Abstract—We consider a bidirectional network in which two users exchange information with the help of a buffer-aided relay. In such a network without direct link between user 1 and user 2, there exist six possible transmission modes, i.e., four point-to-point modes (user 1-to-relay, user 2-to-relay, relay-to-user 1, relay-to-user 2), a multiple access mode (both users to the relay), and a broadcast mode (the relay to both users). Because of the buffering capability at the relay, the transmissions in the network are not restricted to adhere to a predefined schedule, and therefore, all the transmission modes in the bidirectional relay network can be used adaptively based on the instantaneous channel state information (CSI) of the involved links. For the considered network, assuming fixed transmit powers for both the users and the relay, we derive the optimal transmission mode selection policy which maximizes the sum rate. The proposed policy selects one out of the six possible transmission modes in each time slot based on the instantaneous CSI. Simulation results confirm the effectiveness of the proposed protocol compared to existing protocols.

I. INTRODUCTION

In a bidirectional relay network, two users share information via a relay node [1]. Several protocols have been proposed for such a network under the practical half-duplex constraint, i.e., a node cannot transmit and receive at the same time and in the same frequency band. The simplest protocol is the traditional two-way protocol in which the transmission is accomplished in four successive point-to-point phases: user 1-to-relay, relay-to-user 2, user 2-to-relay, and relay-to-user 1. In contrast, the time division broadcast (TDBC) protocol exploits the broadcasting capability of the wireless medium and combines the relay-to-user 1 and relay-to-user 2 phases into one phase, the broadcast phase [2]. Thereby, the relay broadcasts a superimposed codeword, carrying information for both user 1 and user 2, such that each user is able to recover its intended information by self-interference cancellation. To further increase the spectral efficiency, the multiple access broadcast (MABC) protocol was proposed in which the user 1-to-relay and user 2-to-relay phases are also combined into one phase, the multiple-access phase [3]. In the multiple-access phase, user 1 and user 2 simultaneously transmit to the relay, which decodes both messages. For the bidirectional relay network without a direct link between user 1 and user 2, the capacity regions of all six possible transmission modes, i.e., the four point-to-point modes, the multiple access mode, and the broadcast mode, are known [4], [5]. Using this knowledge, a significant research effort has been dedicated to obtaining the achievable rate region of the bidirectional relay network [1]-[8]. Specifically, the achievable rates of most existing protocols for two-hop relay transmission are limited by the instantaneous capacity of the weakest link associated with the relay. The reason for this is the fixed schedule of using the transmission modes which is adopted in all existing protocols, and does not exploit the instantaneous channel state information (CSI) of the involved links. For one-way relaying, an adaptive link selection protocol was proposed in [9] where based on the instantaneous CSI, in each time slot, either the source-relay link or the relay-destination link is selected for transmission. To this end, the relay has to have a buffer for data storage. This strategy was shown to achieve the capacity of the one-way relay channel with fading [10].

Motivated by the protocols in [9] and [10], our goal is to utilize all available degrees of freedom in the three-node half-duplex bidirectional relay network with fading, via a buffer-aided and adaptive mode selection protocol. In particular, given the fixed transmit powers of all three nodes in the bidirectional relay network, we find a policy which selects the optimal transmission mode from the six available modes in each time slot such that the sum rate is maximized. A similar problem was considered in [8]. However, the selection policy in [8] does not use all possible modes, i.e., it only selects between two point-to-point modes and the broadcast mode, and assumes that the transmit powers of all three nodes are identical. We will show that considering only the three modes in [8] is not optimal. In fact, the multiple access mode is selected with non-zero probability in the proposed optimal selection policy. Finally, we note that the advantages of buffering come at the expense of an increased end-to-end delay. Therefore, the proposed protocol is suitable for applications that are not sensitive to delay. The delay analysis of the proposed protocol is beyond the scope of the current work and is left for future research.

The remainder of the paper is organized as follows. In Section II, the system model is presented. In Section III, the average sum rate is investigated and the optimal mode selection policy is provided. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

In this section, we first describe the channel model and review the achievable rates for the six possible transmission modes.

