Cooperative ARQ protocols for Underlay Cognitive Radio Networks

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***Abstract*—In this paper, we derive the throughput of cooperative Automatic Repeat reQuest protocols for underlay cognitive radio networks. Three relay selection techniques are investigated : Opportunistic Amplify and Forward (O-AF), Partial Relay Selection AF (PRS-AF) and Opportunistic Decode and Forward (O-DF). In O-AF, the selected relay should offer the highest SNR of the relaying link (S-R-D) and verify some interference constraint. In PRS-AF, the selected relay offers the highest SNR of the first hop (S-R) and verify the interference constraint. In O-DF, the selected relay should have correctly decoded the packet, verify the interference constraint and offer the highest SNR of the second hop. Theoretical curves are validated using simulation results for different number of relays and interference threshold.**

Keywords*:* Cognitive radio networks, cooperative communications, Automatic Repeat reQuest (ARQ).

I. INTRODUCTION

Wireless communication systems were extensively used during the last decades in order to transmit voice, video and data. Wireless applications become more and more indispensable and the number of wireless users increases ceaselessly. At the same time, the radio spectrum remains a natural limited resource assigned or sold by auction to well identified users endowed with an operating license for a long period of time in a vast geographical zone. Generally, these users with license are little inclined to share their spectrum with other foreign and competitor users. However, it seems that an important portion of the spectrum under license is under-exploited in a proportion going from 15 to 85 per cent [1]. Cognitive Radio (CR) is a practical solution to the shortage of spectrum [2]. Indeed, CR nodes use the concept of dynamic spectrum access and consequently users without license (Secondary Users: SU) can use the spectrum without creating lot of interferences to Primary Users (PU) having a license.

Cognitive radio [1] has emerged as a promising technology to optimize spectrum resources exploitation by using the licensed spectrum in an opportunistic fashion [2-4]. In this technology, any cognitive secondary user may share the spectrum with a licensed primary user as long as the latter fulfills its Quality of Service (QoS) requirement. The protocols settling the coexistence of primary and secondary users are classified into three approaches [5]: (i) interweave approach where the secondary user can operate as long as the primary user is idle and must switch off whenever this latter becomes active. (ii) Overlay approach where the secondary and primary users share simultaneously the spectrum whereas the secondary nodes must implement and perform some techniques in order to cancel or reduce the interference caused at primary receivers. (iii) Finally, an underlay approach where secondary users share the spectrum with the primary one but have to adjust their transmit power to keep the induced interference always below a given allowable threshold. To fulfill the interference constraint, the secondary transmitter has generally to use low transmit power which limits largely the performances of the cognitive radio network and hence this network may suffer from a low throughput and high symbol error probabilities (SEP). A way to enhance the performance of the secondary network is to use relaying. Recently, several works have focused on relaying techniques in underlay cognitive radio network [6]-[18]. In [6], Zou et al. have proposed to select the relay with the largest signal-to-noise ratio (SNR) in relay-destination link under the constraint of satisfying a required primary outage probability. In [7], Chen et al. have proposed a distributed relay selection scheme while considering adaptive modulation and coding and energy states of relay nodes. The same authors have proposed in [8] a relay selection scheme that maximizes the secondary data rate whilst ensuring a minimum required primary data rate. The relay is selected based on a primal-dual priority-index heuristic. In [9], a distributed relay selection concurrently considering the channel states of all related links and residual energy state of the relay nodes have been proposed. In [10], krishna et al. have proposed that relays use beam steering capability to impose a target signal to interference plus noise ratio (SINR) whilst verifying the requirement of primary user. In [11], Lin et al. have used the pricing function in game theory to propose a novel low-interference relay selection derived from the conventional max-min relay selection. In [12], cooperative techniques are used for spectrum sensing and spectrum sharing by using space-time-frequency coding technique that can opportunistically adjust its coding structure by adapting itself to the dynamic spectrum environment. A selective fusion spectrum sensing and best relay data transmission is proposed in [13] for cognitive radio networks with multiple relays. The use of relays for spectrum sensing and secondary transmission is discussed in [14]. Optimal relay selection and power allocation for cognitive two-way relaying

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are presented in [15]. Spectrum leasing for cognitive radio are proposed in [16] to improve spectrum utilization. In [17], the benefits of employing relay station is investigated in large coverage cognitive radio systems. The diversity multiplexing tradeoff is evaluated in [18] for selective cooperation cognitive radio networks.

