EUSIPCO 2013 1569743651

ROBUST 3-WAY TENSOR DECOMPOSITION AND EXTENDED STATE KALMAN FILTERING TO EXTRACT FETAL ECG

Mohammad Niknazar,[1] Hanna Becker,[1,2] Bertrand Rivet,[1] Christian Jutten,[1] and Pierre Comon[1]

[1] GIPSA-lab (UMR CNRS-5216), University of Grenoble, Grenoble, France
[2] I3S (CNRS UMR-7271), Sophia Antipolis, France

ABSTRACT

This paper addresses the problem of fetal electrocardiogram (ECG) extraction from multichannel recordings. The proposed two-step method, which is applicable to as few as two channels, relies on (i) a deterministic tensor decomposition approach, (ii) a Kalman filtering. Tensor decomposition criteria that are robust to outliers are proposed and used to better track weak traces of the fetal ECG. Then, the state parameters used within an extended realistic nonlinear dynamic model for extraction of N ECGs from M mixtures of several ECGs and noise are estimated from the loading matrices provided by the first step. Application of the proposed method on actual data shows its significantly superior performance in comparison to the classic methods.

Index Terms— fetal ECG extraction, underdetermined source separation, robust tensor decomposition, extended Kalman filtering, nonlinear Bayesian filtering.

1. INTRODUCTION

The fetal electrocardiogram (fECG) may provide useful information about the fetus' heart condition for detecting the fetus at risk of damage or death in the uterus. However, extraction of the fECG signal from the mixture of maternal electrocardiogram (mECG) and fECG signals, and other interference sources remains a difficult problem for the biomedical engineering community. This is due to much lower amplitude of fECG compared with mECG.

Among several methods in the literature for multichannel fECG extraction, one can name blind source separation [1], semi-blind source separation [2], adaptive filtering [3, 4], and periodic component analysis (πCA) [5]. All these methods exploit the redundancy of the multichannel ECG recordings to reduce mECG and other interference sources. Nevertheless, even if this reduction has been successful, the exogenous noise cannot be totally canceled in this way [6]. Moreover, they demand several channels to recover weak traces of fetal signal.

On the other hand, one can extract the fECG using a single sensor by singular value decomposition (SVD) [7] or by nonlinear decomposition such as shrinkage wavelet denoising [8] or nonlinear projections [6].

In this paper, the tensor based parallel deflation procedure [9], which can be seen as an extension of the SVD method proposed in [7], is modified to tackle the fECG extraction by proposing criteria that are robust to outliers. Then, the fECG and mECG estimates are improved by a Kalman filtering, whose state parameters are obtained from the loading matrices of the tensor decomposition. The proposed method is applicable to as few as two channels.

The rest of the paper is organized as follows. In section 2 equations and theories supporting our proposed method are described. In section 3 results of the proposed method applied on two sets of data and discussion about the results are presented. Finally, our conclusion is stated in section 4.

2. METHODS

The proposed method is based on two steps: (i) a robust tensor decomposition (subsection 2.1) and (ii) a refined Kalman filtering (subsection 2.2).

2.1. Robust tensor decomposition: deterministic source extraction

The deterministic blind separation of sources having different symbol rates, proposed in [9] has been adopted and modified in this study for fetal ECG extraction. This method, which is also applicable to underdetermined mixtures (i.e. more sources than sensors), assumes that each of the $n = 1, \ldots, N$ sources of interest has periodic symbols. For each source, it then builds a three-way tensor with dimensions space, symbol period, and temporal pattern from measurement data that is recorded with M sensors over a certain time interval. To this end, for the n-th source, L_n symbol periods composed of T_n time samples are identified from the measurements, yielding a data matrix of size $M \times T_n$ for each symbol period. By stacking these matrices along the second dimension of a three-dimensional array, one obtains the tensor $\boldsymbol{Y}^{(n)} \in \mathbb{C}^{M \times L_n \times T_n}$. In the ECG context, due to the quasi-periodic nature of the ECG signal, one can firstly detect ECG R-peaks to identify different beats (ECG symbols). Then the data of the maternal ECG beats comprising a fixed number of time

