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BAYESIAN COMPRESSIVE SENSING USING MONTE CARLO METHODS

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ABSTRACT

The problem of reconstructing a signal from compressively sensed measurements is solved in this work from a Bayesian perspective. The proposed reconstruction solution differs from previous Bayesian methods in that it numerically evaluates the posterior of the sparse solution. This allows the method to utilize any kind of information on the signal without the need to evaluate the posterior in closed form. Specifically, the method uses multi-stage sampling together with a greedy subroutine to efficiently draw information directly from the likelihood and any prior distribution on the signal, including a sparsity prior. The approach is shown to accurately represent the Bayesian belief on the sparse solution based on noisy compressively sensed signals.

Index Terms— Bayesian compressive sensing, sparse reconstruction, Monte Carlo methods

1. INTRODUCTION

Compressive sensing enables the acquisition of possibly high-dimensional signals at sub-Nyquist rate while preserving signal structure [1–4]. One type of compressive acquisition process is agnostic to the signal being sensed. This non-adaptive process has the potential of simplifying receiver design, while at the same time preserving information in any type of acquired signal. Adaptive compressive acquisition methods have also been proposed which either adapt during compressive acquisition [5, 6] or configure the adaptive mechanism before signal acquisition based on prior information [7, 8].

Following compressive acquisition, a reconstruction process can be used to identify the original sparse signal. One type of reconstruction algorithm identifies a single reconstructed signal which best represents the compressed measurements based on a minimum Euclidian distance metric and the assumption of sparsity [9, 10]. Another type of reconstruction method is based on the Bayesian framework, where the posterior density function of the sparse solution is estimated [6]. A point estimate solution can also be obtained by a Bayesian method via maximum a posteriori (MAP) or minimum mean squared error (MMSE) estimation.

Existing Bayesian reconstruction methods combine the measurement likelihood with a sparsity promoting prior in order to estimate the posterior. Since the sparsity prior and likelihood are not conjugate, the resulting posterior cannot be evaluated in closed form. A hierarchical prior is then used to solve the Bayesian problem [6, 11, 12]. Moreover, Bayesian reconstruction methods are able to use adaptivity during the acquisition process and choose to obtain more compressive measurements to improve reconstruction [6]. Adaptivity has also been used in compressive sensing and processing (CSP) with no reconstruction in [7, 8]. However, current Bayesian reconstruction methods are not designed to use prior information on the composition of the signal, which is available in sequential estimation scenarios [7, 8] and configure the sensing mechanism prior to signal acquisition.

The Monte Carlo Bayesian Compressive Sensing method (MC-BCS) proposed in this work differs from existing Bayesian methods in that it estimates the posterior numerically using a sampling and weighting process. This process allows the method to directly utilize the likelihood and any kind of prior information without invoking a hierarchical form to evaluate the posterior, as done in existing Bayesian methods [6]. The ability of the method to effectively use prior information on the composition of the signal is shown to improve reconstruction quality. Moreover, the sparsity prior [6] is shown to be a special case of the prior on signal composition and to be seamlessly integrated in the method. The proposed method can, however, become computationally expensive when reconstructing sparse signals with a large numbers of elements. In order to overcome the dimensionality problem, a multi-stage sampling process [15, 16] involving a greedy-type subroutine [9, 10] is used to efficiently select individual signal elements which accurately represent the original signal.

In Section 2 the complex signal model is described. In Section 3 a connection is drawn between the solution provided by Bayesian and point estimate reconstruction methods and the proposed method. In Section 4 the proposed MC-BCS algorithm is described. The quality of the reconstruction so-

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lution is assessed in Section 5 with conclusions in Section 6.

2. SIGNAL MODEL

Nyquist rate signal:

A deterministic M-dimensional complex signal composed of elementary signals or atoms $\mathbf{s}_l, l \in \mathcal{T}_\jmath$ is given by

$$\grave{\mathbf{s}}_\jmath = \sum_{l \in \mathcal{T}_\jmath} \alpha(l)\mathbf{s}_l = \mathbf{S}\mathbf{x}_\jmath. \tag{1}$$

