# Adaptive Mode Selection in Bidirectional Buffer-aided Relay Networks with Fixed Transmit Powers

*(Invited Paper)*

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***Abstract*—We consider a bidirectional network in which two users exchange information with the help of a buffer-aided relay. In such a network without direct link between user 1 and user 2, there exist six possible transmission modes, i.e., four point-to-point modes (user 1-to-relay, user 2-to-relay, relay-to-user 1, relay-to-user 2), a multiple access mode (both users to the relay), and a broadcast mode (the relay to both users). Because of the buffering capability at the relay, the transmissions in the network are not restricted to adhere to a predefined schedule, and therefore, all the transmission modes in the bidirectional relay network can be used adaptively based on the instantaneous channel state information (CSI) of the involved links. For the considered network, assuming fixed transmit powers for both the users and the relay, we derive the optimal transmission mode selection policy which maximizes the sum rate. The proposed policy selects one out of the six possible transmission modes in each time slot based on the instantaneous CSI. Simulation results confirm the effectiveness of the proposed protocol compared to existing protocols.**

## I. INTRODUCTION

In a bidirectional relay network, two users share information via a relay node [1]. Several protocols have been proposed for such a network under the practical half-duplex constraint, i.e., a node cannot transmit and receive at the same time and in the same frequency band. The simplest protocol is the traditional two-way protocol in which the transmission is accomplished in four successive point-to-point phases: user 1-to-relay, relay-to-user 2, user 2-to-relay, and relay-to-user 1. In contrast, the time division broadcast (TDBC) protocol exploits the broadcasting capability of the wireless medium and combines the relay-to-user 1 and relay-to-user 2 phases into one phase, the broadcast phase [2]. Thereby, the relay broadcasts a superimposed codeword, carrying information for both user 1 and user 2, such that each user is able to recover its intended information by self-interference cancellation. To further increase the spectral efficiency, the multiple access broadcast (MABC) protocol was proposed in which the user 1-to-relay and user 2-to-relay phases are also combined into one phase, the multiple-access phase [3]. In the multiple-access phase, user 1 and user 2 simultaneously transmit to the relay, which decodes both messages. For the bidirectional relay network without a direct link between user 1 and user 2, the capacity regions of all six possible transmission modes, i.e., the four point-to-point modes, the multiple access mode, and the broadcast mode, are known [4], [5]. Using this knowledge, a significant research effort has been dedicated to obtaining the achievable rate region of the bidirectional relay network [1]-[8]. Specifically, the achievable rates of most existing protocols for two-hop relay transmission are limited by the instantaneous capacity of the weakest link associated with the relay. The reason for this is the fixed schedule of using the transmission modes which is adopted in all existing protocols, and does not exploit the instantaneous channel state information (CSI) of the involved links. For one-way relaying, an adaptive link selection protocol was proposed in [9] where based on the instantaneous CSI, in each time slot, either the source-relay link or the relay-destination link is selected for transmission. To this end, the relay has to have a buffer for data storage. This strategy was shown to achieve the capacity of the one-way relay channel with fading [10].

Motivated by the protocols in [9] and [10], our goal is to utilize all available degrees of freedom in the three-node half-duplex bidirectional relay network with fading, via a buffer-aided and adaptive mode selection protocol. In particular, given the fixed transmit powers of all three nodes in the bidirectional relay network, we find a policy which selects the optimal transmission mode from the six available modes in each time slot such that the sum rate is maximized. A similar problem was considered in [8]. However, the selection policy in [8] does not use all possible modes, i.e., it only selects between two point-to-point modes and the broadcast mode, and assumes that the transmit powers of all three nodes are identical. We will show that considering only the three modes in [8] is not optimal. In fact, the multiple access mode is selected with non-zero probability in the proposed optimal selection policy. Finally, we note that the advantages of buffering come at the expense of an increased end-to-end delay. Therefore, the proposed protocol is suitable for applications that are not sensitive to delay. The delay analysis of the proposed protocol is beyond the scope of the current work and is left for future research.

The remainder of the paper is organized as follows. In Section II, the system model is presented. In Section III, the average sum rate is investigated and the optimal mode selection policy is provided. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

## II. SYSTEM MODEL

In this section, we first describe the channel model and review the achievable rates for the six possible transmission modes.

### A. Channel Model

We consider a simple network in which users 1 and 2 exchange information with the help of a relay node, as shown in Fig. 1. We assume that there is no direct link between

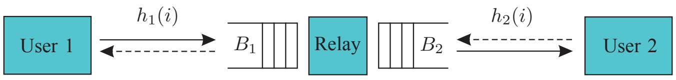


Fig. 1. Three-node bidirectional relay network consisting of two users and a relay.