samples around each R-peak are stacked into the tensor $\boldsymbol{Y}^{(1)}$. The same procedure is repeated to build the fetus tensor $\boldsymbol{Y}^{(2)}$ based on the fetal R-peaks. These tensors $\boldsymbol{Y}^{(n)}$ can then be decomposed into the loading matrices $\mathbf{A}^{(n)} \in \mathbb{C}^{M\times R_n}$, $\mathbf{S}^{(n)} \in \mathbb{C}^{T_n\times R_n}$, and $\mathbf{H}^{(n)} \in \mathbb{C}^{L_n\times R_n}$, which provide estimates of the mixing matrix, the ECG beat amplitude, and the ECG temporal pattern, using the Canonical Polyadic (CP) decomposition according to the following criterion [10]:

$$\min_{\{\hat{\mathbf{A}}^{(n)},\hat{\mathbf{S}}^{(n)},\hat{\mathbf{H}}^{(n)}\}} \sum_{i,j,k} \left\| y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} s_{jr}^{(n)} h_{kr}^{(n)} \right\|_F^2, \quad (1)$$

where $y_{ijk}^{(n)}$ are the entries of $\boldsymbol{Y}^{(n)}$ and R_n is the assumed rank of the n-th source corresponding to the number of components of the n-th ECG. As it has been shown in [9], if $T_n \geq R_n$ and $L_n \geq R_n$, then $M = 2$ sensors are enough to blindly separate R_n components. We are interested in utilizing a minimal number of electrodes. Thereby, although maternal and fetal ECGs can be multidimensional signals, we assume that only 2 electrodes are available. In this case, a direct algorithm that is based on eigenvalue decompositions [11] can be used to compute the CP decomposition.

However, using the classic optimisation criterion (1) to determine the dominant components of the fECG tensor, one fails to find fetal components. Since in the mixture of maternal and fetal ECGs, the mECG signal is much more powerful, it prevents the algorithms to concentrate on the signal of interest, fECG, which has much lower power. In order to overcome this problem, we considered two different solutions that will be presented in the following.

The first idea consists in using a weighted CP decomposition (WCP) for the fECG tensor, that applies a weight on each entry of the tensor to better concentrate on the signal of interest. Therefore, the new criterion is:

$$\min_{\{\hat{\mathbf{A}}^{(n)},\hat{\mathbf{S}}^{(n)},\hat{\mathbf{H}}^{(n)}\}} \sum_{i,j,k} \left\| w_{ijk}^{(n)} \left(y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} s_{jr}^{(n)} h_{kr}^{(n)} \right) \right\|_F^2, \quad (2)$$

where

$$w_{ijk}^{(n)} = \exp\left\{ -\frac{(y_{ijk}^{(n)} - \mu_{ik})^2}{\sigma_{ik}^2} \right\}, \quad n = 1, \ldots, N \quad (3)$$

are the elements of a nonnegative weight tensor, which is of the same size as $\boldsymbol{Y}^{(n)}$. Here, μ_{ik} is the mean of $\boldsymbol{Y}^{(n)}$ over the j-th dimension and σ_{ik} is the median absolute deviation (MAD) estimator of $\boldsymbol{Y}^{(n)}$ over the j-th dimension. Practically, it gives very small weights to values far from the mean, i.e. especially outliers. This method is especially adapted to the application at hand because it exploits the structure of the data to compute weights that discriminate values of the mECG signal in the fECG tensor.

A second, more general solution, that might also be of interest for other applications, consists in considering a different cost function that does not attribute high errors to the values of the mECG signal, which can be regarded as outliers. To this end, we propose to employ a criterion that is based on a Gaussian-shaped cost function and that is defined as:

$$\min_{\{\hat{\mathbf{A}}^{(n)},\hat{\mathbf{S}}^{(n)},\hat{\mathbf{H}}^{(n)}\}} \sum_{i,j,k} \psi \left(y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} s_{jr}^{(n)} h_{kr}^{(n)} \right) \quad (4)$$

with $\psi(u) = 1 - \exp\{-\frac{u^2}{2\sigma^2}\}$. The resulting decomposition is referred to as Gaussian CP (GCP) decomposition in the following. In this case, an error value of about $u = 3\sigma$ between a tensor element and the reconstructed tensor element is treated as an outlier and its effective error value $\psi(u)$ is limited to approximately 1. The parameter σ that adjusts the width of the Gaussian function thus permits to define a threshold between "normal" errors and outliers. The optimal value for σ should thus be chosen according to the data. If available, an estimate of the variance of the data can be used to this end. The criterion (4) can be optimised using a gradient descent algorithm.

