rapidly and the spectrum holes might not remain unoccupied for a long duration. Therefore, it is of paramount importance to identify the spectrum holes *quickly* and consequently the notion of agile spectrum sensing has received extensive research attention. A few research directions that are relevant to the scope of the proposed scheme in this paper are discussed next.

This research was supported in part by the National Science Foundation under Grant DMS-1118605.

A relevant direction is the application of distilled sensing [1] and adaptive sampling [2, 3] in spectrum sensing, in which a thresholding-based approach is proposed. In these approaches the channels corresponding to which the measurements do not satisfy a threshold criteria are discarded recursively. The threshold is designed based on the statistical distributions of the measurements from the busy and idle channels. The approach proposed in this paper concurs with the approaches of [1, 2, 3] in being data-adaptive and is different in being robust against uncertainties about the statistical distributions of the measurements.

Another notable direction is the quickest sequential search approach of [4], in which a wideband spectrum is split into smaller narrowband channels and the cognitive users scan them sequentially one at-a-time. Upon scanning and accumulating enough information about each channel a cognitive user decides whether the channel is a hole or is occupied. If the channel is determined to be a hole, the search is terminated and otherwise the process is carried on until a hole is detected. While this approach is very effective when the spectrum holes are *not* rare, in case of rare spectrum holes providing an accurate decision for each channel substantially lengthens the sensing duration.

In another direction it is assumed that the wideband channel is heavily under-utilized and it is used sparsely. In this approach the cognitive radios exploit the sparsity structure of the wideband channel and construct compressed sensing-based machinery for estimating the power spectral density (PSD) of the wideband channel [5, 6, 7, 8, 9, 10, 11]. These approaches are further extended to also track temporal variations of the spectral occupancy during sensing [12, 13]. Exploiting the sparsity empowers the cognitive radios to sample the signal activity over the channel at a sub-Nyquist rate, which expedites the process of estimating the PSD.

In this paper the focus is on the scenario in which the spectrum holes are rare and spread randomly across a wideband spectrum. The goal is to identify *one or more* rare spectrum holes through 1) designing an information-gathering process for collecting information from the entire spectrum, and 2) delineating optimal decision rules. Designing a quick search process involves a tension between two performance measures, one being the aggregate amount of information accumulated (i.e., the number of observations made) and the other being the reliability (or cost) of the decision. In this paper we design an optimal information-gathering process that maximizes the decision reliability subject to a *hard*[1] constraint on the aggregate number of observations we are allowed to make from the spectrum. The proofs of the results are omitted and can be found in [14].

2. SENSING MODEL

We consider a wideband spectrum consisting of n narrowband channels and adopt a dichotomous statistical model to distinguish between occupied and vacant channels. The set of observations made from channel n is denoted by $\mathcal{X}_n$, which consists of independent and identically distributed (i.i.d.) elements $\mathcal{X}^i \triangleq \{X_1^i, X_2^i, \dots\}$ taking values in $\mathbb{R}$ obeying one of the two hypotheses

$$\begin{array}{ll} \mathsf{H}_0: & X_j^i \sim F_0, \quad j = 1, 2, \dots \\ \mathsf{H}_1: & X_j^i \sim F_1, \quad j = 1, 2, \dots \end{array} \tag{1}$$

where F_0 and F_1 denote the cumulative distribution functions (cdfs) of two distinct distributions on $\mathbb{R}$. The distributions F_0 and F_1 capture the underlying statistical models of the observations taken from the occupied and vacant channels, respectively. For convenience, we assume that F_0 and F_1 have probability density functions (pdfs) f_0 and f_1, respectively. It is also assumed that each channel is vacant with a known probability $\epsilon_n \in (0, 1)$ and independently of the rest. The true hypothesis and a decision about channel i are denoted by $\mathsf{T}_i \in \{\mathsf{H}_0, \mathsf{H}_1\}$ and $\mathsf{D}_i \in \{\mathsf{H}_0, \mathsf{H}_1\}$, respectively,

