PERFORMANCE OF LINEAR CROSSTALK CANCELATION IN FOURTH GENERATION WIRED BROADBAND ACCESS NETWORKS

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ABSTRACT

The International Telecommunication Union (ITU) is currently working on a standard for the next digital subscriber line (DSL) generation, called G.fast. It uses frequencies up to 200 MHz and targets bit-rates of up to 1 Gbps over a single twisted-pair line. Crosstalk coupling represents a major limitation on such high frequencies. Hence, in this work we compare the performance of two prominent linear crosstalk cancellation schemes depending on transmission directions, zero-forcing linear equalization (ZFE) for upstream and diagonalizing precoding (DP) for downstream, when applied in a G.fast context. Both schemes require accurate channel state information. Simulation results reveal that even small estimation errors can severely limit the bit-rates achievable by ZFE and DP in some particular scenarios.

Index Terms— DSL, G.fast, vectoring, estimation errors

1. INTRODUCTION

Digital subscriber line (DSL) is the most widely used technology for providing high-speed data communication services over the wired access network ($\geq$ 360 million customers worldwide) [1]. DSL owes most of its success to its low cost, both, in terms of the service providers' investment as well as from a consumer point of view. Optical fiber promises substantially higher rates at additional costs for digging trenches and fiber installation. However, these costs are disproportional to the current market demand. Hybrid architectures such as fiber-to-the-basement (FTTB), fiber-to-the-curb/cabinet (FTTC), and fiber-to-the-distribution point (FTTD) constitute cost efficient intermediate steps towards fiber-to-the-home (FTTH). Under the working name G.fast the International Telecommunication Union (ITU) is developing a standard for DSL systems bridging the last 250 m to the customer premises at targeted bit-rates of up to 1 Gbps [2]. Short loop lengths come with lower insertion loss (IL)

and thus a higher usable frequency band of up to 200 MHz. However, strong crosstalk coupling at high frequencies potentially causes a degraded signal-to-interference-noise ratio (SINR).

We evaluate the bit-rate performance of two linear crosstalk cancellation ("vectoring") schemes, zero-forcing linear equalization (ZFE) [3] for upstream transmission and diagonalizing precoding (DP) [4] for downstream transmission, according to the current status of the G.fast recommendation [2]. More precisely, we study worst-case and best-case scenarios for ZFE and DP based on different network topologies, cable types, bandwidth profiles, and crosstalk coupling models. Furthermore, since both ZFE and DP depend on accurate channel state information (CSI), we also investigate the influence of CSI estimation errors on their performance. Since it is realistic to expect that the direct channel is estimated correctly, we only assume errors in the estimation of crosstalk magnitude and phase. The influence of estimation errors is analyzed depending on crosstalk coupling strength and deployment scenarios.

The paper is organized as follows. In Sections 2 and 3 we describe our DSL system and performance models. In Section 4 we provide simulation results before drawing our conclusions in Section 5.

2. SYSTEM AND CHANNEL MODEL

We consider a DSL system with U interfering subscriber lines, indexed by $\mathcal{U} = \{1, \ldots, U\}$. For all considered network topologies we assume that the modems are co-located at the distribution point while they may be distributed at the customers' side. G.fast employs time division duplexing (TDD) [2], leaving far-end crosstalk (FEXT) as the main interference source. Furthermore, the standard is based on discrete multi-tone (DMT) modulation. Ideally, data transmission on each tone can therefore be independently modeled as

$$\mathbf{y}^k = \mathbf{H}^k \mathbf{x}^k + \mathbf{z}^k, \tag{1}$$

where $\mathbf{x}^k \in \mathbb{R}^U$ with $x_u^k \sim \mathcal{N}(0, p_u^k)$ and $\mathbf{y}^k \in \mathbb{R}^U$ represent the transmitted and received symbols on tone k, respectively, and $\mathbf{H}^k \in \mathbb{R}^{U \times U}$ denotes the channel matrix on tone k.

Its main diagonal and off-diagonal elements represent the direct and crosstalk channel coefficients, respectively. The term $\mathbf{z}^k \in \mathbb{R}^U$ with elements $z_u^k \sim \mathcal{N}(0, \sigma_u^k)$ represents the background noise on tone k, including alien noise, i.e., also FEXT originating from lines that are not within the same vectoring group.