One may apply one of the above-mentioned methods, which are called WCP and GCP in this paper, to directly estimate fetal ECG as a multidimensional extension of [7]. However, in this case the main drawback is that the ECG dynamics are lost, because all ECG beats have exactly the same temporal pattern, which is stored in $\mathbf{H}$, up to their amplitudes that are stored in $\mathbf{S}$. In order to estimate more realistic ECGs, a realistic ECG model within a Kalman filtering framework is extended to extract fetal ECG.

2.2. EKF Framework for ECG Extraction

In [12], Bayesian filters such as the Extended Kalman Filter (EKF) and Extended Kalman Smoother (EKS) have been proposed for single-channel ECG denoising. The state-space model used in these filters suggests to approximate the PQRST waves by the sum of 5 Gaussian-shaped functions to model realistic synthetic ECGs. This state-space model was then further developed in [13]. In this study, we adopt the developed state-space model and extend it for extraction of several ECGs from multichannel recordings. The model of one ECG signal, in its discrete form with a small sampling period δ, is:

$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\delta) \mathrm{mod}(2\pi) \\ z_{k+1} = -\sum\limits_{i\in\mathcal{W}} \delta \frac{\alpha_{i,k}\omega}{b_{i,k}^2} \Delta\theta_{i,k} \exp(-\frac{\Delta\theta_{i,k}^2}{2b_{i,k}^2}) + z_k + \eta_k^z \\ \alpha_{i,k+1} = \alpha_{i,k} + \eta_k^{\alpha_i} \\ b_{i,k+1} = b_{i,k} + \eta_k^{b_i} \\ \psi_{i,k+1} = \psi_{i,k} + \eta_k^{\psi_i} \end{cases} \quad (5)$$

where θ, z, α_i, b_i, and ψ_i are the state variables in polar coordinates and k denotes the discrete time index. $\mathcal{W} = \{P, Q, R, S, T\}$ is the set of the PQRST waves. The α_i and

b_i correspond to the peak amplitude and width parameters of the Gaussian functions used for modeling each of the ECG waves. $\Delta\theta_{i,k} = (\theta_k - \psi_i)\text{mod}(2\pi)$, in which ψ_i corresponds to the phase of the maximum of the ith Gaussian function. ω is the phase increment and η_k, $\eta_k^{\alpha_i}$, $\eta_k^{b_i}$, and $\eta_k^{\psi_i}$ are random additive noises. The state vector associated with this ECG signal is thus defined by its phase θ_k, amplitude z_k and Gaussian function parameters α_i, b_i, and ψ_i. In addition to the noisy ECG recording, s_k, an observed phase, ϕ_k, is obtained by a linear time wraping of the R-R intervals into $[0, 2\pi)$, leading to the following system:

$$\begin{bmatrix} \phi_k \\ s_k \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \theta_k \\ z_k \end{bmatrix} + \begin{bmatrix} u_k \\ v_k \end{bmatrix}, \tag{6}$$

where u_k and v_k are the corresponding observation noises with zero-mean random variable entries.

With multichannel recordings, representational redundancy of each ECG (fetal or maternal) can be exploited to estimate the information of the desired ECG mixed with the other ECGs and background noise. In order to do so, a linear transform is assumed to decompose M mixed ECG signals into N components. In other words, we assume that maternal and fetal ECGs have N components in total, which are demonstrated in M signals. For N mixed ECG components, the dynamic equations may be written as:

