

# TOA SENSOR NETWORK CALIBRATION FOR RECEIVER AND TRANSMITTER SPACES WITH DIFFERENCE IN DIMENSION

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## ABSTRACT

We study and solve the previously unstudied problem of finding both sender and receiver positions from time of arrival (TOA) measurements when there is a difference in dimensionality between the affine subspaces spanned by receivers and senders. Anchor-free TOA network calibration has uses both in sound, radio and radio strength applications. Using linear techniques and requiring only a minimal number of receivers and senders, an algorithm is constructed for general dimension  $p$  for the lower dimensional subspace. Degenerate cases are determined and partially characterized as when receivers or senders inhabits a lower dimensional affine subspace than was given as input. The algorithm is further extended to the overdetermined case in a straightforward manner. Simulated experiments show good accuracy for the minimal solver and good performance under noisy measurements. An indoor environment experiment using microphones and speakers gives a RMSE of 2.35cm on receiver and sender positions compared to computer vision reconstruction.

## 1. INTRODUCTION

Sound ranging or sound localization has been used since world war I to determine the sound source using a number of microphones at known locations and measuring the time-difference of arrival of sounds. The same mathematical model is today used both for applications based on acoustics and radio and both for signal strength or time-based information such as time of arrival (TOA) or time differences of arrival (TDOA), or a combination thereof. Although such problems have been studied extensively in the literature in the form of localization of e.g. a sound source using a calibrated detector array, the problem of calibration of a sensor array using only measurement has received less attention.

In this paper we study the previously unstudied sensor network calibration problem using only TOA measurements for the particular case where there is a difference in dimensionality between the affine subspaces spanned by senders

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and receivers. We prove that such problems can be solved in closed form using linear techniques and give a solution scheme for general dimensionality  $p$ . Furthermore, the solver is extended to the overdetermined cases, and simulated and real experiments supports the feasibility of the method. The proposed method could be a part of sensor fusion methods and for Simultaneous Localization and Mapping, where a plausible problem is to track a receiver moving in 2D, e.g. on a building floor, when senders are in 3D in unknown positions.

Several previous works dealing with sensor network calibration rely on prior knowledge or extra assumptions of locations of the sensors to initialize the problem, see e.g. [1, 2, 3].

In [4] a far field approximation was utilized to solve the TOA and TDOA case. Initialization of TOA networks using only measurements has been studied in [5, 6], where solutions to the minimal cases of three senders and three receivers in the plane, or six senders and four receivers in 3D are given. Initialization of TDOA networks is studied in [7], where solutions were given to two non-minimal cases of ten senders and five receivers. In [8, 9] a TDOA setup is used for indoor navigation based on non-linear optimization, but the methods can get stuck in local minima and is dependent on initialization.

Of the above contributions, [4, 5, 6, 7] can be said to solve a calibration problem with either minimal or close to minimal data. Minimal algorithms for sensor network calibration are of interest in random sample consensus (RANSAC) schemes to weed out outliers in noisy or incorrectly matched TOA/TDOA measurements. The difference in dimensionality problem we study here is either a degenerate case for the papers above, or requires estimating several extra variables because of the assumption that receivers and senders lie in a same dimensional subspace. Thus the methods in the papers above are ill suited or cannot be applied to the problem at hand.

## 2. PROBLEM FORMULATION

In this paper we study the so called TOA node calibration problem when the dimension of the affine subspaces spanned by receivers and senders are different. We assume that (i) the speed of sound  $v$  is known, and thus all time measurements are transformed to distances by multiplication by  $v$ , (ii) re-

ceivers can distinguish which TOA signal comes from which sender. This can be done in practice by e.g. separating the signals temporally or by frequency.

**Problem 2.1.** Assume receiver coordinates  $\mathbf{r}_i, i = 1, \dots, m$  are embedded in a  $p$ -dimensional affine space  $\Pi$  and sender coordinates  $\mathbf{s}_j, j = 1, \dots, n$  are embedded in a  $d_2$ -dimensional, possibly infinite, space  $\Pi_2$ , where  $\Pi \subset \Pi_2$ . Given absolute distance measurements  $d_{ij} = \|\mathbf{r}_i - \mathbf{s}_j\|_2$ , determine  $\mathbf{r}_i$  and  $\mathbf{s}_j$ .