3. SEQUENTIAL SEARCH

3.1. Sampling Model

With the ultimate objective of identifying $T \in \mathbb{N}$ spectrum holes the proposed sampling procedure is initiated by making rough observations from *all* channels $\mathcal{X}^1, \dots, \mathcal{X}^n$. Based on these rough observations a fraction of the channels that are least-likely spectrum holes are discarded and the rest are retained for more accurate scrutiny. Repeating this procedure successively refines the search support and progressively focuses the sensing resources on the more promising channels. More specifically, at each time a subset of the channels is selected and one measurement is taken from each of these channels. Upon collecting these measurements, the sampling process takes one of the following actions:

A_1 **(Detection):** stops sampling and identifies T channels that have the highest likelihood of being vacant;

A_2 **(Observation):** continues to further observe the same set of channels in order to gather more information about their occupancy; or

A_3 **(Refinement):** discards a portion of the channels permanently and declares that they are most likely occupied and the remaining channels are retained for further scrutiny. By denoting the number of channels retained prior to a refinement action by ℓ, the number of sequences that this action discards is $(1-\alpha)(\ell-T)$ for some $\alpha \in (0, 1)$. Discarding the channels at this rate ensures that at least T channels will be retained for the final detection action (action A_1).

We denote the set of channels observed at time $t \in \mathbb{N}$ by $\mathcal{L}_t$. As we initialize the information-gathering procedure by including all sequences for observation we have $\mathcal{L}_1 = \{1, \dots, n\}$. Also, we denote the stopping time of the procedure, i.e., the time after which detection (action A_1) is performed, by τ. Furthermore, we define the switching function $\psi : \{1, \dots, \tau\} \to \{0, 1\}$ to model actions A_2 (observation) and A_3 (refinement). At each time $1 \le t \le \tau - 1$ we set $\psi(t) = 0$ if we decide in favor of performing observation, while $\psi(t) = 1$ indicates a decision in favor of performing refinement, i.e., $\forall t \in \{1, \dots, \tau - 1\}$:

$$\psi(t) = \left\{ \begin{array}{lll} 0 & \text{action } \mathrm{A}_2 & \text{and } \ \mathcal{L}_{t+1} = \mathcal{L}_t \\ 1 & \text{action } \mathrm{A}_3 & \text{and } \ \mathcal{L}_{t+1} \subset \mathcal{L}_t \end{array} \right. . \tag{2}$$

Let X_t^i denote the observation made from channel $i \in \mathcal{L}_t$ at time t and denote the σ-algebra generated by observation $\{X_1^i, \dots, X_t^i\}$ by

$$\forall i \in \mathcal{L}_t: \quad \mathcal{F}_t^i = \sigma(X_1^i, \dots, X_t^i) . \tag{3}$$

Given $\mathcal{F}_t^i$, we denote the posterior probability that channel i is occupied by $\pi_t^i \triangleq \mathbb{P}(\mathsf{T}_i = \mathsf{H}_1 \mid \mathcal{F}_t^i)$. Invoking the independence among the observations $\{X_1^i, \dots, X_t^i\}$ provides

$$\pi_t^i = \left[1 + \frac{1-\epsilon_n}{\epsilon_n} \prod_{u=1}^{t} \frac{f_0(X_u^i)}{f_1(X_u^i)} \right]^{-1} . \tag{4}$$

By defining the likelihood ratio

$$\Lambda_t^i \triangleq \prod_{u=1}^{t} \frac{f_0(X_u^i)}{f_1(X_u^i)} , \tag{5}$$

we have

$$\pi_t^i = \left[1 + \frac{1-\epsilon_n}{\epsilon_n} \, \Lambda_t^i \right]^{-1} , \tag{6}$$

and the actions A_1, A_2, and A_3 can be formalized as follows.