$$\begin{cases} \theta_{k+1}^{(1)} &= (\theta_k^{(1)} + \omega^{(1)}\delta)\text{mod}(2\pi) \\ z_{k+1}^{(1)} &= -\sum_{i\in\mathcal{W}^{(1)}} \delta \frac{\alpha_{i,k}^{(1)}\omega^{(1)}}{b_{i,k}^{(1)^2}} \\ & \quad \Delta\theta_{i,k}^{(1)} \text{exp}\left(-\frac{\Delta\theta_{i,k}^{(1)^2}}{2b_{i,k}^{(1)^2}} \right) + z_k^{(1)} + \eta_k^{z^{(1)}} \\ \alpha_{i,k+1}^{(1)} &= \alpha_{i,k}^{(1)} + \eta_k^{\alpha_i^{(1)}} \\ b_{i,k+1}^{(1)} &= b_{i,k}^{(1)} + \eta_k^{b_i^{(1)}} \\ \psi_{i,k+1}^{(1)} &= \psi_{i,k}^{(1)} + \eta_k^{\psi_i^{(1)}} \\ & \vdots \\ \theta_{k+1}^{(N)} &= (\theta_k^{(N)} + \omega^{(N)}\delta)\text{mod}(2\pi) \\ z_{k+1}^{(N)} &= -\sum_{i\in\mathcal{W}^{(N)}} \delta \frac{\alpha_{i,k}^{(N)}\omega^{(N)}}{b_{i,k}^{(N)^2}} \\ & \quad \Delta\theta_{i,k}^{(N)} \text{exp}\left(-\frac{\Delta\theta_{i,k}^{(N)^2}}{2b_{i,k}^{(N)^2}} \right) + z_k^{(N)} + \eta_k^{z^{(N)}} \\ \alpha_{i,k+1}^{(N)} &= \alpha_{i,k}^{(N)} + \eta_k^{\alpha_i^{(N)}} \\ b_{i,k+1}^{(N)} &= b_{i,k}^{(N)} + \eta_k^{b_i^{(N)}} \\ \psi_{i,k+1}^{(N)} &= \psi_{i,k}^{(N)} + \eta_k^{\psi_i^{(N)}} \end{cases} \tag{7}$$

The phase observations of N ECG components, $\phi^{(1)}$,...,$\phi^{(N)}$, and M noisy mixtures of the N ECG components, $s^{(1)}$,...,$s^{(M)}$, are related to the state vector at time k as follows:

$$\begin{bmatrix} \phi_k^{(1)} \\ \phi_k^{(2)} \\ \vdots \\ \phi_k^{(N)} \\ s_k^{(1)} \\ s_k^{(2)} \\ \vdots \\ s_k^{(M)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & a_{11} & \dots & a_{1N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & a_{M1} & \dots & a_{MN} \end{bmatrix} \cdot \begin{bmatrix} \theta_k^{(1)} \\ \theta_k^{(2)} \\ \vdots \\ \theta_k^{(N)} \\ z_k^{(1)} \\ z_k^{(2)} \\ \vdots \\ z_k^{(N)} \end{bmatrix}$$

$$+ \begin{bmatrix} u_k^{(1)}, u_k^{(2)}, \dots, u_k^{(N)}, v_k^{(1)}, v_k^{(2)}, \dots, v_k^{(M)} \end{bmatrix}^T \tag{8}$$

where $u_k^{(1)}$,...,$u_k^{(N)}$ and $v_k^{(1)}, ..., v_k^{(M)}$ are the corresponding observation noises. The key step prior to the implementation of the filter is the estimation of the set of state parameters for the n-th ECG component $\{\alpha_i^{(n)}, b_i^{(n)}, \psi_i^{(n)}, \omega^{(n)}\}_{i\in\mathcal{W}}$ as well as the mixing matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \dots & a_{MN} \end{bmatrix}. \tag{9}$$

In order to do so, the loading matrices provided by the previous step (Section 2.1) are used: the mixing matrix is directly defined as the concatenation of the loading matrices $\mathbf{A}^{(n)}$ related to all the ECG components; the state parameters $\{\alpha_i^{(n)}, b_i^{(n)}, \psi_i^{(n)}, \omega^{(n)}\}_{i\in\mathcal{W}}$ are obtained by fitting, for each n, the sum of the Gaussian functions ($i \in \mathcal{W}$) with the loading matrix $\mathbf{H}^{(n)}$; and the variability of the n-th ECG component is estimated using the third loading matrix $\mathbf{S}^{(n)}$, which can be used as the state noises.