A. Channel Model

We consider a simple network in which users 1 and 2 exchange information with the help of a relay node, as shown in Fig. 1. We assume that there is no direct link between
B. Transmission Modes and Their Achievable Rates

In the considered bidirectional relay network only six transmission modes are possible, cf. Fig. 2. The six possible transmission modes are denoted by $M_1, ..., M_6$, and $R_{ij}(i) \geq 0$, $j, j' \in \{1, 2, r\}$, denotes the transmission rate from node $j$ to node $j'$ in the $i$-th time slot. Let $B_1$ and $B_2$ denote two infinite-size buffers at the relay in which the received information from user 1 and user 2 is stored, respectively. Moreover, $Q_j(i), j \in \{1, 2\}$ denotes as the amount of normalized information in bits/symbol available in buffer $B_j$ in the $i$-th time slot. Using this notation, the transmission modes and their respective rates are presented in the following:

$M_1$: User 1 transmits to the relay and user 2 is silent. In this mode, the maximum rate from user 1 to the relay in the $i$-th time slot is given by $R_{11}(i) = C_{11}(i)$, where $C_{11}(i) = C(P_1S_1(i))$. The relay decodes this information and stores it in buffer $B_1$. Therefore, the amount of information in buffer $B_1$ increases to $Q_1(i) = Q_1(i-1) + R_{11}(i)$.

$M_2$: User 2 transmits to the relay and user 1 is silent. In this mode, the maximum rate from user 2 to the relay in the $i$-th time slot is given by $R_{21}(i) = C_{21}(i)$, where $C_{21}(i) = C(P_2S_2(i))$. The relay decodes this information and stores it in buffer $B_2$. Therefore, the amount of information in buffer $B_2$ increases to $Q_2(i) = Q_2(i-1) + R_{21}(i)$.

$M_3$: Both users 1 and 2 transmit to the relay simultaneously. For this mode, we assume that multiple access transmission is used, see [5]. Thereby, the maximum achievable sum rate in the $i$-th time slot is given by $R_{12}(i) + R_{22}(i) = C_r(i)$, where $C_r(i) = C(P_1S_1(i) + P_2S_2(i))$. Since user 1 and user 2 transmit independent messages, the sum rate, $C_r(i)$, can be decomposed into two rates, one from user 1 to the relay and the other one from user 2 to the relay. Moreover, these two capacity rates can be achieved via time sharing and successive interference cancellation. Therefore, in the first $0 \leq t(i) \leq 1$ fraction of the $i$-th time slot, the relay first decodes the codeword received from user 2 and considers the signal from user 1 as noise. Then, the relay subtracts the signal component of user 2 from the received signal and decodes the codeword received from user 1. A similar procedure is performed in the remaining $1 - t(i)$ fraction of the $i$-th time slot but now the relay first decodes the codeword from user 1 and treats the signal of user 2 as noise, and then decodes the codeword received from user 2. Therefore, for a given $t(i)$, we decompose $C_r(i)$ as $C_r(i) = C_{12}(i) + C_{21}(i)$ and the maximum rates from users 1 and 2 to the relay in the $i$-th time slot are $R_{12}(i) = C_{12}(i)$ and $R_{22}(i) = C_{21}(i)$, respectively. $C_{12}(i)$ and $C_{21}(i)$ are given by

$$C_{12}(i) = t(i)C(P_1S_1(i)) + (1 - t(i))C\left(\frac{P_1S_1(i)}{1 + P_2S_2(i)}\right)$$

$$C_{21}(i) = (1 - t(i))C\left(P_2S_2(i)\right) + t(i)C\left(\frac{P_2S_2(i)}{1 + P_1S_1(i)}\right)$$

The relay decodes the information received from user 1 and user 2 and stores it in its buffers $B_1$ and $B_2$, respectively. Therefore, the amount of information in buffers $B_1$ and $B_2$ increase to $Q_1(i) = Q_1(i-1) + R_{12}(i)$ and $Q_2(i) = Q_2(i-1) + R_{22}(i)$, respectively.

$M_4$: The relay transmits the information received from user 2 to user 1. Specifically, the relay extracts the information from buffer $B_2$, encodes it into a codeword, and transmits it to user 1. Therefore, the transmission rate from the relay to user 1 in the $i$-th time slot is limited by both the capacity of the relay-to-user 1 channel and the amount of information stored in buffer $B_2$. Thus, the maximum transmission rate from the relay to user 1 is given by $R_{r1}(i) = \min\{C_{r1}(i), Q_2(i-1)\}$, where $C_{r1}(i) = C(P_rS_1(i))$. Therefore, the amount of information...
A. Achievable Average Sum Rate

