JOINT MMSE TRANSCEIVER DESIGN FOR NOISY MIMO CHANNEL WITH ERASURES

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ABSTRACT

This paper investigates the design of optimal precoder for multiple input multiple output (MIMO) channel subjected to channel erasure and additive noise with known covariance. The subchannel erasure process is modeled as an independent Bernoulli process with known loss probabilities. The MIMO channel is known to the transmitter and receiver. But instantaneous erasure states are only known to receiver while transmitter has the statistical information of the erasures. We search for the optimal redundant linear correlating transform coding to combat against channel impairments due to erasure and noise by minimizing mean square error (MSE). While the transmitter is designed to accommodate all the channel scenarios, the minimum MSE (MMSE) receiver reconstructs the source for each channel realization. We propose a method based on linear matrix inequities (LMI) to solve for optimal precoder which can be efficiently computed by well known convex optimization toolbox. Effectiveness of this method is demonstrated by simulation examples.

Index Terms— MMSE, MIMO, Erasure, LMI, Precoder

1. INTRODUCTION

The emergence of new applications in networked control systems and sensor networks demands new coding and decoding schemes that can be integrated in system design to enhance the reliability of communication in face of channel impairments such as data loss and channel noise. It is well known that the linear correlating transform (CT) is effective in reducing the reconstruction error at the receiver when erasures occur in the channel. For example, multiple description coding (MDC) with correlating transform provides graceful degradation of the reconstructed signal during loss of data [1, 2, 3, 4]. The most frequently studied are the 2×2 and 3×3 transform coders. In [5] and [6], an MMSE based general framework for designing square linear coder and non-square redundant precoder has been presented and solved by using iterative gradient algorithm. The MDC-CT is a form of source coding and ignores the channel characteristics such as channel gains, noise and interference.

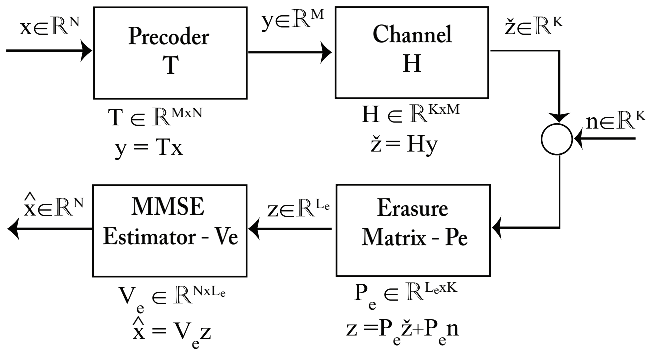

Fig. 1. System Configuration

MMSE based joint precoder design with and without channel state information (CSI) has been studied extensively in the literature. Closed form analytical results based on matrix decomposition [7] and semidifinite programming (SDP) based solutions [8] have been presented. These MIMO precoding designs investigate effect of channel noise and interference under the criteria of bit error rate (BER), signal to noise ratio (SNR), maximum information and maximum capacity. The recent work [9] proposed unitary precoder to minimize MSE for Gaussian erasure channel. In our previous work [10], we proposed an LMI based solution for precoder design for erasure MIMO channel without channel noise.

This paper extends the result of [10] to integrate channel noise. To our best knowledge, redundant joint precoder design for MIMO channel subject to both erasure and noise has not been considered before. Erasure channel is an extreme case of a fading channel and assumes that the transmitted symbols are completely lost or received without error. This concept raises challenges in the problem formulation that minimizes MSE of signal reconstruction, as it effects the invertibility of the combined erasure MIMO channel.

In this work we assume that the MIMO channel is static and known at both transmitter and receiver and subjected to erasure. The receiver has the instantaneous channel erasure state information (ESI) but the transmitter has the statistical information of ESI. MIMO precoding with statistical information at transmitter [11] investigates a similar scenario where the channel is modeled through the channel mean, covariance and random matrices. In contrast, we assume that the erasure states are finite and therefore there is a finite number of erasure MIMO channels. This renders the formulation of MMSE for each erasure scenario and then a weighted average sum of MMSE is minimized. An suboptimal precoder is derived by a rank relaxed semi-definite programming subject to LMI constraints.