Without loss of generality, receivers and senders can be interchanged. We need to consider what we can expect of a solution. First of all, while still fulfilling the distance measurements, a solution can be changed according to

$$\begin{aligned}\bar{\mathbf{r}}_i &= \mathbf{r}_i + \mathbf{t}, \quad \bar{\mathbf{s}}_j = \mathbf{s}_j + \mathbf{t}, \\ \bar{\mathbf{r}}_i &= R\mathbf{r}_i, \quad \bar{\mathbf{s}}_j = R\mathbf{s}_j,\end{aligned}$$

where  $R$  is a rotation and/or mirroring matrix and  $\mathbf{t}$  a translation. Assuming a rotation so that the last coordinate of the receivers  $\mathbf{r}_i$  are 0, the problem has undeterminable degrees of freedom (gauge freedom) of  $p$  from the translation and  $p(p-1)/2$  from the rotation, which gives a total number of  $p + p(p-1)/2$ .

Furthermore, for the senders  $\mathbf{s}_j$  one can only hope to determine the orthogonal projection onto  $\Pi$  and the distance to  $\Pi$ , as a rotation around  $\Pi$  will keep the receivers constant but change the senders, without changing the measurements. Therefore, for practical purposes one can assume that senders are embedded in a  $p+1$  dimensional space. Assuming this is the case, each sender can still be mirrored in  $\Pi$  without changing the distances, giving a sign ambiguity on each sender's last coordinate.

We denote the problem as minimal if the number of solutions  $\mathbf{r}_i, \mathbf{s}_j$  for generic distance measurements  $d_{ij}$  is finite and at least one up to translation, rotation, mirroring in coordinate axis and mirroring of each  $\mathbf{s}_j$  in  $\Pi$ .

There are many ways to account for these ambiguities. We choose to translate and rotate the coordinate system so that (i) the last coordinate of  $\mathbf{r}_i$  is 0. The extra coordinate for sender  $j$  will thus be  $\mathbf{s}_j$ 's last coordinate. The rest of the rotation ambiguity is left free, and fixed first when comparing two solutions using [10]. (ii) The translation is locked by setting  $\mathbf{r}_1 = \mathbf{0}$ . (iii) The mirroring is locked by setting  $\mathbf{s}_j$ 's last coordinate to positive.

### 3. SOLVING THE TOA CALIBRATION PROBLEM

Let  $D = [d_{ij}]_{m \times n}$  be the matrix with the distance measurements, and  $D_2 = [d_{ij}^2]_{m \times n}$  be the matrix with the distance measurements squared. As  $d_{ij}^2 = (\mathbf{r}_i - \mathbf{s}_j)^T(\mathbf{r}_i - \mathbf{s}_j) = \mathbf{r}_i^T \mathbf{r}_i - 2\mathbf{r}_i^T \mathbf{s}_j + \mathbf{s}_j^T \mathbf{s}_j$ , the squared distances can be written as

$$d_{ij}^2 = [1 \quad -2\mathbf{r}_i^T \quad \mathbf{r}_i^T \mathbf{r}_i] [\mathbf{s}_j^T \mathbf{s}_j \quad \mathbf{s}_j^T \quad 1]^T = E_i F_j.$$

This leads to that  $D_2 = EF$  where  $E$  is a matrix with  $E_i$  as the  $i^{\text{th}}$  row and  $F$  is a matrix with  $F_j$  as the  $j^{\text{th}}$  column. This factorization was used in [7]. As the last coordinate of  $\mathbf{r}_i$  only has zeros, the second last column of  $E$  will be zeros, multiplying with the second last row of  $F$ . We can thus remove this row and column from the expression, forming

$$d_{ij}^2 = [1 \quad -2\tilde{\mathbf{r}}_i^T \quad \tilde{\mathbf{r}}_i^T \tilde{\mathbf{r}}_i] [\mathbf{s}_j^T \mathbf{s}_j \quad \tilde{\mathbf{s}}_j^T \quad 1]^T = \bar{E}_i \bar{F}_j \Rightarrow \quad (1)$$

$$D_2 = \bar{E} \bar{F},$$

where  $\tilde{\mathbf{r}}_i, \tilde{\mathbf{s}}_j$  are  $\mathbf{r}_i, \mathbf{s}_j$  with the last coordinate removed,  $\bar{E}$  is an  $m \times (p+2)$  matrix and  $\bar{F}$  is an  $(p+2) \times n$  matrix. This tells us that the rank of  $D_2$  is at most  $p+2$ .