We assume that user 1 and user 2 always have enough information to send in all time slots and that the number of time slots, \(N\), satisfies \(N \to \infty\). Therefore, using \(q_k(i)\), the user 1-to-relay, user 2-to-relay, relay-to-user 1, and relay-to-user 2 average transmission rates, denoted by \(\bar{R}_{1r}, \bar{R}_{2r}, \bar{R}_{r1},\) and \(\bar{R}_{r2}\), respectively, are obtained as

\[
\bar{R}_{1r} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} [q_1(i)C_1r(i) + q_3(i)C_{12r}(i)]
\]

\[
\bar{R}_{2r} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} [q_2(i)C_2r(i) + q_3(i)C_{21r}(i)]
\]

\[
\bar{R}_{r1} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} [q_4(i) + q_6(i)] \min\{C_{r1}(i), Q_2(i-1)\}
\]

\[
\bar{R}_{r2} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} [q_5(i) + q_6(i)] \min\{C_{r2}(i), Q_1(i-1)\}
\]

The average rate from user 1 to user 2 is the average rate that user 2 receives from the relay, i.e., \(\bar{R}_{2r}\). Similarly, the average rate from user 2 to user 1 is the average rate that user 1 receives from the relay, i.e., \(\bar{R}_{r1}\). In the following theorem, we introduce a useful condition for the queues in the buffers of the relay leading to the optimal mode selection policy.

**Theorem 1 (Optimal Queue Condition):** The maximum sum rate, \(\bar{R}_{1r} + \bar{R}_{r2}\), for the considered bidirectional relay network is obtained when the queues in buffers \(B_1\) and \(B_2\) are at the edge of non-absorption. More precisely, the following conditions must hold for the maximum sum rate:

\[
\bar{R}_{1r} = \bar{R}_{r2} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} [q_5(i) + q_6(i)] C_{r2}(i)
\]

\[
\bar{R}_{2r} = \bar{R}_{r1} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} [q_4(i) + q_6(i)] C_{r1}(i)
\]

where \(\bar{R}_{1r}\) and \(\bar{R}_{2r}\) are given by (2a) and (2ab), respectively.

**Proof** Please refer to [11, Appendix A].

Using this theorem, we now derive the optimal transmission mode selection policy.

B. Optimal Mode Selection Protocol

The available degrees of freedom in the considered network are the mode selection variables \(q_i(k)\), \(\forall k, i\) and the time sharing variable in the multiple access mode, \(t(i), \forall i\). In the following, we formulate a problem for optimization of \(q_i(k)\), \(\forall k, i\) and \(t(i), \forall i\) such that the average sum rate, \(\bar{R}_{1r} + \bar{R}_{r2}\), is maximized:

\[
\text{maximize} \quad \bar{R}_{1r} + \bar{R}_{r2}
\]

subject to

\[C_1: \quad \bar{R}_{1r} = \bar{R}_{r2}\]

\[C_2: \quad \bar{R}_{2r} = \bar{R}_{r1}\]

\[C_3: \quad \sum_{k=1}^{6} q_k(i) = 1, \forall i\]

\[C_4: \quad q_k(i)[1 - q_k(i)] = 0, \forall i, k\]

\[C_5: \quad 0 \leq t(i) \leq 1, \forall i\]

(4)

where \(C_1\) and \(C_2\) are due to Theorem 1, \(C_3\) and \(C_4\) guarantee that only one of the transmission modes is selected in each time slot, and \(C_5\) specifies the interval of the time sharing variable. Considering constraints \(C_1\) and \(C_2\), the sum rate, \(\bar{R}_{1r} + \bar{R}_{r2}\), is also given by \(\bar{R}_{1r} + \bar{R}_{r2}\).

As will be seen later, the solution of (4), i.e., the optimal mode selection policy may require coin flips. Therefore, we define \(X_n(i) \in \{0, 1\}\) as the outcomes of the \(n\)-th coin flip in the \(i\)-th time slot. The probabilities of the possible outcomes of the \(n\)-th coin flip are \(Pr\{X_n(i) = 1\} = p_n\) and \(Pr\{X_n(i) = 0\} = 1 - p_n\). In the following theorem, we provide the solution to the maximization problem in (4). To this end, we define three mutually exclusive signal-to-noise ratio (SNR) regions
and each region requires a different optimal selection policy. For given $\Omega_1$, $\Omega_2$, $P_1$, $P_2$, and $P_r$, exactly one of the SNR regions is applicable.