A_1: At the stopping time τ identify the set $\mathcal{U} \subseteq \mathcal{L}_\tau$ as the detector's decision according to

$$\begin{aligned} \mathcal{U} &= \arg \max_{\mathcal{U} \subseteq \mathcal{L}_\tau :\, |\mathcal{U}| = T} \\ &\qquad \mathbb{P}\left(\forall i \in \mathcal{U}: \ \mathsf{T}_i = \mathsf{H}_1 \mid \{\mathcal{F}_\tau^i : \ i \in \mathcal{L}_\tau\} \right) \\ &= \arg \max_{\mathcal{U} \subseteq \mathcal{L}_\tau :\, |\mathcal{U}| = T} \prod_{i \in \mathcal{U}} \pi_\tau^i . \end{aligned} \tag{7}$$

Hence, $\mathcal{U}$ contains the indices of the T smallest elements of the set $\{\Lambda_\tau^i : \ i \in \mathcal{L}_\tau\}$.

[1] By a hard constraint we mean that the *aggregate* number of observations made cannot exceed a specified level.

A$_2$: At time t the decision is to further measure the same set of channels (i.e. $\mathcal{L}_{t+1} = \mathcal{L}_t$), and set

$$\Lambda_{t+1}^i = \Lambda_t^i \cdot \frac{f_0(X_{t+1}^i)}{f_1(X_{t+1}^i)} \, . \tag{8}$$

A$_3$: At time t the decision is to refine $\mathcal{L}_t$ and the set $\mathcal{L}_{t+1}$ is obtained as

$$\mathcal{L}_{t+1} = \arg \max_{\mathcal{L}\subseteq\mathcal{L}_t:\, |\mathcal{L}|=\bar{\alpha}_t} \mathbb{P}\left(\forall i \in \mathcal{L}:\ \mathsf{T}_i = \mathsf{H}_1 \mid \{\mathcal{F}_t^i:\ i \in \mathcal{L}_t\}\right) \, , \tag{9}$$

which indicates that $\mathcal{L}_{t+1}$ contains the indices of the $\bar{\alpha}_t$ smallest elements of the set $\{\Lambda_t^i:\ i \in \mathcal{L}_t\}$.

3.2. Optimal Sampling

Characterizing the experimental design and decision rules relies on an interplay between two performance measures, one being the aggregate number of observations made and the other being the frequency of erroneous detection. The optimal design of the search process involves optimizing a tradeoff between them. For a given stopping time τ and a given sequence of switching functions $\bar{\psi}(\tau) \triangleq \{\psi(1), \psi(2), \dots, \psi(\tau-1)\}$, the probability of erroneous detection, that is the probability that the detected channels include an occupied channel, is

$$\mathsf{P}_n(\tau, \bar{\psi}(\tau)) \ \triangleq\ \mathbb{P}\Big(\big|\{i \in \mathcal{U}:\ \mathsf{T}_i = \mathsf{H}_0\}\big| \neq 0\Big) \, . \tag{10}$$

Our objective is to minimize this detection error probability over all possible stopping times τ, all switching rules $\bar{\psi}$, and all possible splits of the sensing resources between refinement and detection actions subject to two *hard* constraints. One constraint incorporates the aggregate number of available sensing resources and the other one captures the cost of the refinement actions, which is the permanent loss of the sequences discarded after the refinement actions. This optimization problem can be formalized as

$$\mathcal{P}_n(S,K) \triangleq \begin{cases} \inf_{\tau,\bar{\psi}(\tau)} & \mathsf{P}_n(\tau, \bar{\psi}(\tau)) \\ \text{s.t.} & \frac{1}{n}\sum_{t=1}^{\tau-1} |\mathcal{L}_t| \leq S \\ & \sum_{t=1}^{\tau} \psi(t) \leq K \end{cases} \, . \tag{11}$$

where S controls the aggregate sampling budget and K is an upper bound on the number of refinement actions. We solve this problem in the asymptote of large n and characterize:

1. the optimal stopping time and sampling process; and

2. the minimum distance between distributions F_0 and F_1 such that the two hypotheses are guaranteed to be distinguished perfectly, i.e., $\mathcal{P}_n(S,K) \xrightarrow{n\to\infty} 0$.