The paper is organized as follows. In Section 2 we present the system model and derive the primary optimization problem. In Section 3 we present the LMI based solution and obtain the precoder design by rank approximation. The numerical examples are given in Section 4.

2. SYSTEM MODEL

2.1. System Model

Consider the MIMO communication system with M transmitters and K receivers as given in Fig. 1. The signal source $x \in \mathbb{R}^N$ is a memoryless Gaussian random vector with zero mean and known covariance $E(xx^T) = R_x$. The source signal is encoded by a redundant linear transformation, $T \in \mathbb{R}^{M\times N}$, generating correlated descriptions $y \in \mathbb{R}^M$. With $M > N$, the precoder adds redundancy to the transmission. The MIMO communication channel denoted by $H \in \mathbb{R}^{K\times M}$ is the constant channel matrix with known channel parameters and full column rank M ($K \geq M$), $n \in \mathbb{R}^N$ is the additive noise in the channel with zero mean and known covariance $E(nn^T) = R_n$. With M transmitters, the encoded descriptions are transmitted separately. The MIMO channel is subject to erasures. An erasure event occurs when a random subset of the subchannels is erased from the receiver side. The channel erasure state is defined through the matrix $P_e \in \mathbb{R}^{L_e\times K}$, where L_e is the number of subchannels received at the estimator. When there are no erasures the matrix P_e is a $K \times K$ identity matrix. When an erasure event occurs, the respective rows from P_e are removed to denote the erasure. Thus the dimension of P_e varies depending on the channel erasure state. The number of the received descriptions L_e depends on the channel erasure state at the time. With K receivers, there are 2^K channel states. At the receiver, a linear MMSE estimator $V_e \in \mathbb{R}^{N\times L}$ reconstructs the source signal based on the signal received after erasure. In this work, we assume that the estimator has information of the channel erasure states, P_e, during transmission and the channel matrix H. The decoder here is also a matrix with variable dimension. The transmitter only has the channel stochastic information and does not know individual channel realization during the transmission.

In the sequel, we will investigate the design of the precoder T and the MMSE estimator V_e to minimize the signal reconstruction error in the MIMO communication system described above.

2.2. Problem formulation

From Figure 1 the received signal at the estimator is

$$z = P_e HTx + P_e n. \tag{1}$$

Then the estimator output is

$$\hat{x} = V_e z. \tag{2}$$

Since the source x has zero mean and covariance R_x, the received vector also has zero mean and covariance $R_z = E(zz^T) = P_e HTR_x T^T H^T P_e^T + P_e R_n P_e^T$ and cross covariance of $R_{xz} = E(xz^T) = R_x T^T H^T P_e^T$ and $R_{zx} = E(zx^T) = P_e HTR_x$. Define $R_{ne} := P_e R_n P_e^T$. Then the MMSE estimate at the receiver is [12]

$$\begin{aligned}\hat{x} = R_{xz}R_z^{-1}z &= R_x T^T H^T P_e^T (P_e HTR_x T^T H^T P_e^T \\ &\quad + R_{ne})^{-1} z\end{aligned}$$

with the optimal MMSE estimator for a given precoder T given by

$$V_e = R_x T^T H^T P_e^T (P_e HTR_x T^T H^T P_e^T + R_{ne})^{-1}.$$

Then the MMSE is

$$\begin{aligned}D_e &= \mathrm{Tr}(R_x - R_{xz}R_z^{-1}R_{zx}) \\ &= \mathrm{Tr}(R_x - R_x T^T H^T P_e^T (P_e HTR_x T^T H^T P_e^T \\ &\quad + R_{ne})^{-1} P_e HTR_x).\end{aligned} \tag{3}$$

Above derivation is for a specific state of channel erasure matrix P_e. In our work, the precoder is designed to address an arbitrary number of channel states. Since P_e is a stochastic variable that depends on the erasure process, the distortion is also a stochastic variable. Hence the precoder is designed to minimize the weighted sum of the distortion.