There are J such sparse signals with unique sets of atoms $\mathcal{T}_\jmath, \jmath = 1, \ldots, J$ of cardinality $T_\jmath = |\mathcal{T}_\jmath| << M \; \forall \jmath$. Furthermore, nearly orthogonal atoms indexed $l \in \mathcal{L}$ in any sparse signal are assumed to be members of a finite set $\mathcal{L} = \bigcup\{\mathcal{T}_\jmath\}_{\jmath=1}^{J}$ with cardinality $L = |\mathcal{L}| \leq M$. Then, a size $M \times L$ matrix $\mathbf{S}$ is then defined containing L atoms $\frac{1}{\sqrt{\xi_s}}\mathbf{s}_l, l = 1, \ldots, L$, in its columns. Therefore, $\grave{\mathbf{s}}_\jmath = \mathbf{S}\mathbf{x}_\jmath$ with $\mathbf{x}_\jmath = \sum_{l \in \mathcal{T}_\jmath} \alpha(l)\boldsymbol{\delta}_l$ being a sparse vector and $\boldsymbol{\delta}_l$ is a unit impulse at l. A prior probability on the appearance of $\grave{\mathbf{s}}_\jmath$ at the receiver, available in sequential estimation scenarios [7, 8], is given as

$$p(\grave{\mathbf{s}}_\jmath) = p(\mathcal{T}_\jmath), \jmath = 1, \ldots, J. \tag{2}$$

The associated atom probability distribution is then

$$p(l) = \sum_{\jmath : l \in \mathcal{T}_\jmath} p(\mathcal{T}_\jmath) \tag{3}$$

which sums probabilities $p(\mathcal{T}_\jmath)$ over $\jmath$ such that atom $l \in \mathcal{T}_\jmath$. $p(l)$ in (3) then represents the probability that atom l has non-negligible magnitude. The probability $p(l)$ may, for example, correspond to the probability that a radar target reflected signal containing a delay-Doppler corresponding to index l is expected to arrive at the receiver at the next time step. The information is based on prior probability on target state built from past measurements and kinematic information. A special case of prior information on the composition of the signal described in (2) and (3) is the probability on the number of atoms in the signal given by

$$p(T) \propto \sum_{\jmath : |\mathcal{T}_\jmath| = T} p(\mathcal{T}_\jmath) \tag{4}$$

where all probabilities of equal cardinality sets $\mathcal{T}_\jmath$ are combined to build the required probability distribution. If the probability $p(T)$ is heavily skewed towards small numbers of atoms T then $p(T)$ represents a sparsity prior [6].

Stochastic versions of (1) are given by length M vectors

$$\mathbf{r}_\jmath = \sum_{l \in \mathcal{T}_\jmath} \gamma(l)\mathbf{s}_l, \text{ and } \mathbf{r}_\jmath^{\mathfrak{v}} = \sum_{l \in \mathcal{T}_\jmath} \gamma(l)\mathbf{s}_l + \mathfrak{v}. \tag{5}$$

Random variables $\gamma(l), l \in \mathcal{T}_\jmath$ represent atom strength while $\mathfrak{v}$ is a length M zero mean random noise vector. $\mathbf{r}_\jmath$ represents the noiseless vector of the signal which is the one to be reconstructed while $\mathbf{r}_\jmath^{\mathfrak{v}}$ also contains additive noise.

The Compressive Acquisition Matrix:

A configurable acquisition matrix is given by [13]

$$\boldsymbol{\Phi} = \mathbf{Q}\,\mathbf{S}^* \tag{6}$$

composed of the $M \times L$ sparsity dictionary matrix $\mathbf{S}$ and the $C \times L$ matrix $\mathbf{Q}$ which is a partial Fourier compressive acquisition matrix [14] with the restricted isometry property (RIP) [1,2]. The matrix construction in (6) is shown in [13] to increase SNR, when no reconstruction is used, versus using a non-adaptive matrix. In Section 4 this matrix is also shown to improve reconstruction quality. In case a fixed matrix is used then $\boldsymbol{\Phi} = \mathbf{Q}$ where $\mathbf{Q}$ is a $C \times M$ partial Fourier matrix.