4. GAUSSIAN OBSERVATIONS

In this section we provide the solutions of (11) when F_0 and F_1 are Gaussian distributions with different means or different variances.

4.1. Gaussian Mean

The hypothesis-testing problem in this case is

$$\begin{array}{ll} \mathsf{H}_0: & X_j^i \sim \mathcal{N}(\mu_0, 1), \quad j = 1, 2, \dots \\ \mathsf{H}_1: & X_j^i \sim \mathcal{N}(\mu_1, 1), \quad j = 1, 2, \dots \end{array} \tag{12}$$

where without loss of generality $\mu_0 > \mu_1$. Under this setting, the likelihood ratio at time t for the sequences $i \in \mathcal{L}_t$ is

$$\Lambda_t^i = \exp\left\{(\mu_0 - \mu_1)\sum_{u=1}^{t} X_u^i\right\} \cdot \prod_{u=1}^{t} \exp\left\{\frac{\mu_1^2 - \mu_0^2}{2}\right\} \, .$$

By defining $Z_t^i \triangleq \sum_{u=1}^{t} X_u^i$ for all $t \in \{1, \dots, \tau\}$ and $i \in \mathcal{L}_t$, the detection and refinement actions formalized in (7) and (9), respectively, are equivalently given by

$$\mathcal{U} = \text{indices of the } T \text{ smallest elements of } \{Z_\tau^i:\ i \in \mathcal{L}_\tau\} \, ,$$
$$\mathcal{L}_{t+1} = \text{indices of the } \bar{\alpha}_t \text{ smallest elements of } \{Z_t^i:\ i \in \mathcal{L}_t\} \, .$$

On the other hand, by invoking the distribution of X_t^i from (12) we immediately have $\forall t \in \{1, \dots, \tau\}$ and $\forall i \in \mathcal{L}_t$,

$$Z_t^i \mid \mathsf{H}_m \ \sim\ \mathcal{N}\Big(\mu_m \cdot t \ , \ t\Big) \, . \tag{13}$$

Furthermore, let us for each $t \in \{1, \dots, \tau\}$ define

$$\bar{U}_j^t \ \triangleq\ \text{the } j^{th} \text{ smallest element of } \{Z_t^i:\ i \in \mathcal{L}_t^0\} \, , \tag{14}$$
$$U_j^t \ \triangleq\ \text{the } j^{th} \text{ smallest element of } \{Z_t^i:\ i \in \mathcal{L}_t^1\} \, . \tag{15}$$

Given these definitions, the probability $\mathsf{P}(\tau, \bar{\psi}(\tau))$, which is the probability that among the detected channels there is at least one occupied channel, can be equivalently written as

$$\mathsf{P}_n(\tau, \bar{\psi}(\tau)) = \mathbb{P}(|\ \mathcal{U} \cap \mathcal{L}_\tau^0\ |\ \geq 0) = \mathbb{P}\Big(U_T^\tau > \bar{U}_1^\tau\ \Big) \, . \tag{16}$$

Assessing this performance measure involves finding the cardinalities and the distributions of the first and the T^{th} order statistics of the sets of random variables $\{Z_\tau^i:\ i \in \mathcal{L}_t^0\}$ and $\{Z_\tau^i:\ i \in \mathcal{L}_t^1\}$, respectively. Hence, we first assess the variations of the number of occupied and vacant channels, denoted by n_t and $\bar{n}_t$, respectively, throughout the refinement process.