3G evolution toward HSPA and LTE means that packet-switched services will be more frequently used with respect to circuit-switched services. Even voice will be transmitted using packet-switched domain thanks to the success of experiences such as SKYPE. In this paper, we investigate the performance of Automatic Repeat reQuest (ARQ) protocols in cognitive radio networks. HARQ is used to retransmit erroneous received packet by transmitting Negative Acknowledgments (NACK) through a feedback channel. Different protocols were considered in the literature. ARQ consists in coding packets with only an error detection code. HARQ I consists in transmitting packets coded with an error detection and error correction codes. HARQ II consists in transmitting incrementally parity bits. ARQ and HARQ I with Chase Combining (CC) consists in combining with MRC weights all received packets in order to enhance SNR. HARQ using AF relaying has been investigated in [20] for cognitive radio networks in order to reach a given end-to-end error control mechanism. In [21], a study of the spectral efficiency of cognitive networks using adaptive modulation and coding was presented. Cross layer design of cognitive HARQ protocols was investigated in [22] based on the idea of spectrum pooling. In [23], a new cognitive system model is presented in order to eliminate the co-existence penalty. The effect of HARQ feedback on the average rate of unlicensed spectrum sharing channels was presented in [24]. Cognitive radio protocols based on HARQ retransmissions were presented in [25].

In this paper, we derive the performance of cooperative ARQ protocols for cognitive underlay networks using three relay selection techniques : opportunistic Amplify and Forward (AF), partial relay selection and opportunistic Decode and Forward (DF).

The paper is organized as follows. The next section presents the system model. Section III, IV and V derive the average number of transmission of cooperative ARQ using the different relay selection techniques. Section VI presents some numerical and simulation results. Section VII draws some conclusions.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a cognitive radio network composed of a primary and a secondary networks. The primary network contains only a source P_T and a destination P_R. The secondary one contains a source S, a destination D and M relays denoted by $R_1, R_2, ..., R_M$. We consider an underlay network and we assume that the secondary and primary sources share the spectrum. The secondary source should verify some interference constraints in order to not degarde the Quality Of Service (QOS) in the primary network. Besides, the relays are used to help the secondary source to deliver packets to the destination D. The generated interference by these relays to primary destination should be below a given threshold T. In the following, we consider three relay selection techniques : Opportunistic AF relaying, Partial relay selection and Opportunistic DF relaying.

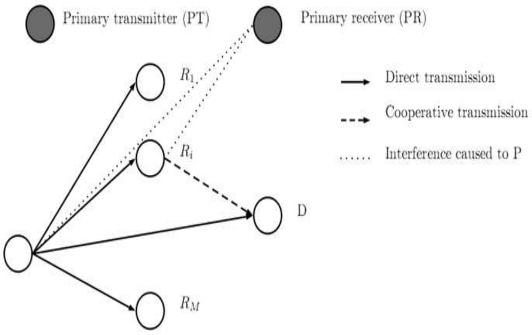

Fig. 1. System model.

A. *Opportunsitic AF relaying*

A subset $\Omega \subset \{1, ..., M\}$ of relays verifying interference constraint is first formed :

$$I_{R_iP_R} = E_{R_i} \left|h_{R_iP_R}\right|^2 < T, \forall i \in \Omega \tag{1}$$

where E_{R_i} is the transmitted energy per symbol by relay R_i and $h_{R_iP_R}$ is the channel coefficient between R_i and P_R. We assume a block Rayleigh fading channel where the channel is constant over each packet and independent from packet to another packet. The power of channel depends on the normalised distance $d_{R_iP_R}$ between R_i and P_R :

$$E\left(\left|h_{R_iP_R}\right|^2\right) = \frac{\gamma}{d^{\beta}_{R_iP_R}}, \tag{2}$$

where β is the pathloss exponent. The same model is adopted for the other links.