Our model of FEXT magnitudes is based on the common 99 % worst-case model [5], scaled by various offsets. Its phase is calculated deterministically based on the mean values of the channel model in [6]. The direct channel magnitude is modeled based on the British Telecom #0 model [5], using two parameterizations according to TP100 [7] and CAD55 [8] cables. The direct channel phase is deterministically defined by the channel model in [6] and the measurements in [9], see [10] for details. While the direct channel is assumed to be estimated perfectly, errors in crosstalk magnitude and phase influence the SNIR as detailed in the following section.

3. PERFORMANCE MODEL

In this section we present the formulas used for computing the SINR and the achievable bit-rates for ZFE and DP as derived in [10]. For simplicity we omit tone indexes in the following, understanding that all parameters are defined on a per-tone basis. Estimated signal at the receiver is calculated by $\hat{y}=H^{-1}y$ in upstream and $\hat{y}=HPx+z$ in downstream with $P=\text{diag}(H)H^{-1}$, where $\text{diag}(H)$ is the diagonal matrix of direct channel gains. The per-tone bit-rate of user u is given by [5]

$$r_u = f_S \cdot \log_2(1 + SINR_u/\Gamma), \qquad (2)$$

where Γ represents the SINR gap, $SINR_u$ and f_S denote the per-tone SINR of user u and the symbol frequency, respectively. Without crosstalk cancellation and assuming perfect CSI the SINR of user u is computed as [5]

$$SINR_u^{FEXT} = \frac{|H_{u,u}|^2 p_u}{\sum_{m \epsilon \mathcal{U} \backslash u} |H_{u,m}|^2 p_m + \sigma_u}, \qquad (3)$$

where $H_{u,m}$ is the channel gain from user m to user u, and $H_{u,u}$ is the direct channel gain of user u. The SINR of an ideal vectoring system is only limited by the background noise and given as [5]

$$SINR_u^{ideal} = \frac{|H_{u,u}|^2 p_u}{\sigma_u}. \qquad (4)$$

We model the error in crosstalk CSI estimation by the frequency-flat normalized expression

$$\xi_{u,m} = \frac{\delta_{u,m}}{|H_{u,m}|^2} \cdot 100[\%], \qquad (5)$$

for any $u,m \epsilon \mathcal{U}$, $u \neq m$, where $\delta_{u,m}$ is the expected value of the channel coefficient's squared error magnitude. The SINR achieved after ZFE under imperfect CSI is obtained as

$$SINR_u^{ZFE} = \frac{p_u}{\eta_u^{ZFE} + \tilde{\sigma}_u}, \qquad (6)$$

where η_u^{ZFE} is a residual FEXT term due to estimation errors and $\tilde{\sigma}_u$ represents an enhanced background noise. The SINR under DP and imperfect CSI is obtained as

$$SINR_u^{DP} = \frac{|H_{u,u}|^2 p_u}{\beta(\eta_u^{DP} + \sigma_u)}, \qquad (7)$$

where η_u^{DP} is again a residual FEXT term and $\beta \geq 1$ represents a power scaling factor that ensures compliance with the spectral mask [4].

Expressions (2) to (7) are used in the next section for performance evaluation. Background noise enhancement and power scaling have been shown to result in a negligible SINR degradation for frequencies $\leq 30\,\text{MHz}$ [3],[4]. However, we show that on frequencies up to 200 MHz, which is in fact the operational environment G.fast is targeted at, the performance of both schemes degrades especially under imperfect CSI.

4. SIMULATION RESULTS

To encompass the wide range of bit-rates achievable by the studied vectoring schemes in practical scenarios, we carefully design extreme channel instances. We consider different network topologies with varying loop lengths, two different bandwidth profiles, and three different FEXT levels based on offsets of 0 dB, -6 dB, and -9 dB from the worst-case model. We also consider two cable types, the TP100 [7] model which provides optimistic (low) values of insertion loss, and the CAD55 model which leads to rather pessimistic (high) values of insertion loss [8]. Furthermore, various values of crosstalk channel estimation errors are considered as listed in Table 1. For reference, estimation errors in commercial VDSL2 equipment due to outdated CSI have been found to be in similar rage [11]. In Section 4.1 we consider bus network topologies, while in Section 4.2 tree topologies are assumed that lead to lower crosstalk couplings. All simulations were conducted in our xDSL simulator [12] with simulation parameters listed in Table 1.The shown results are based on the assumption that the link is only utilized in a single transmission direction.