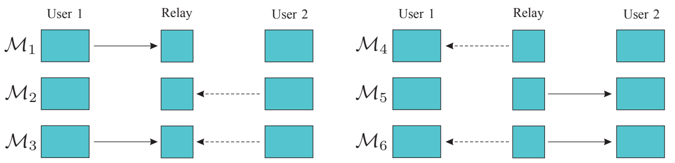


Fig. 2. The six possible transmission modes in the considered bidirectional relay network.

user 1 and user 2, and thus, user 1 and user 2 communicate with each other only through the relay node. We assume that all three nodes in the network are half-duplex. Furthermore, we assume that time is divided into slots of equal length and each node transmits codewords which span one time slot except in the multiple access mode in which the nodes may transmit codewords which span a fraction of the time slot. The duration of one time slot is normalized to one. We assume that the users-to-relay and relay-to-users channels are all impaired by AWGN having zero mean and unit variance, and block fading, i.e., the channel coefficients are constant during one time slot and change from one time slot to the next. Moreover, the channel coefficients are assumed to be reciprocal such that the user 1-to-relay and user 2-to-relay channels are identical to the relay-to-user 1 and relay-to-user 2 channels in each time slot, respectively. Let $h_1(i)$ and $h_2(i)$ denote the channel coefficients between user 1 and the relay and between user 2 and the relay in the $i$-th time slot, respectively. Furthermore, let $S_1(i) = |h_1(i)|^2$ and $S_2(i) = |h_2(i)|^2$ denote the squares of the channel coefficient amplitudes in the $i$-th time slot. $S_1(i)$ and $S_2(i)$ are assumed to be ergodic and stationary random processes with means $\Omega_1 = E\{S_1\}$ and $\Omega_2 = E\{S_2\}$[1], respectively, where $E\{\cdot\}$ denotes expectation. Since the noise is AWGN, in order to achieve the capacity in each mode, each node has to transmit codewords which are Gaussian distributed. Therefore, the transmitted codewords of user 1, user 2, and the relay are composed of symbols which are modeled as Gaussian random variables with variances $P_1, P_2$, and $P_r$, respectively, where $P_j$ is the transmit power of node $j \in \{1, 2, r\}$. For ease of notation, we define $C(x) \triangleq \log_2(1+x)$. In the following, we introduce the transmission modes and their achievable rates.

### B. *Transmission Modes and Their Achievable Rates*

In the considered bidirectional relay network only six transmission modes are possible, cf. Fig. 2. The six possible transmission modes are denoted by $\mathcal{M}_1, ..., \mathcal{M}_6$, and $R_{jj'}(i) \geq 0,\ j, j' \in \{1, 2, r\}$, denotes the transmission rate from node $j$ to node $j'$ in the $i$-th time slot. Let $B_1$ and $B_2$ denote two infinite-size buffers at the relay in which the received information from user 1 and user 2 is stored, respectively. Moreover, $Q_j(i),\ j \in \{1, 2\}$ denotes as the amount of normalized information in bits/symbol available in buffer $B_j$ in the $i$-th time slot. Using this notation, the transmission modes and their respective rates are presented in the following:

$\mathcal{M}_1$: User 1 transmits to the relay and user 2 is silent. In this mode, the maximum rate from user 1 to the relay in the $i$-th time slot is given by $R_{1r}(i) = C_{1r}(i)$, where $C_{1r}(i) = C(P_1 S_1(i))$. The relay decodes this information and stores it in buffer $B_1$. Therefore, the amount of information in buffer $B_1$ increases to $Q_1(i) = Q_1(i-1) + R_{1r}(i)$.

$\mathcal{M}_2$: User 2 transmits to the relay and user 1 is silent. In this mode, the maximum rate from user 2 to the relay in the $i$-th time slot is given by $R_{2r}(i) = C_{2r}(i)$, where $C_{2r}(i) = C(P_2 S_2(i))$. The relay decodes this information and stores it in buffer $B_2$. Therefore, the amount of information in buffer $B_2$ increases to $Q_2(i) = Q_2(i-1) + R_{2r}(i)$.