The best relays in Ω is selected using the SNR of AF relaying link :

$$R_{sel} = \arg\max_{i\in\Omega} \Gamma_{SR_iD}, \tag{3}$$

where [26]

$$\Gamma_{SR_iD} = \frac{\Gamma_{SR_i}\Gamma_{R_iD}}{\Gamma_{SR_i} + \Gamma_{R_iD} + 1}. \tag{4}$$

This SNR corresponds to the SNR of the entire link $S - R_i - D$ and depends on the SNR of the first hop Γ_{SR_i} and that of the second hop Γ_{R_iD}.

B. *Partial relay selection*

Here, the relays use also AF relaying. However, the selected relay in Ω offers the highest SNR of the first hop Γ_{SR_i} [27].

$$R_{sel} = \arg\max_{i\in\Omega} \Gamma_{SR_i}, \tag{5}$$

Partial relay selection can be easily implemented since it requires only the SNR of the first hop which can be measured at each relay. However, opportunistic relaying outperforms partial relay selection [28]. The diversity order of opportunistic relays equal to $M+1$ in case where the destination combines the signals from source and selected relay. However, the diversity order of partial relay selection is equal to two for any number of relays [28].

C. Opportunistic DF relaying

A subset is formed $C \subset \Omega$ of relays having correctly decoded and verifying interference constraints. The best relay is then selected using the SNR of the second hop

$$R_{sel} = \arg\max_{i \in C} \Gamma_{R_iD}, \tag{6}$$

III. COOPERATIVE ARQ USING OPPORTUNISTIC AF RELAYING

We assume that each packet contains k data bits and n_p parity bits. The first transmission is made by the source and the remaining ones by a selected relay. If all relays generate a lot of interference, the source retransmits the packet. The average number of transmissions is given by

$$\begin{aligned} T_r &= 1 + Pbloc_{SD} + Pbloc_{SD} \sum_{i=1}^{+\infty} Pbloc_{Ret}^{i} \qquad (7) \\ &= 1 + Pbloc_{SD} + Pbloc_{SD} \\ &\times \sum_{i=1}^{+\infty} \sum_{\theta \subset \{1,\dots,M\}} Pbloc_{Ret}^{i} | \Theta = \theta \; p(\Theta = \theta) \end{aligned}$$

where $Pbloc_{SD}$ is the BLock Error Probability (BLEP) of the direct link, $Pbloc_{Ret}|\Theta = \theta$ is the BLEP during a retransmission when the set of relays verifying interference constraint is $\theta, p(\Theta = \theta)$ is the probability that all relays in θ verify interference constraints

$$p(\Theta = \theta) = \prod_{i \in \theta} p(I_{R_iP_R} < T) \prod_{j \notin \theta} p(I_{R_jP_R} > T) \tag{8}$$

where

$$p(I_{R_iP_R} < T) = 1 - e^{-\frac{T}{\overline{\overline{I}}_{R_iP_R}}} \tag{9}$$

Assuming a BPSK modulation, the BLEP of the direct link is given by

$$Pbloc_{SD} = 1 - \int \left[1 - Q(\sqrt{2\gamma})\right]^{k+n_p} f_{\Gamma_{SD}}(\gamma) d\gamma \tag{10}$$

where $f_{\Gamma_{SD}}(\gamma)$ is the Probability Density Function (PDF) of the SNR of the direct link

$$f_{\Gamma_{SD}}(\gamma) = \frac{1}{\overline{\overline{\Gamma}}_{SD}} e^{-\frac{\gamma}{\overline{\overline{\Gamma}}_{SD}}} \tag{11}$$