Table 1: Simulation parameters.

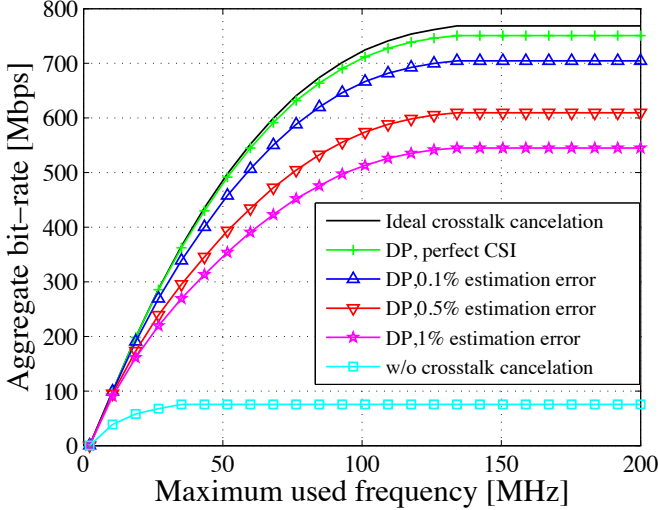|	
Transmit PSD	$-76\,\text{dBm/Hz}$
Start (min) frequency	2.2 MHz, 17.664 MHz, 30 MHz
End (max) frequency	200 MHz
SNR Gap	10.75 dB
Tone width	$12 * 4.3125\,\text{kHz}$
Min/max bits per tone	1 bit/12 bits
Background noise	$-140\,\text{dBm/Hz}$
Symbol frequency	48000 symbols/s
Estimation errors	0.1 %, 0.5 %, 1 %
Total loop lengths	50 m, 100 m, 200 m

Fig. 1: Achievable bit-rates for DP with perfect CSI and different values of estimation error (200 m loop length).

Fig. 2: Achievable bit-rates for DP with perfect CSI and different values of estimation error (50 m loop length).

Table 2: Bit-rate loss [%] with perfect CSI and 0.1 % estimation error (in brackets) respectively.

		Start frequency		
		2.2 MHz	17.664 MHz	30 MHz
200 m	ZFE	2.3 (8)	3 (9.6)	3.5 (10)
	DP	2.3 (8.5)	3 (10)	3.5 (10.5)
100 m	ZFE	1.4 (20)	1.6 (22)	1.7 (24)
	DP	1.4 (21)	1.6 (23)	1.7 (25)
50 m	ZFE	0.41 (26.4)	0.47 (28.8)	0.5 (31)
	DP	0.41 (27)	0.47 (29)	0.5 (31.5)

4.1. Results based on bus network topologies

The first considered scenario is a bus topology with 12 co-located users. We consider TP100 cables, different loop lengths, and different bandwidth profiles. Under perfect CSI the performance of both, ZFE and DP, improves as the loop length decreases. As an example, in case of 200 m long loops and no crosstalk cancelation each user can achieve 75.5 Mbps. Applying DP for downstream transmission the bit-rate increases to 750 Mbps, which is only 2.3 % below the rate of 768 Mbps achievable by ideal crosstalk cancelation, see Fig. 1. Decreasing the loop length to 50 m, the losses by DP compared to ideal vectoring become even smaller (2.36 Gbps versus 2.37 Gbps, corresponding to a loss of 0.4 %), see Fig. 2. Estimation error of only 0.1 % causes a 27 % loss in bit-rate compared to ideal crosstalk cancelation while for 1 % of estimation error the loss in bit-rate reaches 51 %. Furthermore, longer loops (e.g., 200 m long) have shown to be relatively less sensitive to estimation errors. In case of 0.1 % error we have an 8.5 % loss in bit-rate, while for 1 % of estimation error the bit-rate loss is 29 % compared to ideal vectoring. The impact of estimation errors on the performance of ZFE and DP regarding the different bandwidth profiles is shown in Table 2. We see that increasing the start frequency there are larger bit-rate losses compared to ideal vectoring under both, perfect and imperfect CSI. This is explicable by the better channel conditions at low frequencies. Moreover, while the performance of ZFE and DP differs little in case of perfect CSI (see Table 2), in case of estimation errors DP yields lower bit-rates than ZFE, see Table 2.