$\mathcal{M}_3$: Both users 1 and 2 transmit to the relay simultaneously. For this mode, we assume that multiple access transmission is used, see [5]. Thereby, the maximum achievable sum rate in the $i$-th time slot is given by $R_{1r}(i) + R_{2r}(i) = C_r(i)$, where $C_r(i) = C(P_1 S_1(i) + P_2 S_2(i))$. Since user 1 and user 2 transmit independent messages, the sum rate, $C_r(i)$, can be decomposed into two rates, one from user 1 to the relay and the other one from user 2 to the relay. Moreover, these two capacity rates can be achieved via time sharing and successive interference cancellation. Thereby, in the first $0 \leq t(i) \leq 1$ fraction of the $i$-th time slot, the relay first decodes the codeword received from user 2 and considers the signal from user 1 as noise. Then, the relay subtracts the signal component of user 2 from the received signal and decodes the codeword received from user 1. A similar procedure is performed in the remaining $1 - t(i)$ fraction of the $i$-th time slot but now the relay first decodes the codeword from user 1 and treats the signal of user 2 as noise, and then decodes the codeword received from user 2. Therefore, for a given $t(i)$, we decompose $C_r(i)$ as $C_r(i) = C_{12r}(i) + C_{21r}(i)$ and the maximum rates from users 1 and 2 to the relay in the $i$-th time slot are $R_{1r}(i) = C_{12r}(i)$ and $R_{2r}(i) = C_{21r}(i)$, respectively. $C_{12r}(i)$ and $C_{21r}(i)$ are given by

$$C_{12r}(i) = t(i) C\left(P_1 S_1(i)\right) + (1 - t(i)) C\left(\frac{P_1 S_1(i)}{1 + P_2 S_2(i)}\right) \quad \text{(1a)}$$

$$C_{21r}(i) = (1 - t(i)) C\left(P_2 S_2(i)\right) + t(i) C\left(\frac{P_2 S_2(i)}{1 + P_1 S_1(i)}\right) \quad \text{(1b)}$$

The relay decodes the information received from user 1 and user 2 and stores it in its buffers $B_1$ and $B_2$, respectively. Therefore, the amount of information in buffers $B_1$ and $B_2$ increase to $Q_1(i) = Q_1(i-1) + R_{1r}(i)$ and $Q_2(i) = Q_2(i-1) + R_{2r}(i)$, respectively.

$\mathcal{M}_4$: The relay transmits the information received from user 2 to user 1. Specifically, the relay extracts the information from buffer $B_2$, encodes it into a codeword, and transmits it to user 1. Therefore, the transmission rate from the relay to user 1 in the $i$-th time slot is limited by both the capacity of the relay-to-user 1 channel and the amount of information stored in buffer $B_2$. Thus, the maximum transmission rate from the relay to user 1 is given by $R_{r1}(i) = \min\{C_{r1}(i), Q_2(i-1)\}$, where $C_{r1}(i) = C(P_r S_1(i))$. Therefore, the amount of information

[1] In the paper, we drop time index $i$ in expectations for notational simplicity.

in buffer $B_2$ decreases to $Q_2(i){=}Q_2(i{-}1){-}R_{r1}(i)$.

$\mathcal{M}_5$: This mode is identical to $\mathcal{M}_4$ with user 1 and 2 switching places. The maximum transmission rate from the relay to user 2 is given by $R_{r2}(i) = \min\{C_{r2}(i), Q_1(i-1)\}$, where $C_{r2}(i) = C(P_r S_2(i))$ and the amount of information in buffer $B_1$ decreases to $Q_1(i){=}Q_1(i{-}1){-}R_{r2}(i)$.

$\mathcal{M}_6$: The relay broadcasts to both user 1 and user 2 the information received from user 2 and user 1, respectively. Specifically, the relay extracts the information intended for user 2 from buffer $B_1$ and the information intended for user 1 from buffer $B_2$. Then, based on the scheme in [4], it constructs a superimposed codeword which contains the information from both users and broadcasts it to both users. Thus, in the $i$-th time slot, the maximum rates from the relay to users 1 and 2 are given by $R_{r1}(i) = \min\{C_{r1}(i), Q_2(i-1)\}$ and $R_{r2}(i) = \min\{C_{r2}(i), Q_1(i-1)\}$, respectively. Therefore, the amount of information in buffer $B_1$ and $B_2$ decrease to $Q_1(i) = Q_1(i-1) - R_{r2}(i)$ and $Q_2(i) = Q_2(i-1) - R_{r1}(i)$, respectively.

Our aim is to develop an optimal selection policy which selects one of the six possible transmission modes in each time slot such that, given $P_1$, $P_2$, and $P_r$, the average sum rate of both users is maximized. To this end, we introduce six binary variables $q_k(i) \in \{0,1\},\ k = 1,...,6$, where $q_k(i)$ indicates whether or not transmission mode $\mathcal{M}_k$ is selected in the $i$-th time slot. In particular, $q_k(i) = 1$ if mode $\mathcal{M}_k$ is selected and $q_k(i) = 0$ if it is not selected in the $i$-th time slot. Moreover, since in each time slot only one of the six transmission modes can be selected, only one of the mode selection variables is equal to one and the others are zero, i.e., $\sum_{k=1}^{6} q_k(i) = 1$ holds.