Compressively Sensed Signal:

A projection of the signal in (1) on a $C \times M$ acquisition matrix $\boldsymbol{\Phi}$ yields compressed deterministic sequences $\grave{\mathbf{g}}_\jmath, \jmath = 1, \ldots, J$ of dimensionality $C << M$ given, using (1), by

$$\grave{\mathbf{g}}_\jmath = \boldsymbol{\Phi}\grave{\mathbf{s}}_\jmath = \sum_{l \in \mathcal{T}_\jmath} \boldsymbol{\Phi}\mathbf{s}_l = \sum_{l \in \mathcal{T}_\jmath} \alpha(l)\mathbf{g}_l, \; \mathbf{g}_l = \boldsymbol{\Phi}\mathbf{s}_l. \tag{7}$$

Similarly, a projection of the signal in (5) yields the compressively acquired signal at the compressive receiver

$$\mathbf{h}_\jmath = \boldsymbol{\Phi}\mathbf{r}_\jmath^{\mathfrak{v}}. \tag{8}$$

3. THE GOAL OF RECONSTRUCTION AND CONNECTION TO PRIOR WORK

A Bayesian solution estimates the posterior distribution of the reconstructed signal given compressively sensed measurements and any prior information available on the signal to be received [7, 8]. The goal of the numerical based Bayesian reconstruction presented in this work is to identify a set of N reconstructed signals $\grave{\mathbf{s}}_n$ along with probabilities ω_n as

$$\grave{\mathbf{s}}_n = \sum_{l \in \mathcal{T}_n} \alpha(l)\mathbf{s}_l, \quad \omega_n \quad \text{for } n = 1, \ldots, N \tag{9}$$

with $\grave{\mathbf{s}}_n$ as in (1). Moreover, ω_n reflects the belief that the signal accurately represents the original noiseless signal $\mathbf{r}_\jmath$ in (5). Given the compressed measurements in (8), then

$$\omega_n = p(\grave{\mathbf{s}}_n|\mathbf{h}_\jmath) \propto p(\mathbf{h}_\jmath|\grave{\mathbf{s}}_n)p(\grave{\mathbf{s}}_n) \text{ for } n = 1, \ldots, N \tag{10}$$

represents a discrete version of the posterior, proportional to the likelihood $p(\mathbf{h}_\jmath|\grave{\mathbf{s}}_n)$ and prior $p(\grave{\mathbf{s}}_n)$ in (2) supported on $n = 1, \ldots, N$. The likelihood is based on the Euclidian distance [6] or equivalently the cross-correlation between templates $\grave{\mathbf{g}}_\jmath$ in (7) and the compressively received waveform $\mathbf{h}_\jmath$ in (8) defined later in (19). The compressed measurement likelihood has been shown in [13] to become more highly peaked when either the number of compressed measurements increases or when prior information is included in the acquisition process, with the potential to improve the overall performance of Bayesian reconstruction.

The probability distribution $p(\grave{\mathbf{s}}_n)$ in (2) on the other hand, represents any prior information on the composition of $\grave{\mathbf{s}}_n$, including that of sparsity. The proposed method differs from current Bayesian methods which restrict $p(\grave{\mathbf{s}}_n)$ to be a sparsity

prior [6]. Moreover, $p(\grave{\mathbf{s}}_n)$ is used both when compressively acquiring the measurements as explained in [8,13] and during reconstruction as explained in Section 4.1.

A point estimate can be derived from the posterior in (9) using the MAP as

$$\hat{\mathbf{s}}_{MAP} = \grave{\mathbf{s}}_n \text{ for } n = \underset{n}{\text{argmax}}\ \omega_n \tag{11}$$

or the minimum mean squared error (MMSE) as

$$\hat{\mathbf{s}}_{MMSE} = \sum_{n=1}^{N} \omega_n \grave{\mathbf{s}}_n. \tag{12}$$

Point estimate methods [9, 10] identify a sparse solution $\mathbf{x}_{opt}$ composed of atoms with indices $l \in \mathcal{T}_{opt}$ and magnitudes $\alpha_{opt}(l)$ as in (1) similarly to the MAP solution in (11) as

$$\min_{\mathbf{x}_{opt}} \{||\mathbf{h}_\jmath - \mathbf{\Phi S} \mathbf{x}_{opt}||_2^2 + \varrho ||\mathbf{x}_{opt}||_1\} \tag{13}$$

i.e. having a minimum Euclidian distance or equivalently maximum correlation as in with the compressed measurements and also being sparse. Sparsity on the solution is enforced using the l_1-norm of $\mathbf{x}_{opt}$ in (1). Moreover, the parameter ϱ balances between the Euclidean norm for minimizing noise energy and the l_1-norm for promoting sparsity.