3. RESULTS

The results of the proposed method on two sets of actual data have been presented and compared with the results of πCA and FastICA methods. The WCP, GCP, EKSWCP and EKSGCP labels denote results of the first and second proposed approaches for tensor decomposition without and with the Kalman filtering stage, respectively.

3.1. DaISy Dataset

The DaISy fetal ECG database [14] consists of a single dataset of cutaneous potential recording of a pregnant woman. A total of 8 channels (5 abdominal and 3 thoracic) are available, sampled at 250 Hz and lasting 10 seconds.

Fig.1 shows the results of fECG extraction using only the first and second channels of this dataset. The mECG and fECG tensors are of sizes $2 \times 12 \times 184$ and $2 \times 22 \times 113$ and the

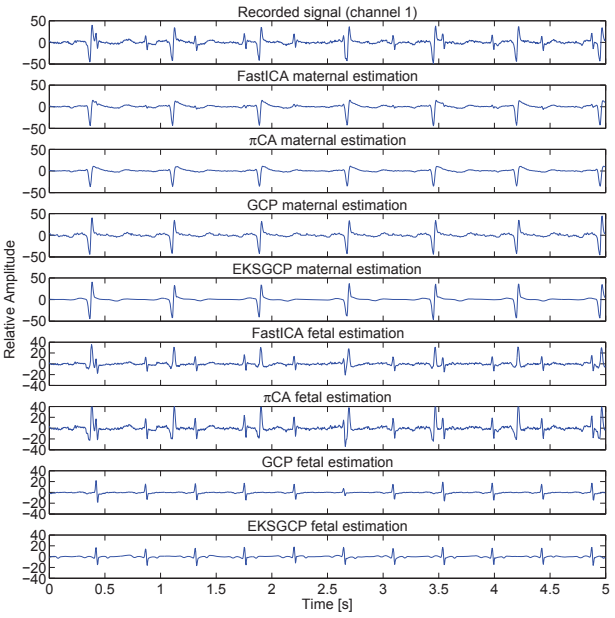

Fig. 1. mECG and fECG extraction by FastICA, πCA, GCP and EKSGCP on the first and second channels of DaISy data.

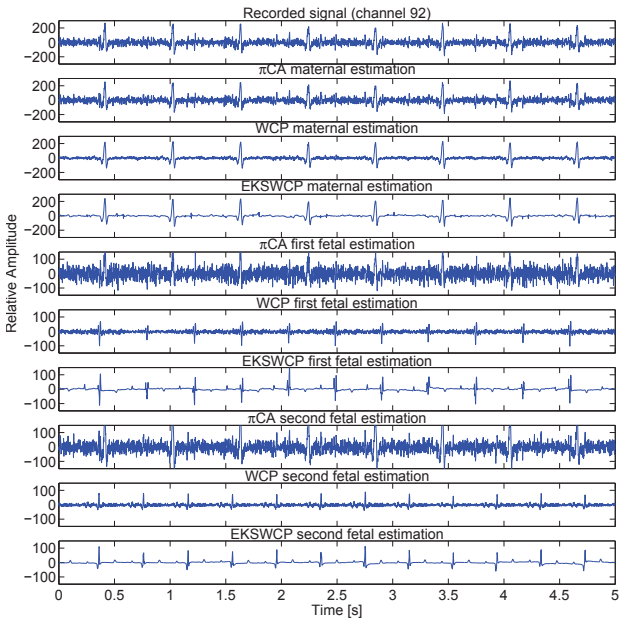

Fig. 2. Maternal and fetal MCG extraction by πCA, WCP and EKSWCP on the 92th and 116th channels of twin MCG data.

Table 1. Maternal and fetal R-peak values on fECG estimate of DaISY dataset (mean + standard deviation (SD)).