So far we based our simulations on TP100 cables, while the CAD55 cables additionally considered in the following lead to higher values of insertion loss. Fig. 3 shows the impact
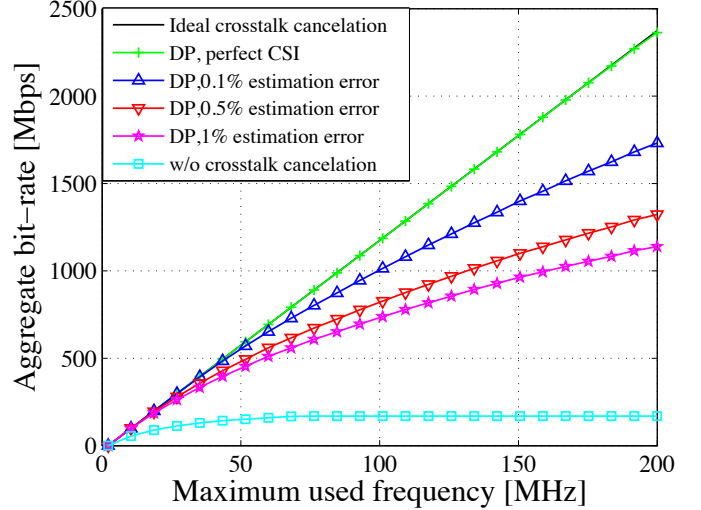
of estimation errors on the performance of DP regarding these two cable types and a loop length of 100 m. Under perfect CSI the performance of both schemes degrades[1] using the lower quality cable (higher insertion loss), while the influence of estimation errors fades. In Fig. 4 we see how the bit-rate loss increases with the number of users. The presented results are for TP100 cables, the 2.2 MHz/200 MHz bandwidth profile, and a 50 m loop length. However, assuming perfect CSI both, ZFE and DP, achieve bit-rate losses that are below 0.5 % even for 16 users. Summarizing the presented results, scenarios with good channel quality (low IL), high-FEXT (e.g., 50 m loops with TP100 cables) and a high number of co-located users have the highest bit-rate losses under estimation errors. On the other hand, scenarios where background noise is the main performance limitation (e.g., loops with high IL) are less impaired by estimation errors. Instead of considering all users co-located, in the following section we discuss results based on more realistic tree network topologies.

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[1]Results for ZFE are qualitatively similar to that under DP in Fig. 3 and therefore omitted.
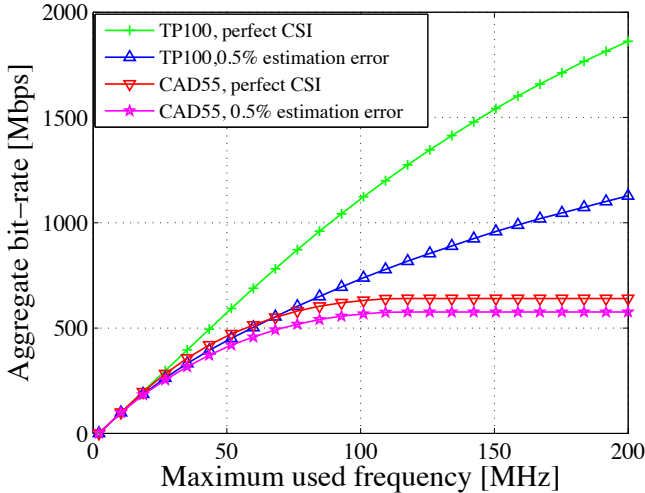
Fig. 3: Achievable bit-rates for DP with perfect and imperfect CSI for different cable types (TP100 and CAD 55, 100 m loop length).
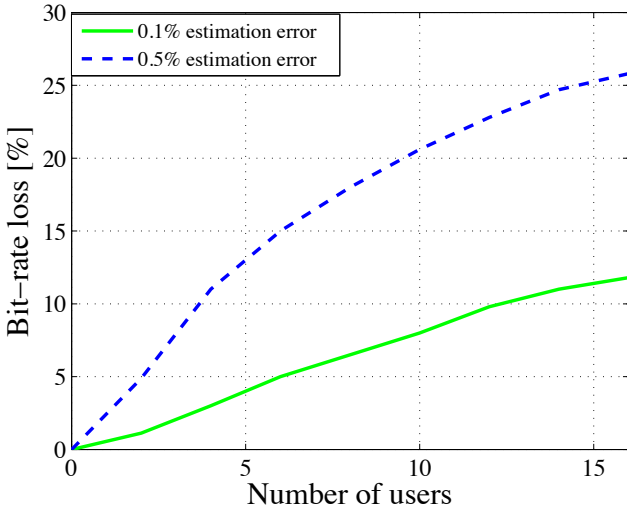