Lemma 1 (Refinement Performance) *Let $\bar{n}_t = |\mathcal{L}_t^0|$ and $n_t = |\mathcal{L}_t^1|$ denote the number of occupied and vacant channels retained up to time t. For any arbitrary $\delta \in (0,1)$ and for sufficiently large n, the event $n_\tau \geq (1-\delta)n_1$ holds almost surely if*

$$(\mu_0 - \mu_1)^2 \ =\ \omega\left(n^{-\varepsilon_n}\right) \, , \tag{17}$$

where ε_n is defined as

$$\varepsilon_n \triangleq \frac{\ln n\epsilon_n}{\ln n} \ . \tag{18}$$

Therefore, when the condition in (17) is satisfied, the refinement actions almost surely discard no more than a fraction δ of the vacant channels, for any arbitrary $\delta \in (0,1)$. Therefore, the final ratio of the number of vacant channels to that of the occupied channels increases dramatically throughout the refinement actions.

Besides the performance of the refinement actions, the overall detection reliability also depends on the performance of the detection action (A_1). The next lemma describes a necessary and sufficient condition that guarantees asymptotically error-free detection. Note that this lemma does not restrict itself to any specific performance for the outcome of the refinement actions and applies to any arbitrary sequence of refinement and observation actions captured by $\bar{\psi}(\tau)$.

Lemma 2 (Detection Performance) *For a given stopping time τ and switching sequence $\bar{\psi}(\tau)$, the detection error probability $\mathsf{P}_n(\tau, \bar{\psi}(\tau))$ tends to zero in the asymptote of large n if and only if*

$$r_{\rm m} \ > \ \frac{(1-\sqrt{\varepsilon_n})^2}{\tau} \ , \tag{19}$$

where we have defined

$$r_{\rm m} \triangleq \frac{(\mu_0-\mu_1)^2}{2\ln n} \ . \tag{20}$$

Therefore, this lemma, conditionally on the value of the stopping time, which is stochastic, provides a necessary and sufficient condition on the distance between distributions F_0 and F_1 (captured by $(\mu_0-\mu_1)^2$) for achieving asymptotically optimal detection performance. In the next lemma we show that the stochastic stopping time is upper bounded by a constant.

Lemma 3 *The sampling stopping time τ is upper bounded by S/α^K for sufficiently large n.*

Combining the results of Lemmas 2 and 3 and replacing the stopping time τ in (19) with its upper bound from Lemma 3 provides that

$$\frac{(\mu_0-\mu_1)^2}{2\ln n} \ > \ \frac{(1-\sqrt{\varepsilon_n})^2}{S/\alpha^K} \ ,$$

is a *necessary* condition for ensuring asymptotically error-free detection. This clearly imposes a more stringent condition on the distance $(\mu_0-\mu_1)^2$ than (17), which is a *sufficient* condition for retaining at least a fraction $(1-\delta)$ of the rare events throughout the refinement cycles. As a result, irrespective of the optimal value of the stopping time τ, ensuring asymptotically error-free detection imposes the requirement that the refinement process retains at least a fraction $(1-\delta)$ of the spectrum holes.

4.2. Gaussian Variance

In this section we analyze the performance of detection and refinement in the Gaussian variance hypothesis testing problem. The presentation of the results follows the same flow as the problem of testing the mean, while the proofs, provided in [14], are entirely different. The problem of interest can be posed as

$$\begin{array}{ll} \mathsf{H}_0: & X_j^i \sim \mathcal{N}(0, A_0), \quad j=1,2,\dots \\ \mathsf{H}_1: & X_j^i \sim \mathcal{N}(0, A_1), \quad j=1,2,\dots \end{array} \tag{21}$$

where $A_0 > A_1$ and $A_0, A_1 \in \mathbb{R}^+$. By following the same line of arguments and posing the detection error problem in a form similar to (16), the evolution of the number of occupied and vacant channels throughout the refinement cycles can be characterized as follows in the next lemma.

Lemma 4 (Refinement Performance) *Let $\bar{n}_t = |\mathcal{L}_t^0|$ and $n_t = |\mathcal{L}_t^1|$ denote the number of occupied and vacant channels, respectively, that are retained up to time t. For sufficiently large n, the event $n_\tau = n_1$ holds almost surely if*

$$\frac{A_0}{A_1} \ = \ \omega\left(\varepsilon_n \ln n\right) , \tag{22}$$

where ε_n is defined in (18).