4. THE RECONSTRUCTION PROCESS

In order to obtain the Bayesian solution in (9), a simple algorithm would assign a weight $\omega_\jmath$ to each signal $\grave{\mathbf{s}}_\jmath, \jmath = 1, \ldots, J$ in (1). However, this would be computationally expensive due to the large number of unique combinations of atoms J composing an equal number of solutions. Instead, a multi-stage proposal is used to first sample individual atoms most likely to be found in the signal and then merge the resulting good quality atoms into $N << J$ signals which accurately represent the noiseless signal in (5). This sampling method is based on multi-stage and likelihood sampling particle filtering methods [15–17] combined with a greedy method [9, 10] to emphasize weaker atoms in the signal.

4.1. Multi-stage atom sampling

For constructing each solution n, compressed templates $\mathbf{g}_{\tilde{l}} = \mathbf{\Phi s}_{\tilde{l}}, \tilde{l} = 1, \ldots, L$ are first formed as in (7). Then, for each atom index $t = 1, \ldots, T_{max}$ single atom correlations with the residue are found as

$$a(l) = \frac{\mathbf{g}_l^* \mathbf{h}_t^{res}}{|\mathbf{g}_l|^2},\ l = 1, \ldots, L. \tag{14}$$

Then the value of $a(l)$ both indicates the proposed atom amplitude and the single element likelihood ratio

$$\Lambda(a(l)) = \frac{p(a(l)|\mathbf{g}_l)}{p(a(l)|0)},\ l = 1, \ldots, L. \tag{15}$$

The type of the likelihood probability distribution $p(a(l)|\mathbf{g}_l)$ depends on the distribution of the atom strength which is scaled by $\frac{C}{M}$ (see [13]) and the additive noise in (5). Also, $p(a(l)|0)$ denotes the noise-only likelihood. Next, an atom is sampled as

$$l_t \sim \{\Lambda(a(\tilde{l}))p(\tilde{l})\}_{\tilde{l}=1}^{L} \tag{16}$$

where the prior distribution on individual atoms $p(l)$ in (3) was considered. The sampled atom is associated with magnitude $\hat{\alpha}_n(t) = a(l_t)$ found in (14) and a sampling bias of $b_t = \Lambda(\hat{\alpha}_n(t))p(l_t)$. For sampling the next atom, atom index t of solution n is incremented by 1 and the residue is updated similarly to [10]

$$\mathbf{h}_t^{res} = \mathbf{h}_{t-1}^{res} - \hat{\alpha}_n(t-1)\mathbf{g}_{l_t} \tag{17}$$

where the initial compressed signal residue is defined as $\mathbf{h}_t^{res} = \mathbf{h}_\jmath$, with $\mathbf{h}_\jmath$ as in (8). Therefore, using this greedy subroutine the sampled atom is removed from the measurements to emphasize the rest of the atoms for the next atom sampling step. The process of identifying atom amplitudes in (14) and sampling from the resulting single atom distribution as in (16) is repeated until the residue remains nearly unchanged as $\Delta_{res} = |||\mathbf{h}_t^{res}||_2^2 - ||\mathbf{h}_{t-1}^{res}||_2^2| < \Theta_{res}$ with Θ_{res} an assigned threshold found to work well in the numerical analysis of 5. Different stopping criteria [9], including ones related to the probability of false alarm or lower bounds in tracking error, could be applied in future work. It is noted that the use of a peaked prior in (16) leads to a more efficient sampling of atoms. Therefore, the overall number of proposed solutions N required decreases thus reducing computational expense.

Table 1. The MC-BCS Algorithm

- Obtain measurements $\mathbf{h}_\jmath = \mathbf{\Phi r}_\jmath^{\mathfrak{v}}$ (8)
- For each solution $n = 1, \ldots, N$
  - Let $\mathbf{h}_1^{res} = \mathbf{h}_\jmath$ (8)
  - For $t = 1, \ldots, T_{max}$
    - Calculate $a(l) = \frac{\mathbf{g}_l^* \mathbf{h}_t^{res}}{|\mathbf{g}_l|^2},\ l = 1, \ldots, L$ (14)
    - Calculate likelihood $\Lambda(a(l)),\ l = 1, \ldots, L$ (15)
    - Sample index $l_t \sim \{\Lambda(\alpha(\tilde{l}))p(\tilde{l})\}_{\tilde{l}=1}^{L}$ (16)
    - Update residue $\mathbf{h}_t^{res} = \mathbf{h}_{t-1}^{res} - \hat{\alpha}_n(t-1)\mathbf{g}_{l_{t-1}}$ (17)
    - If $\Delta_{res} < \Theta_{res}$ break loop and set $T_n = t$
  - Form solutions $\grave{\mathbf{s}}_n = \sum_{t=1}^{T_n} \hat{\alpha}_n(t)\mathbf{s}_{l_t}$ (18)
  - Calculate weights $\omega_n = \frac{\Lambda(\beta_n|\grave{\mathbf{g}}_n)p(\grave{\mathbf{s}}_n)}{\prod_{t=1}^{T_n} b_t}$ (21)