Fig. 4: DP bit-rate loss compared to ideal crosstalk cancelation with 0.1% and 0.5% estimation error and $50\,m$ loop length.

4.2. Results based on tree network topologies

We consider a tree topology with three branches that have a total loop length of $50\,m$, $100\,m$, and $200\,m$ and X_1, X_2, and X_3 users, respectively, see Fig. 5. The coupling length takes values of $1\,m$, $10\,m$ or $50\,m$. Fig. 6 shows bit-rates for the shortest lines in the tree topology with $U = 12$ users ($X_1 = X_2 = X_3 = 4$), a $50\,m$ coupling length, TP100 cables, and the $2.2\,MHz/200\,MHz$ bandwidth profile. Differently to Section 4.1, for this tree topology the performance of ZFE and DP under perfect CSI differs, recall also Table 2. ZFE
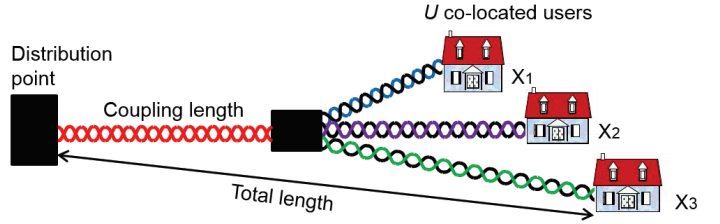

Fig. 5: Tree topology.

incurs a bit-rate loss of 1.5% compared to ideal vectoring while in case of DP the loss is 2%. This is due to the fact that the FEXT is not the same for upstream and downstream transmission as it was for the bus topology. Considering the users connected through the two longer tree branches, losses are even higher due to the longer loop length. ZFE incurs 4.6% bit-rate loss while DP loses 5.4% compared to ideal vectoring. However, we can see that under perfect CSI the losses of both schemes are negligible.

Under imperfect CSI, the performance of ZFE and DP changes in accordance with the conclusions drawn in Section 4.1. Users with the longest loop lengths are most sensitive to estimation errors under ZFE since they have the weakest direct channel and receive high FEXT noise from other users. For example, the bit-rate loss of ZFE with 0.5% estimation error varies between 32% and 57% among users. Since the FEXT from the short lines into the long lines is highly attenuated the power scaling penalty as well as bit-rate loss are low. As FEXT is attenuated by the insertion loss similarly as the direct channels, users with long loops have lower bit-rate losses compared to shorter ones under DP with imperfect CSI. Considering again 0.5% estimation error, the losses under DP vary between 47% and 15% among users.

All results discussed so far were obtained using the 99% worst-case model. In the following we assume a less conservative FEXT coupling, based on offsets of $-6\,dB$ and $-9\,dB$, respectively. We note that, as intuitively expected, the bit-rate loss of DP and ZFE now becomes smaller. Considering the same tree topology as before, under perfect CSI, ZFE and DP have a 1% and 1.4% bit-rate loss for the $-6\,dB$ offset and 0.5% and 0.7% loss for the $-9\,dB$ offset, respectively. Lower FEXT also alleviates the impact of estimation errors, as shown in Fig. 7.