If Θ is the empty set, the retransmission is made by the source and

$$Pbloc_{Ret} | \Theta = \emptyset = Pbloc_{SD}. \tag{12}$$

Otherwise, the BLEP is computed similarly to (10) using the SNR PDF of the best AF relaying link:

$$f_{\Gamma_{SR_{sel}D}}(\gamma) = \sum_{i \in \theta} f_{\Gamma_{SR_iD}}(\gamma) \prod_{j \in \theta, j \neq i} F_{\Gamma_{SR_iD}}(\gamma), \tag{13}$$

$F_{\Gamma_{SR_iD}}(\gamma)$ is the Cumulative Distribution Function of the SNR.
The statistics of the SNR of the relaying link are given by [29]

$$\begin{aligned} f_{\Gamma_{SR_iD}}(\gamma) = 2e^{-(\lambda_i+\mu_i)\gamma} \Big[\lambda_i\mu_i(2\gamma+1)K_0\left(2\sqrt{\lambda_i\mu_i\gamma(\gamma+1)}\right) \\ + (\lambda_i + \mu_i)\sqrt{\lambda_i\mu_i\gamma(\gamma+1)}K_1(2\sqrt{\lambda_i\mu_i\gamma(\gamma+1)})\Big], \end{aligned} \tag{14}$$

$$F_{\Gamma_{SR_iD}}(\gamma) = 1 - 2e^{-(\lambda_i+\mu_i)\gamma}\sqrt{\lambda_i\mu_i\gamma(\gamma+1)}K_1(2\sqrt{\lambda_i\mu_i\gamma(\gamma+1)}), \tag{15}$$

where $\lambda_i = \frac{1}{\overline{\overline{\Gamma}}_{SR_i}}$, $\mu_i = \frac{1}{\overline{\overline{\Gamma}}_{R_iD}}$ and K_v is the v-th order modified Bessel function of the second kind.

The normalized throughput of ARQ can be written as

$$Thr = \frac{k}{(k+n_p)T_r}. \tag{16}$$

IV. COOPERATIVE ARQ USING PARTIAL RELAY SELECTION

The average number of transmissions can be written similarly to the previous section

$$\begin{aligned} T_r = 1 + Pbloc_{SD} + Pbloc_{SD} \sum_{i=1}^{+\infty} [Pbloc_{SD}^{i} P(\Theta = \emptyset) \\ + \sum_{\theta \subset \{1,\dots,M\}, \theta \neq \emptyset} \sum_{k \in \theta} p_k Pbloc_{SR_kD}^{i} \; p(\Theta = \theta)] \end{aligned} \tag{17}$$

where $p_k = P(R_{sel} = R_k | \Theta = \theta)$ is the probability that R_k is the selected relay in subset θ [28]

$$p_k = \sum_{i \in I_k} \sum_{n=0}^{2^{N-2}-1} \frac{(-1)^{\zeta(n)}}{1 + \frac{\overline{\overline{\Gamma}}_{SR_i}}{\overline{\overline{\Gamma}}_{SR_k}} + \overline{\Gamma}_{SR_i} \sum_{p=1}^{N-2} \frac{\varepsilon_n(p)}{\overline{\overline{\Gamma}}_{SR_{l(k,i,p)}}}}, \tag{18}$$

where $I_k = \theta \backslash \{k\}, N = |\theta|$ is the number of relays verifying interference constraints, $(\varepsilon_n(1), ..., \varepsilon_n(N-2))$ is the binary representation of n, $\zeta(n) = \sum_{p=1}^{N-2} \varepsilon_n(p)$ and $\{l(k,i,p\}_{p=1}^{N-2}$ is the set of relays indices from θ except i and k.

When R_k is the selected relay, the BLEP is given by

$$Pbloc_{SR_kD} = 1 - \int \left[1 - Q(\sqrt{2\gamma})\right]^{k+n_p} f_{\Gamma_{SR_kD}}(\gamma) d\gamma \tag{19}$$

where the PDF of Γ_{SR_kD} is given in (14).