In the proposed framework, we assume that one node (e.g. the relay node) is responsible for deciding which transmission mode is selected based on the full CSI of both links and the proposed protocol, cf. Theorem 2. Then, it sends its decision to the other nodes and transmission in time slot $i$ begins. Furthermore, for each node to be able to decode their intended messages and adapt their transmission rates, all three nodes have to have full CSI of both links.

## III. Adaptive Mode Selection

In this section, we first investigate the achievable average sum rate of the network. Then, we formulate a maximization problem whose solution yields the maximum average sum rate.

### A. Achievable Average Sum Rate

We assume that user 1 and user 2 always have enough information to send in all time slots and that the number of time slots, $N$, satisfies $N \to \infty$. Therefore, using $q_k(i)$, the user 1-to-relay, user 2-to-relay, relay-to-user 1, and relay-to-user 2 average transmission rates, denoted by $\bar{R}_{1r}$, $\bar{R}_{2r}$, $\bar{R}_{r1}$, and $\bar{R}_{r2}$, respectively, are obtained as

$$\bar{R}_{1r} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} [q_1(i) C_{1r}(i) + q_3(i) C_{12r}(i)] \tag{2a}$$

$$\bar{R}_{2r} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} [q_2(i) C_{2r}(i) + q_3(i) C_{21r}(i)] \tag{2b}$$

$$\bar{R}_{r1} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} [q_4(i) + q_6(i)] \min\{C_{r1}(i), Q_2(i-1)\} \tag{2c}$$

$$\bar{R}_{r2} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} [q_5(i) + q_6(i)] \min\{C_{r2}(i), Q_1(i-1)\} \tag{2d}$$

The average rate from user 1 to user 2 is the average rate that user 2 receives from the relay, i.e., $\bar{R}_{r2}$. Similarly, the average rate from user 2 to user 1 is the average rate that user 1 receives from the relay, i.e., $\bar{R}_{r1}$. In the following theorem, we introduce a useful condition for the queues in the buffers of the relay leading to the optimal mode selection policy.

*Theorem 1 (Optimal Queue Condition):* The maximum sum rate, $\bar{R}_{r1} + \bar{R}_{r2}$, for the considered bidirectional relay network is obtained when the queues in buffers $B_1$ and $B_2$ are at the edge of non-absorbtion. More precisely, the following conditions must hold for the maximum sum rate:

$$\bar{R}_{1r} = \bar{R}_{r2} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} [q_5(i) + q_6(i)]\, C_{r2}(i) \tag{3a}$$

$$\bar{R}_{2r} = \bar{R}_{r1} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} [q_4(i) + q_6(i)]\, C_{r1}(i) \tag{3b}$$

where $\bar{R}_{1r}$ and $\bar{R}_{2r}$ are given by (2aa) and (2ab), respectively.

**Proof** Please refer to [11, Appendix A].

Using this theorem, we now derive the optimal transmission mode selection policy.

### B. Optimal Mode Selection Protocol

The available degrees of freedom in the considered network are the mode selection variables $q_k(i),\ \forall k, i$ and the time sharing variable in the multiple access mode, $t(i),\ \forall i$. In the following, we formulate a problem for optimization of $q_k(i),\ \forall k, i$ and $t(i),\ \forall i$ such that the average sum rate, $\bar{R}_{r1} + \bar{R}_{r2}$, is maximized:

$$\begin{aligned}
&\underset{q_k(i), t(i),\ \forall i,k}{\text{maximize}} && \bar{R}_{1r} + \bar{R}_{2r} \\
&\text{subject to} && \text{C1}: \quad \bar{R}_{1r} = \bar{R}_{r2} \\
& && \text{C2}: \quad \bar{R}_{2r} = \bar{R}_{r1} \\
& && \text{C3}: \quad \textstyle\sum_{k=1}^{6} q_k(i) = 1,\ \forall i \\
& && \text{C4}: \quad q_k(i)[1 - q_k(i)] = 0,\ \forall i, k \\
& && \text{C5}: \quad 0 \le t(i) \le 1,\ \forall i.
\end{aligned} \tag{4}$$

where C1 and C2 are due to Theorem 1, C3 and C4 guarantee that only one of the transmission modes is selected in each time slot, and C5 specifies the interval of the time sharing variable. Considering constraints C1 and C2, the sum rate, $\bar{R}_{r1} + \bar{R}_{r2}$, is also given by $\bar{R}_{1r} + \bar{R}_{2r}$.

As will be seen later, the solution of (4), i.e., the optimal mode selection policy may require coin flips. Therefore, we define $X_n(i) \in \{0,1\}$ as the outcomes of the $n$-th coin flip in the $i$-th time slot. The probabilities of the possible outcomes of the $n$-th coin flip are $\Pr\{X_n(i) = 1\} = p_n$ and $\Pr\{X_n(i) = 0\} = 1 - p_n$. In the following theorem, we provide the solution to the maximization problem in (4). To this end, we define three mutually exclusive signal-to-noise ratio (SNR) regions and each region requires a different optimal selection policy. For given $\Omega_1$, $\Omega_2$, $P_1$, $P_2$, and $P_r$, exactly one of the SNR regions is applicable.