**Theorem 2 (Mode Selection Policy):** The optimal mode selection policy which maximizes the average sum rate of the considered three-node half-duplex bidirectional relay network with AWGN and block fading is given by

$$q_{k^*}(i) = \begin{cases} 1, & k^* = \arg\max_{k=1, \ldots, 6} \{I_k(i)\Lambda_k(i)\} \\ 0, & \text{otherwise} \end{cases}$$

(5)

where $\Lambda_k(i)$ is referred to as the selection metric, given by

$$\Lambda_k(i) = (1 - \mu_1)C_{1r}(i) \tag{6a}$$

$$\Lambda_2(i) = (1 - \mu_2)C_{2r}(i) \tag{6b}$$

$$\Lambda_3(i) = (1 - \mu_1)C_{1r}(i) + (1 - \mu_2)C_{2r}(i) \tag{6c}$$

$$\Lambda_4(i) = \mu_2C_{1r}(i) \tag{6d}$$

$$\Lambda_5(i) = \mu_1C_{2r}(i) \tag{6e}$$

$$\Lambda_6(i) = \mu_1C_{2r}(i) + \mu_2C_{1r}(i) \tag{6f}$$

and $I_k(i) \in \{0, 1\}$ is a binary indicator variable which determines whether $M_k$ is a possible candidate for selection in the $i$-th time slot, i.e., mode $M_k$ cannot be selected if $I_k(i) = 0$, and $\mu_1$ and $\mu_2$ are decision thresholds. For the values of $I_k(i)$, the optimal values of $\mu_1$ and $\mu_2$ in (6), and the optimal value of $l(i)$ in $C_{1r}(i)$ and $C_{2r}(i)$ in (6c), we distinguish three mutually exclusive SNR regions:

**SNR Region $\mathcal{R}_1$:** SNR region $\mathcal{R}_1$ is characterized by $\{P_2 > P_r \text{ AND } \omega_1 < 1\}$ OR $\{P_2 < P_r \text{ AND } \omega_2 < 1\}$ OR $\{P_2 > P_r \text{ AND } \omega_3 < \omega_2< \omega_1\}$. The threshold $\mu_1$ is defined as

$$\mu_1 = \frac{\omega_1 - \omega_2}{\omega_1 - \omega_3}$$

where the coin flip probabilities are $p_1 = \omega_1, p_2 = \omega_2, p_3 = (1 - \omega_3)^2, p_4 = \frac{1 - \omega_2}{\omega_2(1 - \omega_3)^2}, p_5 = \frac{1 - \omega_1}{\omega_1(1 - \omega_2)^2}$, and for $p_6$, we have the freedom to choose any value in the interval $[\omega_3^\circ, \omega_3^\circ]$. The SNR region $\mathcal{R}_1$ is characterized by

$$\{P_2 > P_r \text{ AND } \omega_3 < \omega_1\} \text{ OR } \{P_2 < P_r \text{ AND } \omega_2 < 1\} \text{ OR } \{P_2 > P_r \text{ AND } \omega_3 < \omega_2 < \omega_1\}$$

with $\omega_1 = \frac{E\{q[C_r] \}}{E\{q[C_{1r}]\}}, \omega_2 = \frac{E\{q[C_{2r}]\}}{E\{q[C_r]\}},$ and $\omega_3 = \frac{E\{q[C_{1r}]\}}{E\{q[C_{2r}]\}},$ where $q(i)$ is defined as

$$q(i) = \begin{cases} 1, & \text{if } \begin{cases} P_2 > P_r \text{ AND } \Omega_3 \leq \Lambda_6 \end{cases} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

In (8), if $P_2 > P_r$, the threshold $\mu_2^*$ is chosen such that $E\{1 - q[C_{2r}]\} = E\{q[C_{1r}]\}$ holds, whereas if $P_2 < P_r$, $\mu_1^*$ is chosen such that $E\{q[C_r] = E\{1 - q[C_{2r}]\}\}$ holds.

For this SNR region, $(i) = 0, \forall i$, and $I_k(i) = 0$, for $k = 1, 2, 3, 5, 6$, and the thresholds $\mu_1$ and $\mu_2$ are as follows

If $P_2 > P_r$ then $\mu_1 = \mu_1^*$ and $I_0(i) = 0$

$$\begin{cases} I_0(i) = 0, & \text{if } P_2 > P_r \text{ AND } \Omega_3 \leq \Lambda_6 \end{cases} \tag{8}$$

where $t(i)$ is given by

$$t(i) = \begin{cases} 0, & \text{if } t \leq 0 \\ 1, & \text{if } t \leq 0 \text{ AND } t < 1 \\ 0, & \text{otherwise} \end{cases}$$

and $\mu^*$ is chosen such that $E\{q[C_{1r}] = E\{1 - q[C_{2r}]\}$ holds. Moreover, if $0 < t < 1$, we obtain $\mu_1 = \mu_2 = \mu^*$. Otherwise, if $t \leq 0$ or $t \geq 1$, we obtain $\mu_1 > \mu_2$ and $\mu_1 \leq \mu_2$, respectively, and $\mu_1$ and $\mu_2$ are chosen such that $C_1$ and $C_2$ in (4) hold.