We form the matrix  $S = [s_{ij}]_{(m-1) \times n}$ ,  $i = 1, \dots, m-1$

$$\begin{aligned}s_{ij} &= d_{i+1,j}^2 - d_{1,j}^2 \\ &= (\mathbf{r}_{i+1} - \mathbf{s}_j)^T(\mathbf{r}_{i+1} - \mathbf{s}_j) - (\mathbf{r}_1 - \mathbf{s}_j)^T(\mathbf{r}_1 - \mathbf{s}_j) \\ &= \tilde{\mathbf{r}}_{i+1}^T \tilde{\mathbf{r}}_{i+1} - 2\tilde{\mathbf{r}}_{i+1}^T \tilde{\mathbf{s}}_j \\ &= [-2\tilde{\mathbf{r}}_{i+1}^T \quad \tilde{\mathbf{r}}_{i+1}^T \tilde{\mathbf{r}}_{i+1}] [\tilde{\mathbf{s}}_j^T \quad 1]^T = \tilde{E}_i \tilde{F}_j,\end{aligned} \quad (2)$$

as  $\mathbf{r}_1 = \mathbf{0}$  and  $\mathbf{r}_i$ 's last coordinate is 0. By using the first receiver we have eliminated the quadratic constraints on the senders, effectively forming equations in our unknowns  $\mathbf{r}_i$  and  $\mathbf{s}_j$  which only depends on the  $p$  first coordinates, i.e. their orthogonal projection in  $\Pi$ . The equations are also linear in  $\mathbf{s}_j$ .

We note that  $S = \tilde{E} \tilde{F}$ , where  $\tilde{E}$  is an  $(m-1) \times (p+1)$  matrix where the  $i^{\text{th}}$  row is  $\tilde{E}_{i+1}$  and  $\tilde{F}$  is an  $(p+1) \times n$  matrix where the  $j^{\text{th}}$  column is  $\tilde{F}_j$ . This tells us that  $S$  is at most of rank  $p+1$ .

#### 3.1. Solving in $\Pi$

We seek a factorization  $S = \tilde{E} \tilde{F}$  such that  $\tilde{F}$  has a last row of ones, and the quadratic constraints in each row of  $\tilde{E}$  is fulfilled (2). Assuming that  $S$  has rank  $p+1$ , we start by doing the compact singular value decomposition (svd)

$$S = \underbrace{U}_{X_0} \underbrace{\Sigma V^T}_{Y_0} = X_0 Y_0, \quad (3)$$

where  $U, \Sigma$  and  $V$  are  $(m-1) \times (p+1)$ ,  $(p+1) \times (p+1)$  and  $n \times (p+1)$  respectively. We continue by expanding

$$S = \underbrace{X_0 B_0^{-1}}_{X_1} \underbrace{B_0 Y_0}_{Y_1} = X_1 Y_1, \quad (4)$$

where  $B_0$ 's last row is chosen so that  $Y_1$  will have a last row of ones. This is done by solving a linear system of equations with  $p+1$  unknowns, which tells us that we need  $p+1$  senders so that we can solve the system uniquely. This can always be done as  $Y_0$  has full rank, implied by the assumption that  $S$  has rank  $p+1$ .

The other  $p$  rows of  $B_0$  are calculated with Gram-Schmidt so that  $B_0$  is a unitary real matrix times a scalar. Then the condition number of  $B_0$  will be 1 so that  $X_0 B_0^{-1}$  does not lose unnecessary precision.  $Y_1$  does now have the right properties suggested by (2) but  $X_1$  does not. We continue by expanding

$$S = \underbrace{X_1 B_1}_{\tilde{E}} \underbrace{B_1^{-1} Y_1}_{\tilde{F}} = \tilde{E} \tilde{F}, \quad B_1 = \begin{bmatrix} A & \mathbf{b} \\ \vec{0} & 1 \end{bmatrix}, \quad (5)$$

where  $A$  is a general invertible  $p \times p$  matrix,  $\mathbf{b}$  is a general  $p \times 1$  matrix and  $\vec{0}$  is a vector of zeros. This implies that  $B_1^{-1}$  has the same restrictions as  $B_1$  which gives the most general form for preserving the last row of ones and rank in  $Y_1$  when forming  $B_1^{-1} Y_1$ . It remains to determine  $A$  and  $\mathbf{b}$ .