*Theorem 2 (Mode Selection Policy):* The optimal mode selection policy which maximizes the average sum rate of the considered three-node half-duplex bidirectional relay network with AWGN and block fading is given by

$$q_{k^*}(i) = \begin{cases} 1, & k^* = \underset{k=1,\dots,6}{\arg\max}\{\mathcal{I}_k(i)\Lambda_k(i)\} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $\Lambda_k(i)$ is referred to as the selection metric, given by

$$\Lambda_1(i) = (1-\mu_1)C_{1r}(i) \tag{6a}$$
$$\Lambda_2(i) = (1-\mu_2)C_{2r}(i) \tag{6b}$$
$$\Lambda_3(i) = (1-\mu_1)C_{12r}(i) + (1-\mu_2)C_{21r}(i) \tag{6c}$$
$$\Lambda_4(i) = \mu_2 C_{r1}(i) \tag{6d}$$
$$\Lambda_5(i) = \mu_1 C_{r2}(i) \tag{6e}$$
$$\Lambda_6(i) = \mu_1 C_{r2}(i) + \mu_2 C_{r1}(i) \tag{6f}$$

and $\mathcal{I}_k(i) \in \{0,1\}$ is a binary indicator variable which determines whether $\mathcal{M}_k$ is a *possible candidate* for selection in the $i$-th time slot, i.e., mode $\mathcal{M}_k$ cannot be selected if $\mathcal{I}_k(i) = 0$, and $\mu_1$ and $\mu_2$ are decision thresholds. For the values of $\mathcal{I}_k(i)$, the optimal values of $\mu_1$ and $\mu_2$ in (6), and the optimal value of $t(i)$ in $C_{12r}(i)$ and $C_{21r}(i)$ in (6c), we distinguish three mutually exclusive SNR regions:

**SNR Region $\mathcal{R}_1$:** SNR region $\mathcal{R}_1$ is chractarized by $\{P_2 > P_r \text{ AND } \omega_1 < 1\}$ OR $\{P_2 < P_r \text{ AND } \omega_2 < 1\}$ OR $\{P_2 = P_r \text{ AND } \omega_3^l < \omega_3^u\}$ with $\omega_1 = \frac{E\{qC_{r2}\}}{E\{(1-q)[C_r - C_{2r}]\}}$, $\omega_2 = \frac{E\{qC_{2r}\}}{E\{(1-q)C_{r1}\}}$, $\omega_3^l = \frac{E\{C_{r2}\}}{E\{C_r\}}$, and $\omega_3^u = \frac{E\{C_{r1}\}}{E\{C_{r1}+C_{2r}\}}$, where $q(i)$ is defined as

$$q(i) = \begin{cases} 1, & \text{if } \left\{ P_2 > P_r \text{ AND } \Lambda_3 \le \Lambda_6 \Big|^{t(i)=0,\ \forall i}_{\mu_1=1,\mu_2=\mu_2^*} \right\} \\ & \text{OR} \left\{ P_2 < P_r \text{ AND } \Lambda_3 \ge \Lambda_6 \Big|^{t(i)=0,\ \forall i}_{\mu_1=\mu_1^*,\mu_2=0} \right\} \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

In (7), if $P_2 > P_r$, $\mu_2^*$ is chosen such that $E\{(1-q)C_{2r}\} = E\{qC_{r1}\}$ holds, whereas if $P_2 < P_r$, $\mu_1^*$ is chosen such that $E\{q[C_r - C_{2r}]\} = E\{(1-q)C_{r2}\}$ holds.

For this SNR region, $t(i) = 0$, $\forall i$, and $\mathcal{I}_k(i) = 0$, for $k = 1, 4$, $\forall i$, whereas $\mathcal{I}_k(i)$ for $k = 2, 3, 5, 6$, and the thresholds $\mu_1$ and $\mu_2$ are as follows

$$\text{If } P_2 > P_r \text{ then} \begin{cases} \mu_1 = 1 \\ \mu_2 = \mu_2^* \end{cases} \text{ and } \begin{cases} \mathcal{I}_5(i) = 0 \\ \mathcal{I}_3(i) = 1 - \mathcal{I}_2(i) = X_1(i) \\ \mathcal{I}_6(i) = 1 \end{cases}$$

$$\text{If } P_2 < P_r \text{ then} \begin{cases} \mu_1 = \mu_1^* \\ \mu_2 = 0 \end{cases} \text{ and } \begin{cases} \mathcal{I}_2(i) = 0, \\ \mathcal{I}_6(i) = 1 - \mathcal{I}_5(i) = X_2(i) \\ \mathcal{I}_3(i) = 1 \end{cases}$$