	Maternal R-peak value	Fetal R-peak value
Original mixture	43.66±2.38	17.68±2.37
FastICA	31.30±2.29	13.09±1.91
πCA	41.39±2.68	19.21±2.17
WCPD	-0.88±0.83	16.65±1.26
GCF	-0.90±0.91	16.04±2.72
EKSWCP	0.29±1.40	17.54±0.99
EKSGCP	0.17±1.46	16.19±1.11

Table 2. Maternal and fetal R-peak values on the **first** fetal MCG estimate of twin MCG dataset (mean + SD).

	Maternal R-peak value	First fetal R-peak value	Second fetal R-peak value
Original mixture	210.08±31.42	66.04±40.74	74.97±29.27
πCA	159.72±25.79	63.15±36.77	21.28±24.39
WCPD	-3.44±10.86	55.85±13.98	-2.57±8.37
GCF	-3.74±7.00	46.79±29.92	-3.08±9.55
EKSWCP	1.39±6.77	71.22±28.12	0.20±6.75
EKSGCP	1.94±8.10	65.48±33.29	1.06±8.85

chosen values for mECG and fECG ranks are equal to 2 and 1, respectively. πCA and FastICA methods demand several channels to recover the weak pattern of fECG, so as it is seen, if only two electrodes are available, they fail to extract fECG, whereas GCP and EKSGCP do not. There is neither ground truth nor golden standard on actual fetal ECG recordings to be used as the reference for comparing the performance of the different methods. Nevertheless, in order to quantize the performance of each method on actual data, the mean values of the contaminating and desired ECGs have been measured at their R-peak positions in the estimated ECG. This can provide an estimate of residual of the contaminating mECG in the estimated fECG. If the contaminating mECG has been successfully canceled, the values of this measure should be low, meanwhile, the values of the estimated fECG at its R-peak positions should be close to values of the corresponding points in the original mixture. Table 1 shows values of this measure on the fECG estimated by the different methods. Although GCP (or WCP) and EKSGCP (or EKSWCP) provided close quantitative results, it should be noted that valuable inter-beat dynamics of mECG and fECG are lost in the GCP (or WCP) estimate, because as it was explained in the previous section all beats of the reconstructed ECGs have exactly the same temporal pattern up to their amplitudes.

3.2. Twin MCG Dataset

Due to the morphological similarity of the ECG and the magnetocardiogram (MCG), the proposed method is also directly applicable to MCG recordings. Since the built tensor is weighted according to the signal of interest, as long as two

sources are not exactly synchronous, they can be separated even if their symbol rates are the same. This enables the method to separate twin cardiac signals even if heart rates are approximately equal. This discrimination is also provided by the Gaussian function in the second approach. The dataset used in this subsection consists of three sets of twin MCGs and other signals, in arrays of 208 channels recorded over 30 minutes, with a sampling rate of 1025 Hz [1]. The presented results have been achieved on a typical couple of channels (indexed 92 and 116) of namely the q00002252 dataset.

One maternal MCG and two fetal MCGs, there are three sources to be extracted, while two channels are to be utilized. FastICA and πCA methods are not applicable to underdetermined mixtures. Nevertheless, since in the πCA algorithms, the signal of interest is already selected, it is possible to apply πCA algorithms three times so that each time the covariance matrix is made according to the desired source. This way, all three MCGs can be estimated. The mECG and fECG ranks considered in the proposed method are 2 and 1, respectively. The three tensors are constructed with parameters $L_1 = 15$, $T_1 = 619$, $L_2 = 22$, $T_2 = 440$, $L_3 = 23$, and $T_3 = 408$. To suppress the large amount of noise that is present in the data, we also used the WCP decomposition for the mECG tensor. Fig.2 presents the results of πCA, WCP and EKSWCP in extraction of the maternal and two fetal MCG signals from two sensors. Here again, πCA method fails to track periodic patterns related to the fetal components due to their low power and insufficient number of the utilized electrodes. Nevertheless, WCP and EKSWCP could recover weak traces of fetal MCG features. The maternal and fetal R-peak values on the first fetal MCG estimate, are presented in Table 2.