This is done by enforcing the quadratic constraints of the rows of the left matrix in (2) on  $X_1 B_1$ . We denote row  $i$  of  $X_1$  as  $[\mathbf{v}_i \ v_{i,p+1}]$  where  $\mathbf{v}_i$  is a vector of length  $p$  and  $v_{i,p+1}$  is the last element. The constraints then translate to

$$\mathbf{v}_i A A^T \mathbf{v}_i^T = 4(\mathbf{v}_i \mathbf{b} + v_{i,p+1}). \quad (6)$$

As  $A A^T = A R R^T A$  for any rotation and/or mirroring matrix  $R$ , and this  $R$  equates exactly to a rotation/mirroring of  $\mathbf{r}_i$  and  $\mathbf{s}_j$  in  $\Pi$ , we need only to solve (6) for the symmetric matrix  $C = A^T A$ . This gives  $p + p(p+1)/2$  unknowns from  $\mathbf{b}$  and  $C$ , which tells us that we need  $1 + p + p(p+1)/2$  receivers to be able to solve the linear equations (6) in the unknowns  $C$  and  $\mathbf{b}$  uniquely, as well as the system having full rank. The extra receiver comes from losing a row in  $S$  by subtracting the first row of  $D_2$  in (2).

When  $C$  and  $\mathbf{b}$  has been determined, an  $A$  can be calculated from e.g. cholesky factorization in  $C$ . Now the left and right hand factorization of  $S$ ,  $\tilde{E} = X_1 B_1$  and  $\tilde{F} = B_1^{-1} Y_1$  can be calculated.  $\tilde{\mathbf{r}}_{i+1}$  can then be calculated as the first  $p$  elements of row  $i$  of  $\tilde{E}$  divided by  $-2$ , and  $\tilde{\mathbf{s}}_j$  are the first  $p$  elements in respective column of  $\tilde{F}$ , according to (2). The last coordinate  $\mathbf{s}_j$  can then be recovered by using (1) and the last coordinate of  $\mathbf{r}_i$  is 0. Accordingly, we have the following algorithm:

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#### Algorithm 1.

*Input:* Dimension  $p$  of receiver space, TOA measurement matrix  $D = [d_{ij}]$  of size  $m = 1 + p + p(p+1)/2$  by  $n = p + 1$ .

*Output:* Receiver and sender positions  $\mathbf{r}_i$  and  $\mathbf{s}_j$

*Postconditions:* (i)  $S$  has rank  $p + 1$ , (ii) linear system  $\mathbf{v}_i C \mathbf{v}_i^T = 4(\mathbf{v}_i \mathbf{b} + v_{i,p+1})$  has full rank, (iii)  $C$  is positive definite

1. Set  $S := [d_{i+1,j}^2 - d_{1,j}^2]$
2. Calculate the svd  $S = U \Sigma V^T$  and set  $X_0$  to first  $p + 1$  columns of  $U$  and  $Y_0$  to first  $p + 1$  rows of  $\Sigma V^T$
3. Calculate  $B_0$  such that  $B_0 Y_0$  has last row of 1's and such that  $B_0$  is unitary times a scalar