$$\text{If } P_2 = P_r \text{ then} \begin{cases} \mu_1 = 1 \\ \mu_2 = 0 \end{cases} \text{ and } \begin{cases} \mathcal{I}_2(i) = X_3(i)[1 - X_4(i)] \\ \mathcal{I}_3(i) = X_3(i)X_4(i) \\ \mathcal{I}_5(i) = [1 - X_3(i)][1 - X_5(i)] \\ \mathcal{I}_6(i) = [1 - X_3(i)]X_5(i) \end{cases}$$

where the coin flip probabilities are $p_1 = \omega_1, p_2 = \omega_2, p_4 = \frac{(1-p_3)\omega_3^l}{p_3(1-\omega_3^l)}, p_5 = \frac{p_3(1-\omega_3^u)}{(1-p_3)\omega_3^u}$, and for $p_3$, we have the freedom to choose any value in the interval $[\omega_3^l, \omega_3^u]$.

**SNR Region $\mathcal{R}_2$:** SNR region $\mathcal{R}_2$ is charactarized by $\{P_1 > P_r \text{ AND } \omega_1 < 1\}$ OR $\{P_1 < P_r \text{ AND } \omega_2 < 1\}$ OR $\{P_1 = P_r \text{ AND } \omega_3^l < \omega_3^u\}$ with $\omega_1 = \frac{E\{qC_{r1}\}}{E\{(1-q)[C_r - C_{1r}]\}}$, $\omega_2 = \frac{E\{qC_{1r}\}}{E\{(1-q)C_{r2}\}}$, $\omega_3^l = \frac{E\{C_{r1}\}}{E\{C_r\}}$, and $\omega_3^u = \frac{E\{C_{r2}\}}{E\{C_{r2}+C_{1r}\}}$, where $q(i)$ is defined as

$$q(i) = \begin{cases} 1, & \text{if } \left\{ P_1 > P_r \text{ AND } \left[\Lambda_3 \le \Lambda_6\right]^{t(i)=1,\ \forall i}_{\mu_1=\mu_1^*,\mu_2=1} \right\} \\ & \text{OR} \left\{ P_1 < P_r \text{ AND } \left[\Lambda_3 \ge \Lambda_6\right]^{t(i)=1,\ \forall i}_{\mu_1=0,\mu_2=\mu_2^*} \right\} \\ 0, & \text{otherwise} \end{cases}. \tag{8}$$

In (8), if $P_1 > P_r$, the threshold $\mu_1^*$ is chosen such that $E\{(1-q)C_{1r}\} = E\{qC_{r2}\}$ holds, whereas if $P_1 < P_r$, the threshold $\mu_2^*$ is chosen such that $E\{q[C_r < C_{1r}]\} = E\{(1 < q)C_{r1}\}$ holds.

For this SNR region, $t(i) = 1$, $\forall i$, and $\mathcal{I}_k(i) = 0$, for $k = 2, 5$, $\forall i$, whereas $\mathcal{I}_k(i)$ for $k = 1, 3, 4, 6$, and the thresholds $\mu_1$ and $\mu_2$ are as follows

$$\text{If } P_1 > P_r \text{ then} \begin{cases} \mu_1 = \mu_1^* \\ \mu_2 = 1 \end{cases} \text{ and } \begin{cases} \mathcal{I}_4(i) = 0 \\ \mathcal{I}_3(i) = 1 - \mathcal{I}_1(i) = X_1(i) \\ \mathcal{I}_6(i) = 1 \end{cases}$$

$$\text{If } P_1 < P_r \text{ then} \begin{cases} \mu_1 = 0 \\ \mu_2 = \mu_2^* \end{cases} \text{ and } \begin{cases} \mathcal{I}_1(i) = 0, \\ \mathcal{I}_6(i) = 1 - \mathcal{I}_4(i) = X_2(i) \\ \mathcal{I}_3(i) = 1 \end{cases}$$

$$\text{If } P_1 = P_r \text{ then} \begin{cases} \mu_1 = 0 \\ \mu_2 = 1 \end{cases} \text{ and } \begin{cases} \mathcal{I}_1(i) = X_3(i)[1 - X_4(i)] \\ \mathcal{I}_3(i) = X_3(i)X_4(i) \\ \mathcal{I}_4(i) = [1 - X_3(i)][1 - X_5(i)] \\ \mathcal{I}_6(i) = [1 - X_3(i)]X_5(i) \end{cases}$$

where the coin flip probabilities are $p_1 = \omega_1, p_2 = \omega_2, p_4 = \frac{(1-p_3)\omega_3^l}{p_3(1-\omega_3^l)}, p_5 = \frac{p_3(1-\omega_3^u)}{(1-p_3)\omega_3^u}$, and for $p_3$, we have the freedom to choose any value in the interval $[\omega_3^l, \omega_3^u]$.