V. COOPERATIVE ARQ USING OPPORTUNISTIC DF RELAYING

The average number of transmissions for DF relaying is given by

$$T_r = 1 + Pbloc_{SD} + Pbloc_{SD} \sum_{i=1}^{+\infty} [Pbloc_{SD}^{i} P(C = \emptyset)$$

$$+ \sum_{\theta \subset \{1,\ldots,M\}, \theta \neq \emptyset} Pbloc^{i}_{R_{sel}D} | C = \theta \; p(C = \theta)] \qquad (20)$$

where $p(C = \theta)$ is the probability that relays in set θ have correctly decoded and verify interference constraints

$$p(C = \theta) = \prod_{i \in \theta} [1 - Pbloc_{SR_i}] \, p(I_{R_i P_R} < T) \qquad (21)$$

$$\prod_{j \notin \theta} \left[1 - (1 - Pbloc_{SR_j}) p(I_{R_j P_R} < T)\right]$$

The BLEP when R_{sel} is selected is given by

$$Pbloc_{R_{sel}D} | C = \theta = 1 - \int \left[1 - Q(\sqrt{2\gamma})\right]^{k+n_p} f_{\Gamma_{R_{sel}D}}(\gamma) d\gamma \qquad (22)$$

where

$$f_{\Gamma_{R_{sel}D}}(\gamma) = \sum_{i \in \theta} \frac{e^{-\frac{\gamma}{\overline{\Gamma}_{R_i D}}}}{\overline{\Gamma}_{R_i D}} \prod_{j \in \theta, j \neq i} \left[1 - e^{-\frac{\gamma}{\overline{\Gamma}_{R_j D}}}\right] \qquad (23)$$

VI. NUMERICAL AND SIMULATION RESULTS

In this section, we present some theoretical and simulation results for packets composed of $k = 190$ data bits, n_p=10 parity bits and BPSK modulation. The normalized distance between the secondary source and the k-th relay is d_{SR_k}=0.2+0.1*(k-1), the distance between R_k and secondary destination $d_{R_k D} = 1 - d_{SR_k}$, $d_{SD} = 1$, the distance between the relays and primary receiver is $d_{R_k P_R} = 1 = d_{SP_R}$. We assume that the source transmits only when the generated interference to primary receiver is below the threshold T.

Fig. 2 shows the throughput of ARQ when opportunistic AF relaying is used and $K = 3$ relays. The threshold T is varied and takes the following values $T = 0.2, 1, +\infty$. We verify that the performance are close to the absence of cooperation $K = 0$ when the threshold T is low. The performance of the secondary network improves when the threshold increases however this implies some performance degradation in the primary network. We also notice that theoretical curves are in a good agreement with simulation results. We have simulated 5000 packets transmissions. Each packet is transmitted until successful reception at D.

Fig.3 shows the throughput of ARQ for opportunistic AF and a threshold $T = 1$. In this figure, we have varied the number of relays. We notice that the performance improves when K increases which is due to spatial diversity. A good match between theoretical and simulation results is also observed for $K = 0$, 3, 5 and 8.

Fig. 4 compares the performance of opportunistic AF, partial relay selection and opportunistic DF for $T = 1$ and $K = 3$ relays. We observe that opportunistic DF outperforms opportunistic AF by 0.6 dB for Thr=0.5. This is due to the fact that AF protocol amplifies the noise with the useful signal. However, AF is less complex than DF since decoding is not performed at the relays. Partial relay selection offers the worst performance, the gap is equal to 1 dB with respect to opportunistic AF for Thr=0.5. However, partial relay selection requires less Channel State Information (CSI) with respect to the other protocols since only the SNR of the first hop is needed.