4. Set  $X_1 := X_0 B_0^{-1}$ ,  $Y_1 := B_0 Y_0$ ,  $\mathbf{v}_i$  to the first  $p$  elements of row  $i$  of  $X_1$  and  $v_{i,p+1}$  to the last.
  5. Solve for the unknowns in the symmetric matrix  $C$  and vector  $\mathbf{b}$  the  $m - 1$  linear equations  $\mathbf{v}_i C \mathbf{v}_i^T = 4(\mathbf{v}_i \mathbf{b} + v_{i,p+1})$
  6. Calculate the cholesky decomposition  $C = A A^T$  and set  $B_1 := \begin{bmatrix} A & \mathbf{b} \\ \vec{0} & 1 \end{bmatrix}$
  7. Set  $\tilde{E} := X_1 B_1$ ,  $\tilde{F} := B_1^{-1} Y_1$ ,  $\tilde{\mathbf{r}}_{i+1}$  to the  $p$  first elements in row  $i$  of  $\tilde{E}$  divided by  $-2$ , and  $\tilde{\mathbf{s}}_j$  to the first  $p$  elements of column  $j$  of  $\tilde{F}$
  8. Solve for  $\mathbf{s}_{j,z}$  to the positive sign solution,  $d_{1j}^2 = [\tilde{\mathbf{s}}_j \ \mathbf{s}_{j,z}]^T [\tilde{\mathbf{s}}_j \ \mathbf{s}_{j,z}]$
  9. Set  $\mathbf{r}_1 := \vec{0}$ ,  $\mathbf{r}_i := [\tilde{\mathbf{r}}_i \ 0]$  and  $\mathbf{s}_j = [\tilde{\mathbf{s}}_j \ \mathbf{s}_{j,z}]$
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### 3.2. Minimal cases

By counting the degrees of freedom of the problem we get  $pm + (p+1)n - p - p(p-1)/2$ . The positive terms come from the coordinates of  $\mathbf{r}_i$  and  $\mathbf{s}_j$  respectively, and the negative terms come from the gauge freedom in Section 2. We have  $mn$  measurements, whereas the algorithm needs that  $m = 1 + p + p(p+1)/2$  and  $n = p + 1$ , giving  $mn = (1 + p + p(p+1)/2)(p+1)$  measurements. Comparing the number of measurements to the degrees of freedom, we see that they are equal. This tells us that the algorithm is minimal for all sizes of dimension  $p$  of the subspace  $\Pi$ .

Note that the algorithm tells us that the problem has essentially only one solution: Throughout the algorithm, we do not lose any solutions due to specific choices of parameters except equivalent solutions up to gauge freedom and mirroring in  $\Pi$ , according to section 2.1.

Assuming that the receivers and senders are in a same dimensional subspace with dimension  $p + 1$ , we can use the algorithm to just solve in  $\Pi$ , skipping step 8 and 9 in the algorithm. Comparing degrees of freedom to measurements now suggests an overdetermined system. Solving this problem in 3D we need 10 receivers and 4 senders, which is on par with [7] which also utilizes linear techniques, whereas [6] solves the minimal case of 6 receivers and 4 senders, but involves solving systems of polynomials.

### 3.3. Degenerate cases

**Theorem 3.1.** *Degenerate cases for the minimal algorithm are when i) the assumption that  $S$  is of full rank, i.e.  $p + 1$ , does not hold or ii) step 5 gives a linear system that does not have full rank.*

*i) happens iff  $X_0$  or  $Y_0$  in the svd in step 2 is rank deficient.  $Y_0$  is rank deficient iff the projection onto  $\Pi$  of the senders,  $\tilde{\mathbf{s}}_j$ , lie in an even lower dimensional affine subspace than  $\Pi$ .  $X_0$  is rank deficient if receivers  $\mathbf{r}_i$  span a lower dimensional subspace than  $\Pi$ .*

*Proof.* Looking at the assumptions for the algorithm to work, the only ones are the ones mentioned in (i) and (ii). Thus they constitute the degenerate cases.

The matrix  $S$  is trivially rank deficient iff  $X_0$  or  $Y_0$  is.  $Y_0$  is rank deficient iff  $\tilde{F}$  is rank deficient, as  $\tilde{F}$  is  $Y_0$  multiplied with invertible matrices.  $\tilde{F}$  is rank deficient iff  $\exists a_1, \dots, a_{p+1} \in \mathbb{R}$ , not all = 0, such that

$$\sum_{i=1}^p a_i \tilde{F}_i = a_{p+1} \mathbf{1} \quad (7)$$

as  $\tilde{F}$  has a last row of ones. As each row  $i$  of  $\tilde{F}$ ,  $\tilde{F}_i$ , consists of the  $i^{\text{th}}$  coordinates of  $\tilde{\mathbf{s}}_j$ , (7) is equivalent to that  $\tilde{\mathbf{s}}_j$  lie in an affine space with dimension  $< p$ .