**SNR Region $\mathcal{R}_0$:** The SNR region is $\mathcal{R}_0$ if and only if it is not $\mathcal{R}_1$ or $\mathcal{R}_2$. For SNR region $\mathcal{R}_0$, $\mathcal{I}_k(i) = 0$ for $k = 1, 2, 4, 5$ and $\mathcal{I}_k(i) = 1$ for $k = 3, 6$, $\forall i$, and $t(i)$ is given by

$$t(i) = \begin{cases} 0, & \text{if } t \le 0 \\ t, & \text{if } 0 < t < 1 \\ 1, & \text{if } t \ge 1 \end{cases} \tag{9}$$

where $t = \frac{E\{q(C_r - C_{2r})\} - E\{(1-q)C_{r2}\}}{E\{q(C_r - C_{1r} - C_{2r})\}}$ and $q(i)$ is given by

$$q(i) = \begin{cases} 1, & \text{if } \frac{C_r(i)}{C_{r1}(i) + C_{r2}(i)} \ge \frac{\mu^*}{1-\mu^*} \\ 0, & \text{if otherwise} \end{cases} \tag{10}$$

where $\mu^*$ is chosen such that $E\{qC_r\} = E\{(1-q)(C_{r1} + C_{r2})\}$ holds. Moreover, if $0 < t < 1$, we obtain $\mu_1 = \mu_2 = \mu^*$. Otherwise, if $t \le 0$ or $t \ge 1$, we obtain $\mu_1 \ge \mu_2$ and $\mu_1 \le \mu_2$, respectively, and $\mu_1$ and $\mu_2$ are chosen such that C1 and C2 in (4) hold.

**Proof** Please refer to [11, Appendix B].

We note that in the proposed optimal policy, regardless of the SNR region, the multiple access and broadcast modes are always selected with non-zero probability. Only in the extreme case, when the average SNR of one of the links is much larger than the average SNR of the other link, some of the point-to-point modes are also selected in the optimal policy. Also, the mode selection metric $\Lambda_k(i)$ includes two parts. The first part is the capacity of mode $\mathcal{M}_k$, and the second part are the decision thresholds $\mu_1$ and/or $\mu_2$. The capacity is a function of the instantaneous CSI, but the decision thresholds $\mu_1$ and $\mu_2$ depend only on the statistics of the channels. Hence, the decision thresholds, $\mu_1$ and $\mu_2$, can be obtained offline and used as long as the channel statistics remain unchanged.

**Remark:** To find the optimal values of $\mu_1$ and $\mu_2$, we need one/two-dimensional searches over $[0\,,\,1]$. Specifically, in SNR regions $\mathcal{R}_1$ and $\mathcal{R}_2$, at least one of the thresholds $\mu_1$ and $\mu_2$ is given. Therefore, at most one one-dimensional search is required. In SNR region $\mathcal{R}_0$, if $0 \le \omega \le 1$, we obtain $\mu_1 = \mu_2 = \mu^*$, therefore, a one-dimensional search is required to determine $\mu^*$. However, if $\omega < 0$ or $\omega > 1$, we need a two-dimensional search to find $\mu_1$ and $\mu_2$.

## IV. Simulation Results

In this section, we evaluate the maximum average sum rate in the considered bidirectional relay network for Rayleigh fading. Thus, channel gains $S_1(i)$ and $S_2(i)$ follow exponential distributions with averages $\Omega_1$ and $\Omega_2$, respectively. Furthermore, the transmit powers of user 1 and user 2 are identical and set to $P_1 = P_2 = 10$ dB, and $\Omega_2 = 0$ dB. The number of time slots used in the simulations is $N = 10^6$.

Fig. 3 illustrates the maximum achievable sum rate versus $\Omega_1$. In order to simulate all possible cases introduced in Theorem 2, we consider $P_r = 5, 10, 15$ dB, which yields the solid lines in Fig. 3. As the value of $\Omega_1$ varies from $-10$ dB to 10 dB in Fig. 3, the SNR region changes from $\mathcal{R}_2$ to $\mathcal{R}_0$ and then from $\mathcal{R}_0$ to $\mathcal{R}_1$. Since in SNR region $\mathcal{R}_2$ the bottleneck is the relay-to-user 1 link, reducing $\Omega_1$ reduces the sum rate. On the other hand, in SNR region $\mathcal{R}_1$, the bottleneck link is the relay-to-user 2 link, and therefore, for large $\Omega_1$, the maximum average sum rate saturates.