4.2. Reconstructed signals and weighting

Following the individual proposal of atoms, multi-element reconstructed signals are constructed as

$$\grave{\mathbf{s}}_n = \sum_{t=1}^{T_n} \hat{\alpha}_n(t)\mathbf{s}_{l_t} \tag{18}$$

for each n. Compressed templates $\grave{\mathbf{g}}_n = \mathbf{\Phi}\grave{\mathbf{s}}_n$ are then formed similarly to (7). The correlation of proposed compressed templates with the compressed measurement in (8) is

$$\beta_n = \grave{\mathbf{g}}_n^* \mathbf{h}_\jmath \tag{19}$$

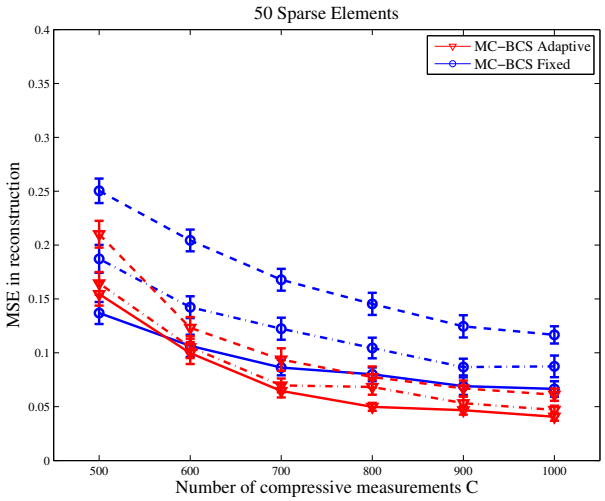

Fig. 1. MC-BCS MSE performance for 50 atoms for SNRs $18, 20, 22$dB with dash, dash-dotted and solid lines respectively.

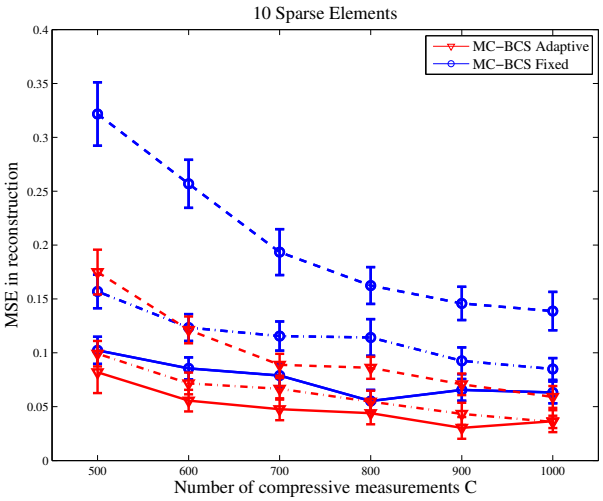

Fig. 2. MC-BCS MSE performance for 10 atoms for SNRs $18, 20, 22$dB with dash, dash-dotted and solid lines respectively.

and the likelihood ratio is identified as

$$\Lambda(\beta_n|\grave{\mathbf{g}}_n) = \frac{p(\beta_n|\grave{\mathbf{g}}_n)}{p(\beta_n|0)},\ n = 1, \ldots, N. \tag{20}$$

The likelihood $p(\beta_n|\grave{\mathbf{g}}_n)$ using correlation is equivalent to the Euclidean distance based likelihood, via the parallelogram identity, which is in line with the estimation goal in (13). The weight of each of the the reconstructed signals n are given as

$$\omega_n = \frac{\Lambda(\beta_n|\grave{\mathbf{g}}_n)p(\grave{\mathbf{s}}_n)}{\prod_{t=1}^{T_n} b_t}, \tag{21}$$

which take into account the bias b_t in proposing atoms in (16) [16] and the prior on the signal structure given by (2), and correspond to (10). As a special case the prior represents a sparsity prior in (4) where the above formulation penalizes solutions which do not conform to sparsity constraints. A single point estimate can be found as (11) or (12). In order to assess the reconstruction performance the mean squared error (MSE) in reconstruction is given by

$$\mathcal{E} = \sum_{n=1}^{N} \omega_n \frac{||\grave{\mathbf{s}}_n - \mathbf{r}_j||_2^2}{||\grave{\mathbf{s}}_n||_2 ||\mathbf{r}_j||_2}. \tag{22}$$

where the noiseless signal in (5) was used. The algorithm is described in Table 1.