Therefore, when the scaling law in (22) is satisfied, the proportion of the vacant channels to the occupied channels increases after the refinement actions. In the next lemma we provide a necessary and sufficient condition on the scaling of A_0/A_1 that ensures perfect identification of the T sequences generated by F_1 (rare events).

Lemma 5 (Detection Performance) *For a given stopping time τ and switching sequence $\bar{\psi}(\tau)$, and conditionally on n_τ and $\bar{n}_\tau$, the detection error probability $\mathsf{P}_n(\tau, \bar{\psi}(\tau))$ tends to zero in the asymptote of large n if and only if*

$$\xi_{\rm v} \ > \ \frac{2(1-\varepsilon_n)}{\tau} \ , \tag{23}$$

where we have defined

$$\xi_{\rm v} \triangleq \frac{\ln \frac{A_0}{A_1}}{\ln n} \ . \tag{24}$$

By comparing the scaling laws offered by Lemmas 4 and 5 we find that the scaling law *necessary* for making a reliable detection, irrespective of τ and $\bar{\psi}(\tau)$, dominates the one that is *sufficient* for maintaining $n_\tau = n_1$ almost surely. In other words, in order to perform reliable detection the refinement action (A_2) retains all the rare events almost surely.

4.3. Optimal Switching Sequence

Given the performance of the refinement action offered by Lemmas 1 and 4, and the detection action given by Lemmas 2 and 5, in this section we provide the optimal choices of the stopping time and the switching sequence. Given the discussions at the end of Sections 4.1 and 4.2, irrespective of the discrepancies in the analysis and the ensuring scaling laws in the mean and variance settings, these two settings conform in the fact that targeting at error-free detection forces the refinement process to retain *almost* all of the vacant channels. Primarily due to such similar behavior of the refinement process in both settings, the optimal choices of the stopping time and the switching sequence turn out to be exactly the same in both settings. The optimal choices of the stopping time and switching sequence, which are the minimizers of the error probability, are given in the next theorem.

Theorem 1 (Stopping Time) *The optimal switching sequence for achieving* $\mathcal{P}_n(S,K) \xrightarrow{n\rightarrow\infty} 0$ *satisfies*

$$\forall t \in \{1,\dots,K^*\}: \quad \psi(1)=1, \quad \textit{and} \;\; \forall t > K^*: \quad \psi(t)=0\,, \tag{25}$$

where

$$K^* = \begin{cases} K\,, & \textit{if } \alpha \leq 1-\frac{1}{S} \\ 0\,, & \textit{if } \alpha > 1-\frac{1}{S} \end{cases} . \tag{26}$$

Also the optimal stopping time is

$$\tau = \begin{cases} K+s(K)\,, & \textit{if } \alpha \leq 1-\frac{1}{S} \\ S\,, & \textit{if } \alpha > 1-\frac{1}{S} \end{cases} , \tag{27}$$

where

$$s(K) \;\triangleq\; \left\lfloor S\cdot\alpha^{-K} + \frac{1-\alpha^{-K}}{1-\alpha} \right\rfloor . \tag{28}$$

This theorem demonstrates that when α is large (close to 1) optimal sampling does not involve any refinement action and adaptive sampling does not offer any gain over the non-adaptive sampling procedure. However, for sufficiently large S, over a wide range of α the optimal sampling procedure involves refinement actions and spectrum sensing becomes adaptive. The gains of this optimal sensing procedure over non-adaptive sensing procedures are discussed in detail in [14].

5. CONCLUSION

In this paper we have provided a search algorithm for spectrum sensing, where the objective is to spot an arbitrary fraction of rare spectrum holes within a wideband spectrum. The main idea of the proposed data-adaptive spectrum sensing procedure is to gradually adjust the measurement process using information gleaned from the previous measurements.

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