**Proof** Please refer to [11, Appendix B].

We note that in the proposed optimal policy, regardless of
the SNR region, the multiple access and broadcast modes are always selected with non-zero probability. Only in the extreme case, when the average SNR of one of the links is much larger than the average SNR of the other link, some of the point-to-point modes are also selected in the optimal policy. Also, the mode selection metric $A_k(i)$ includes two parts. The first part is the capacity of mode $M_k$, and the second part are the decision thresholds $\mu_1$ and $\mu_2$. The capacity is a function of the instantaneous CSI, but the decision thresholds $\mu_1$ and $\mu_2$ depend only on the statistics of the channels. Hence, the decision thresholds, $\mu_1$ and $\mu_2$, can be obtained offline and used as long as the channel statistics remain unchanged.

**Remark:** To find the optimal values of $\mu_1$ and $\mu_2$, we need one/two-dimensional searches over $[0, 1]$. Specifically, in SNR regions $R_1$ and $R_2$, at least one of the thresholds $\mu_1$ and $\mu_2$ is given. Therefore, at most one one-dimensional search is required. In SNR region $R_0$, if $0 \leq \omega \leq 1$, we obtain $\mu_1 = \mu_2 = \mu^*$, therefore, a one-dimensional search is required to determine $\mu^*$. However, if $\omega < 0$ or $\omega > 1$, we need a two-dimensional search to find $\mu_1$ and $\mu_2$.

### IV. Simulation Results

In this section, we evaluate the maximum average sum rate in the considered bidirectional relay network for Rayleigh fading. Thus, channel gains $S_1(i)$ and $S_2(i)$ follow exponential distributions with averages $\Omega_1$ and $\Omega_2$, respectively. Furthermore, the transmit powers of user 1 and user 2 are identical and set to $P_1 = P_2 = 10$ dB, and $\Omega_2 = 0$ dB. The number of time slots used in the simulations is $N = 10^6$.

Fig. 3 illustrates the maximum achievable sum rate versus $\Omega_1$. In order to simulate all possible cases introduced in Theorem 2, we consider $P_1 = 5, 10, 15$ dB, which yields the solid lines in Fig. 3. As the value of $\Omega_1$ varies from $-10$ dB to $10$ dB in Fig. 3, the SNR region changes from $R_0$ to $R_2$ and then from $R_0$ to $R_1$. Since in SNR region $R_2$ the bottleneck is the relay-to-user 1 link, reducing $\Omega_1$ reduces the sum rate. On the other hand, in SNR region $R_1$, the bottleneck link is the relay-to-user 2 link, and therefore, for large $\Omega_1$, the maximum average sum rate saturates.

As performance benchmarks, we consider the traditional two-way, the TDBC [2], the MABC [3], and the buffer-aided protocol in [8]. Since in [8], the protocol is derived for $P_1 = P_2 = P_r$, in Fig. 3, we only show the sum rates of the benchmark schemes for $P_r = 10$ dB. From the comparison in Fig. 3, we observe that a considerable gain is obtained by adaptive mode selection compared to the traditional two-way, TDBC, and MABC protocols. Furthermore, a small gain is obtained compared to the buffer-aided protocol in [8] which selects only the user 1-to-relay, user 2-to-relay, and broadcast modes adaptively. Finally, we note that our protocol is derived for given $\Omega_1$, $\Omega_2$, $P_1$, $P_2$, and $P_r$, and without a constraint on the total average power consumed by all nodes. The problem with a total average power constraint would lead to a different solution.

### V. Conclusion

We have derived the optimal transmission mode selection policy for sum rate maximization of the three-node half-duplex bidirectional buffer-aided relay network with fading links and fixed transmit powers at the nodes. The proposed selection policy determines the optimal transmission mode based on the instantaneous CSI of the involved links in each time slot and their long-term statistics. Simulation results confirmed that the proposed selection policy outperforms the existing protocols from the literature in terms of average sum rate.

### References