5. NUMERICAL ASSESSMENT OF THE QUALITY OF RECONSTRUCTION

Simulation results were obtained over different numbers of compressed measurements C from 500 to 1000. The Nyquist dimensionality complex signals were chosen as time-frequency shifted Björck constant amplitude zero-autocorrelation (CAZAC) sequences [8, 18, 19] of prime length 1999 and with total length of $M = 2100$. The SNR took values of $18, 20$ and 22dB with atom energy and random strength variance set to 1. The SNR was defined as the ratio of

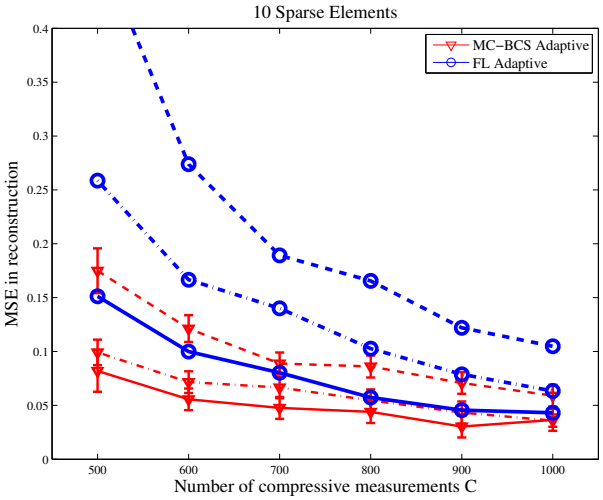

Fig. 3. MC-BCS compared to FL MSE performance for SNRs $18, 20, 22$dB with dash, dash-dotted and solid lines respectively.

the individual atom energy to the noise variance. The likelihood was Rayleigh distributed due to the zero mean complex Gaussian signal strength and noise terms. The atom prior distribution in (3) was chosen to be uniform over $L = 1000$ of the atoms. In the case no prior information is used then the resulting atom dictionary $\mathbf{S}$ is ignored when constructing the acquisition matrix in (6) and when executing the reconstruction algorithm. The number of atoms in each solution was allowed to increase to $T_{max} = 100$ until $\Theta_{res} = 0.01$. The number of solutions used was set to $N = 50$ and 100 Monte Carlo trials were taken.

In Figure 1 and Figure 2 the MSE in (22) was plotted versus the number of compressive measurements for 50 and 10 atoms in the received signal respectively. In all figures the performance improves as the SNR increases from 18 to 22dB where the different SNRs are shown with different line

style. It is, moreover, observed that performance improves as the number of compressive measurements increases due to the more concentrated single atom likelihood which results, as mentioned in Section 3 and shown in [13]. The overall performance improves when prior information is used in the adaptive matrix as shown in [13] and mentioned in Section 4 and also due to the sampling step in (16) which includes the prior. The results show that with the proposed method the use of prior makes the algorithm more robust to an increase in the number of true atoms over using a fixed agnostic acquisition matrix. Moreover, in Figure 3 the Fast Laplace (FL), in [12] and [11] for complex signals, with settings providing best performance, was compared with the MC-BCS for 10 atoms. Both algorithms used the adaptive matrix. The results show that the MCBCS, due to its use of the adaptive prior during atom proposal and estimation and not only in the adaptive matrix as in FL, is improved in performance over FL.

6. CONCLUSIONS

In this paper the MC-BCS method was proposed which numerically evaluates the posterior distribution of the sparse reconstruction solution. The method is able to utilize any kind of information available in the received signal to estimate the posterior without the need of a closed form solution as in existing Bayesian methods [6, 12]. Therefore, the proposed method is able to accommodate realistic measurements associated with non-Gaussian probability distributions. Goals for future research are the identification of large numbers of atoms in the compressively sensed signal, which currently proves to be computationally expensive, and the application of the method in realistic sequential estimation applications accurately described by non-Gaussian, non-linear models.

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