5. DISCUSSION

Accurate channel estimation is crucial for optimal performance of ZFE and DP. Under perfect CSI, both schemes show negligible bit-rate losses compared to ideal vectoring. However, considering perfect CSI is highly unrealistic especially on high frequencies where FEXT coupling is strong. From the presented results we see that the impact of the estimation errors increases with the FEXT coupling, which

6. CONCLUSION

We analyze the performance of two linear crosstalk cancelation schemes (zero-forcing and diagonalizing precoding) in a G.fast compliant setup. We show that under perfect channel state information both schemes show negligible bit-rate losses compared to ideal crosstalk cancelation regardless of the considered network topology. However, even small estimation errors in crosstalk coupling e.g., $0.1\,\%$ severely limit their performance, thus requiring estimation methods with higher accuracy for G.fast.

7. REFERENCES

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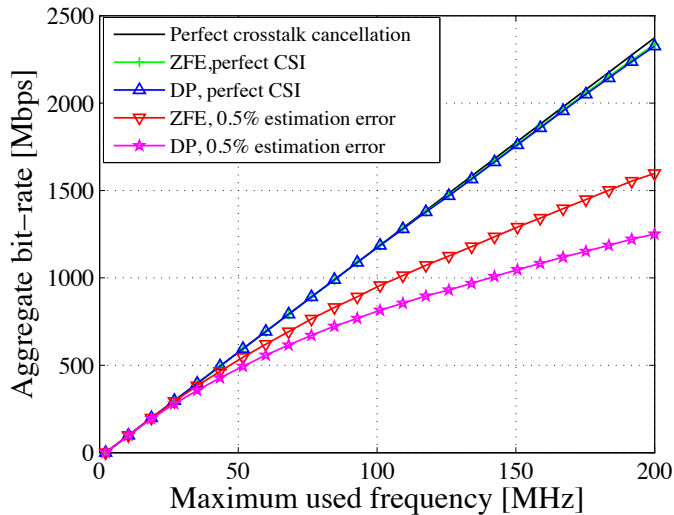[12] T. Nordström, D. Bengtsson, "The FTW xDSL simulation tool," version 3.1, 2008.

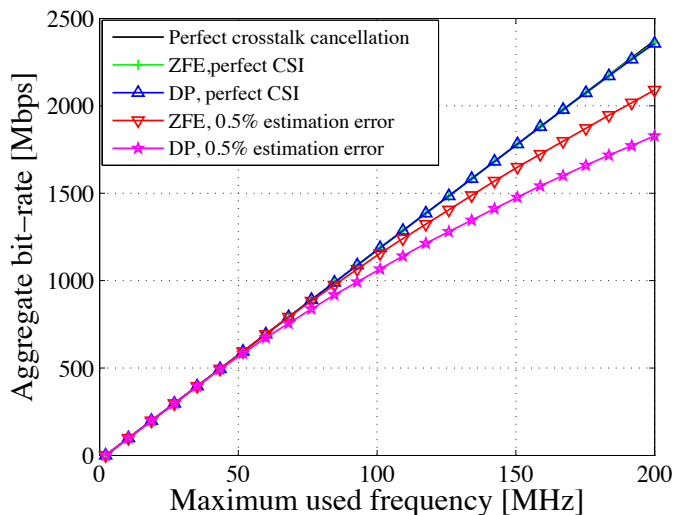**Fig. 6**: ZFE and DP performance for tree topology with $50\,\text{m}$ coupling length ($99\,\%$ worst-case model).

Fig. 7: ZFE and DP performance for tree topology with $50\,\text{m}$ coupling length ($99\,\%$ worst-case model - $9\,\text{dB}$).

tendentially increases with frequency. Consequently, ZFE and DP are less sensitive to estimation errors when used in frequency bands of previous DSL generations ($\leq 30\,\text{MHz}$), see also [10]. However, in this work we consider higher frequencies (up to $200\,\text{MHz}$) and show that ZFE and DP yield high bit-rate losses under imperfect CSI. We demonstrate that even with $99.9\,\%$ accurate CSI estimation (i.e. only $0.1\,\%$ estimation error) bit-rate losses are considerably high for some particular scenarios which are characterized by strong FEXT coupling, short loop lengths, and a high number of co-located users. Furthermore, analysis of more advanced vectoring schemes like minimum mean square error equalization or Tomlinson-Harashima precoding is planned for future work.