The matrix  $X_0$  is rank deficient iff  $\tilde{E}$  is rank deficient, as  $\tilde{E}$  is  $X_0$  multiplied with invertible matrices.  $\tilde{E}$  is rank deficient if the first  $p$  columns do not span a  $p$ -dimensional space. As column  $k$  contains the  $k^{\text{th}}$  coordinates for  $\mathbf{r}_i$ , this gives that  $\mathbf{r}_i$  spans a lower dimensional subspace than  $\Pi$ .  $\square$

Note that the degenerate cases characterized in i) is inherent to the problem, not the algorithm. If the receivers or senders lie in a lower dimensional subspace than assumed, there are fewer degrees of freedoms to estimate than assumed.

A special case, resulting in complex solutions, is when the resulting matrix  $C$  in step 5 is not positive definite. If this happens, then there exists no real solution for  $A$  such that  $C = AA^T$ , and  $A$  will have to be complex, for example solved by eigenvalue decomposition  $C = Q^T D Q = Q^T \sqrt{D} \sqrt{D} Q = AA^T$ , giving complex solutions  $\mathbf{r}_i$  and  $\mathbf{s}_j$  fulfilling the measurements in  $D$ . This can also happen for the  $p + 1^{\text{th}}$  coordinate in  $\mathbf{s}_j$ , if the projection  $\tilde{\mathbf{s}}_j$  is larger than  $d_{1j}^2$ , which is used to calculate the last coordinate in step 8

### 3.4. More receivers and senders

When having more than  $1 + p + p(p + 1)/2$  receivers,  $p + 1$  senders and measurements  $d_{ij}$  possibly corrupted by noise, the algorithm can be expanded by i) approximating  $S$  in (3) by the closest rank  $p + 1$ -matrix in Frobenius norm, by setting the singular values after the  $p + 1$  first to zero, ii) taking the linear least squares solution to the system of equations resulting from (4) and (6) respectively. This will not be an optimal solution in any formal way, but will give a good initial solution, which can serve e.g. as an initial estimate for further non-linear optimization techniques.

From here on the extended algorithm will be used. Note that when having only the minimum numbers of receivers and senders, they are equivalent.

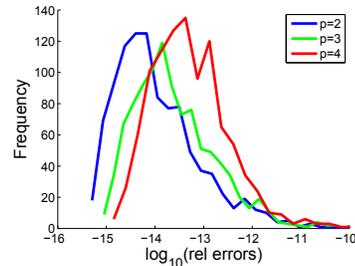
## 4. SIMULATED EXPERIMENTS

For all experiments, ground truth receivers  $\mathbf{r}_{i,gt}$  and senders  $\mathbf{s}_{j,gt}$  have been simulated, and from there a distance matrix  $D_{gt}$  has been calculated, which serves as input. The receivers,

residing in the subspace  $\Pi$  of dimension  $p$ , has been drawn from a uniform distribution over a unit cube centered around the origin. The senders  $\mathbf{s}_{j,gt}$ , residing in  $\Pi_2$  with dimension  $p + 1$ , have their distances from the origin drawn from a uniform distribution  $U(0.1\sqrt{(p)}, \sqrt{(p)})$  and then uniformly distributed over the sphere with that radius.

To be able to evaluate the quality of the solution,  $\mathbf{r}_i$ ,  $\mathbf{s}_j$  are rotated, mirrored and translated so that  $\sum_i \|\mathbf{R}(\mathbf{r}_i - \mathbf{t}) - \mathbf{r}_{i,gt}\|_2^2 + \sum_j \|\mathbf{R}(\mathbf{s}_j - \mathbf{t}) - \mathbf{s}_{j,gt}\|_2^2$  is minimized, where  $\mathbf{R}$  and  $\mathbf{t}$  is a rotation/mirroring and translation in  $\Pi$  respectively. Finding  $\mathbf{R}$  and  $\mathbf{t}$  is done by using [10]. The relative error of the solution is then calculated as  $\|\mathbf{r}_i - \mathbf{r}_{i,gt}\| / \|\mathbf{r}_{i,gt}\|$  and  $\|\mathbf{s}_j - \mathbf{s}_{j,gt}\| / \|\mathbf{s}_{j,gt}\|$ .