As performance benchmarks, we consider the traditional two-way, the TDBC [2], the MABC [3], and the buffer-aided protocol in [8]. Since in [8], the protocol is derived for $P_1 = P_2 = P_r$, in Fig. 3, we only show the sum rates of the benchmark schemes for $P_r = 10$ dB. From the comparison in Fig. 3, we observe that a considerable gain is obtained by adaptive mode selection compared to the traditional two-way, TDBC, and MABC protocols. Furthermore, a small gain is obtained compared to the buffer-aided protocol in [8] which selects only the user 1-to-relay, user 2-to-relay, and broadcast modes adaptively. Finally, we note that our protocol is derived for given $\Omega_1$, $\Omega_2$, $P_1$, $P_2$, and $P_r$, and without a constraint on the total average power consumed by all nodes. The problem with a total average power constraint would lead to a different solution.

## V. Conclusion

We have derived the optimal transmission mode selection policy for sum rate maximization of the three-node half-duplex bidirectional buffer-aided relay network with fading links and fixed transmit powers at the nodes. The proposed selection policy determines the optimal transmission mode based on the instantaneous CSI of the involved links in each time slot and their long-term statistics. Simulation results confirmed that the proposed selection policy outperforms the existing protocols from the literature in terms of average sum rate.

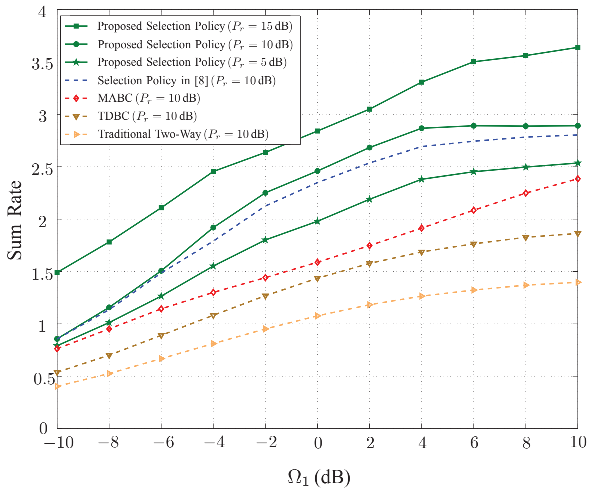


Fig. 3. Maximum achievable sum rate versus $\Omega_1$.

## References

[1] S. J. Kim, N. Devroye, P. Mitran, and V. Tarokh, “Achievable Rate Regions and Performance Comparison of Half Duplex Bi-Directional Relaying Protocols,” *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6405 –6418, Oct. 2011.

[2] Y. Wu, P. A. Chou, and S.-Y. Kung, “Information Exchange in Wireless Networks with Network Coding and Physical-Layer Broadcast,” in *Proc. 39th Ann. Conf. Inf. Sci. Syst.*, March 2005.

[3] P. Popovski and H. Yomo, “Bi-directional Amplification of Throughput in a Wireless Multi-Hop Network,” in *Proc. IEEE VTC*, vol. 2, May 2006, pp. 588–593.

[4] T. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, “Broadcast Capacity Region of Two-Phase Bidirectional Relaying,” *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 454 –458, Jan. 2008.

[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, John and Sons, Incorporated, 1991.

[6] R. F. Wyrembelski, T. J. Oechtering, and H. Boche, “Decode-and-Forward Strategies for Bidirectional Relaying,” in *Proc. IEEE PIMRC*, Sept. 2008, pp. 1 –6.

[7] P. Popovski and H. Yomo, “Physical Network Coding in Two-Way Wireless Relay Channels,” in *Proc. IEEE ICC*, June 2007, pp. 707 –712.

[8] H. Liu, P. Popovski, E. de Carvalho, and Y. Zhao, “Sum-Rate Optimization in a Two-Way Relay Network with Buffering,” *IEEE Commun. Let.*, vol. 17, no. 1, pp. 95 –98, Jan. 2013.

[9] N. Zlatanov, R. Schober, and P. Popovski, “Buffer-Aided Relaying with Adaptive Link Selection,” *IEEE J. Select. Areas Commun.*, vol. 31, no. 8, pp. 1 –13, Aug. 2013.

[10] N. Zlatanov and R. Schober, “Capacity of the State-Dependent Half-Duplex Relay Channel Without Source-Destination Link,” *Submitted IEEE Transactions on Information Theory*, 2013. [Online]. Available: http://arxiv.org/abs/1302.3777

[11] V. Jamali, N. Zlatanov, A. Ikhlef, and R. Schober, “Adaptive Mode Selection in Bidirectional Buffer-aided Relay Networks with Fixed Transmit Powers,” *Submitted in part to EUSIPCO’13*, 2013. [Online]. Available: http://arxiv.org/abs/1303.3732