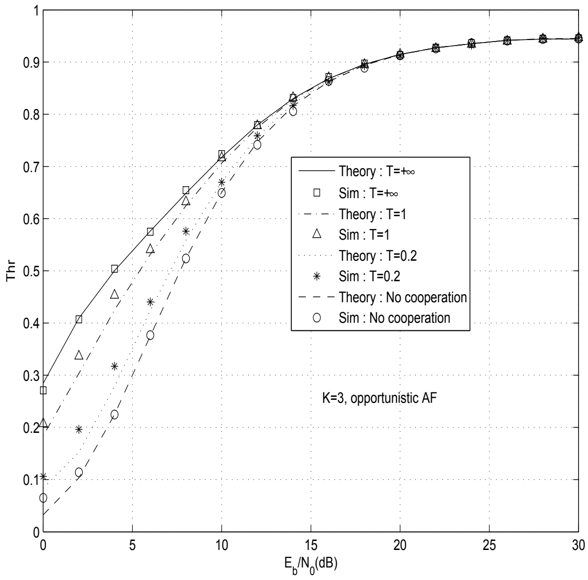

Fig. 2. Throughput for different values of the threshold T : Opportunistic AF, K=3 relays

VII. CONCLUSIONS AND PERSPECTIVES

In this paper, we have derived the throughput of cooperative ARQ protocols for underlay cognitive radio networks. Three relay selection techniques were considered. We have shown that opportunistic DF outperforms opportunistic AF. Partial relay selection offers the worst performance. As expected, the performance improves as the number of relays and the interference threshold increase. As a perspective, we will investigate the performance of HARQ I and HARQ II for cognitive radio networks. Reactive relay selection will be also investigated.

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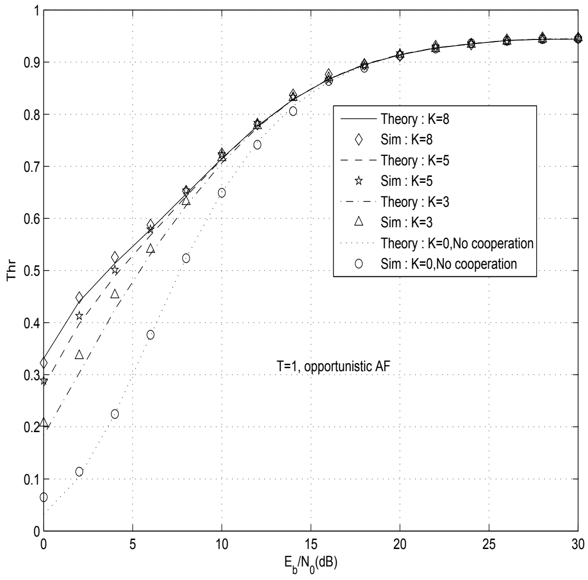

Fig. 3. Throughput for different number of relays : Opportunistic AF, T=1.

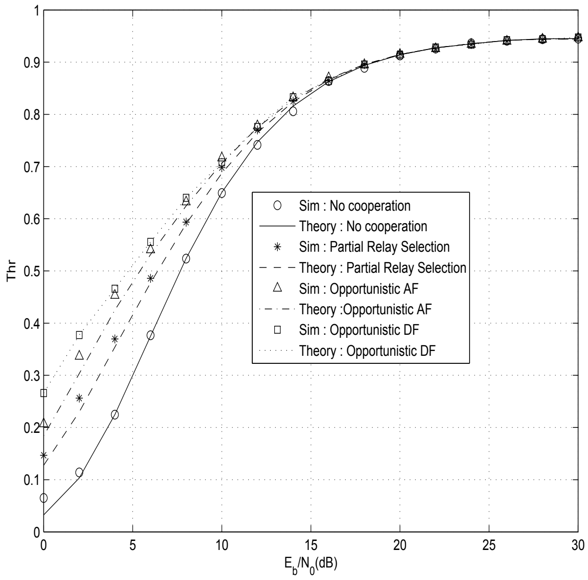

Fig. 4. Throughput for different relaying protocols : $T = 1$ and $K = 3$.

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