4. CONCLUSIONS

The number of utilized channels is a key feature of a monitoring system that can affect the system's price, convenience and portability. Classical multichannel methods for fECG extraction need several sensors to recover the weak fECG signal. In order to utilize a minimal number of electrodes, two robust criteria for deterministic tensor decomposition have been employed to better track weak traces of fECG, then a nonlinear Bayesian filtering framework has been extended and used to improve the fECG and mECG estimates. The proposed method, which needs only two sensors to successfully recover several components of ECG signals performs significantly better than more classical methods. Perspectives include deep comparison between tensor decomposition methods and application of the proposed method on other datasets.

[1] This dataset has been provided by Dr. Dirk Hoyer, from the Biomagnetic Center of the Department of Neurology, in Friedrich Schiller University, Jena, Germany.

Acknowledgement

This work has been partly supported by the European project ERC-2012-AdG-320684-CHESS. H. Becker has been supported by the Conseil Régional PACA and by CNRS.

5. REFERENCES

[1] L. de Lathauwer, B. de Moor, and J. Vandewalle, "Fetal electrocardiogram extraction by blind source subspace separation," *IEEE Trans. Biomed. Eng.*, vol. 47, no. 5, May 2000.

[2] J.-F. Cardoso, "Multidimensional Independent Component Analysis," in *Proc. of ICASSP '98*, vol. 4, pp. 1941–1944, May 1998.

[3] A. G. Favret, "Computer matches filter location of fatal R-waves," *Med Biol Eng*, vol. 6, no. 5, pp. 467–475, September 1968.

[4] B. Widrow, Jr. Glover, J.R., J.M. McCool, J. Kaunitz, C.S. Williams, R.H. Hearn, J.R. Zeidler, Jr. Eugene Dong, and R.C. Goodlin, "Adaptive noise cancelling: Principles and applications," *Proceedings of the IEEE*, vol. 63, no. 12, pp. 1692–1716, 1975.

[5] R. Sameni, C. Jutten, and M. B. Shamsollahi, "Multichannel Electrocardiogram Decomposition using Periodic Component Analysis," *IEEE Trans. Biomed. Eng.*, August 2008.

[6] M. Richter, T. Schreiber, and D.T. Kaplan, "Fetal ECG extraction with nonlinear state-space projections," *IEEE Trans. Biomed. Eng*, vol. 45, no. 1, pp. 133–137, 1998.

[7] P.P. Kanjilal, S. Palit, and G. Saha, "Fetal ECG extraction from single-channel maternal ECG using singular value decomposition," *IEEE Trans. Biomed. Eng*, vol. 44, no. 1, pp. 51–59, January 1997.

[8] F. Mochimaru, Y. Fujimoto, and Y. Ishikawa, "Detecting the fatal electrocardiogram by wavelet theory-based methods," *Progess in Biomedical Research*, vol. 7, no. 3, pp. 185–193, September 2002.

[9] A. Almeida, P. Comon, and X. Luciani, "Deterministic blind separation of sources having different symbol rates using tensor-based parallel deflation," in *Proceedings of the 9th international conference on Latent variable analysis and signal separation*, Berlin, Heidelberg, 2010, LVA/ICA'10, pp. 362–369, Springer-Verlag.

[10] P. Comon, X. Luciani, and A. L. F. De Almeida, "Tensor decompositions, alternating least squares and other tales," *Jour. Chemometrics*, vol. 23, pp. 393–405, August 2009.

[11] S. Leurgans, R. T. Ross, and R. B. Abel, "A decomposition for three-way arrays," *SIAM Jour. Matrix Anal. Appl.*, vol. 14, no. 4, pp. 1064–1083, October 1993.

[12] R. Sameni, M. B. Shamsollahi, C. Jutten, and G. D. Clifford, "A nonlinear Bayesian filtering framework for ECG denoising," *IEEE Trans. Biomed. Eng.*, vol. 54, no. 12, pp. 2172–2185, December 2007.

[13] O. Sayadi and M.B. Shamsollahi, "ECG denoising and compression using a modified extended Kalman filter structure," *IEEE Trans. Biomed. Eng.*, vol. 55, no. 9, pp. 2240–2248, September 2008.

[14] B. de Moor, P. de Gersem, B. de Schutter, and W. Favoreel, "DAISY: A database for identification of systems," *Journal A, Special Issue on CACSD (Computer Aided Control Systems Design*, vol. 38, no. 3, pp. 4–5, September 1997.