For the minimal solver, 1000 experiments were run for  $p = 2, 3, 4$  each. Histograms of relative errors can be seen in figure 1. As seen the errors are small. The mean computational time of the algorithm over these runs was 3.0 ms, run on a computer using Intel Core 2 Duo CPU with two 2.8 GHz processors, implemented in Matlab.

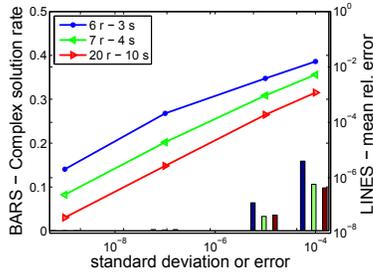


**Fig. 1.** Histograms of relative errors for the minimal solver.

In figure 2 the relative errors of 1000 runs,  $p = 2$ , for different number of receivers and senders, are plotted against additive white Gaussian noise on the measurements  $D$ . The errors are only calculated for the real solutions, as the complex ones do not have a physical meaning, though they fulfill the measurements  $D$  if not overdetermined. Complex solutions correspond to the special cases described in Section 3.3.

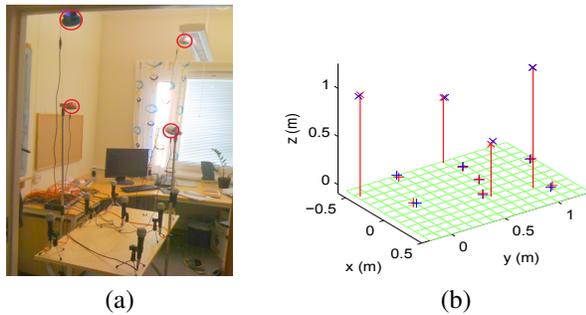
## 5. REAL DATA

Using seven Shure SV100 microphones and four Roxcore portable speakers, all connected to a Fast Track Ultra 8R sound card in an indoor environment, see figure 3, TOA measurements were obtained by matching sounds from different speakers to sound flanks recorded from different microphones. The sounds were separated temporally so that the matching of which sound came from which speaker could be done. Matching was done using the beginning of emitted sounds, thus ignoring reflections as there exist a direct path between speakers and microphones. Microphones were placed on a table, i.e. a plane, and speakers throughout the room, so that  $p = 2$ .



**Fig. 2.** Relative errors for 1000 simulations each plotted against additive Gaussian noise as graph, done for different number of receivers ( $r$ ) and senders ( $s$ ) with  $p = 2$ . Box plot shows the percentage of complex solutions.

To have a basis for comparison, a computer vision reconstruction of microphones and speakers were made, based on cell phone images of the experimental setup. The computer vision solution is then up to rotation, translation and scale. By running the algorithm for  $p = 2$  and aligning the solution with the computer vision solution, we got a root mean square error of 2.35 cm. See figure 3 for a visualization.



**Fig. 3.** (a) Experiment with speakers and microphones. Speakers are emphasized with circles. (b) shows the reconstructed solution using the algorithm (red) versus a reconstruction done with computer vision techniques (blue). Speakers are  $x$  and microphones are  $+$ .

## 6. CONCLUSIONS

In this paper we have solved the previously unsolved TOA calibration problem when receivers and senders are in different dimensional affine subspaces, for general dimensions. The primary interesting cases are when the lower dimensional subspace is on a line or in a plane, whereas higher dimensional solutions is for now of theoretical interest. The difference in dimensionality problem is an important degenerate case for previous papers focusing on the TOA calibration problem. We solve the minimal case and it is shown to be  $1 + p + p(p + 1)/2$  receivers inhabiting a subspace of dimension  $p$ , and  $p + 1$  senders inhabiting a higher dimensional subspace. Only one solution exists up to gauge freedom and

mirroring of senders in the subspace. We determine the degenerate cases and they are partially characterized. As a by-product we show how to solve the TOA calibration problem for same dimensional case for general dimensions, but this is not a minimal solver. The algorithm is extended in a straightforward manner to be able to handle more than a minimal configuration of receivers and senders, and can serve as an initial estimate for non-linear optimization.

Simulated and real experiments support the feasibility of the algorithm. Experiments on the minimal solver show small relative errors, and overdetermined systems handles additive noise well.

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