$$D = \sum_{e=1}^{E} w_e \times D_e.$$

The weighting w_e can be arbitrary and can be selected based on the optimization requirement. In this paper we select the weighting such that w_e is the inverse of the probability of the channel in the erasure state. Let λ be the probability of erasure of a subchannel. Then $w_e = \frac{1}{\lambda^{K-L_e}\times(1-\lambda)^{L_e}}$. Another point to note is that the precoder design formulation excludes the case where all the sub-channels are lost as it is impossible to improve the performance by precoding for such scenario. Hence the objective function for the precoder design is

$$\begin{aligned}D &= \sum_{e=1}^{E} w_e \times D_e = \sum_{e=1}^{E} w_e \mathrm{Tr}[R_x - R_x T^T H^T P_e^T \\ &\quad (P_e HTR_x T^T H^T P_e^T + R_{ne})^{-1} P_e HTR_x]\end{aligned} \tag{4}$$

where E is the number of channel scenarios considered for the optimization problem, and the maximal value of E is $E_{MAX} = 2^K - 1$. Thus the optimal full rank precoder for MMSE estimation can be obtained from solving the following optimization problem

$$\min_T D = \min_T \mathrm{Tr}\Big[\sum_{e=1}^{E} w_e \,\big(R_x - R_x T^T H^T P_e^T (P_e H T R_x T^T H^T P_e^T + R_{ne})^{-1} P_e H T R_x\big)\Big] \quad (5)$$

with transmission power constraint $\mathrm{Tr}(TR_xT^T) \leq P_{TX}$, $P_{TX} \in \mathbb{R}$ and $0 < P_{TX} < \infty$.

3. LMI FORMULATION

The cost function in (5) is neither a convex nor concave function of the optimization variable T, and its direct minimization is difficult. Therefore we propose to optimize an upper bound of the cost function given in (4). Taking R_x out and following [13], it can be shown that

$$D_e \leq \mathrm{Tr}(R_x)\mathrm{Tr}\big(I_N - R_x^{\frac{1}{2}} T^T H^T P_e^T (P_e H T R_x T^T H P_e + R_{ne})^{-1} P_e H T R_x^{\frac{1}{2}}\big). \quad (6)$$

Excluding $Tr(R_x)$ as it is a constant, the optimization problem is reduced to

$$\min_T \tilde{D} = \min_T \mathrm{Tr}\Big[\sum_{e=1}^{E} w_e \,\big(I_N - R_x^{\frac{1}{2}} T^T H^T P_e^T (P_e H T R_x T^T H^T P_e^T + R_{ne})^{-1} P_e H T R_x^{\frac{1}{2}}\big)\Big] \quad (7)$$

$$\text{subject to} \quad \mathrm{Tr}(TR_xT^T) \leq P_{TX}.$$

The problem (7) is equivalent to (5) when $R_x = I_N$ but not convex with respect to the precoder matrix T . To circumvent this difficulty, (7) is reformulated by introducing the new matrix variables F and W_e such that

$$F = TR_xT^T \quad (8)$$

$$W_e = (P_e H T R_x T^T H^T P_e^T + R_{ne})^{-1} = (P_e H F H^T P_e^T + R_{ne})^{-1}. \quad (9)$$

Then using $\mathrm{Tr}(A + B) = \mathrm{Tr}(A) + \mathrm{Tr}(B)$, $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$ and simple algebraic operations, the weighted distortion $\tilde{D}$ can be written as

$$\begin{aligned}\tilde{D} &= \sum_{e=1}^{E} w_e \,\mathrm{Tr}(I_N) - \sum_{e=1}^{E} w_e \,\mathrm{Tr}\big(R_x^{\frac{1}{2}} T^T H^T P_e^T (P_e H T R_x T^T H^T P_e^T + R_{ne})^{-1} P_e H T R_x^{\frac{1}{2}}\big) \\ &= \sum_{e=1}^{E} w_e N - \sum_{e=1}^{E} w_e \mathrm{Tr}\Big[W_e(W_e^{-1} - R_{ne})\Big] \\ &= \Big[\sum_{e=1}^{E} w_e (N - L_e)\Big] + \sum_{e=1}^{E} w_e \mathrm{Tr}(W_e R_{ne}). \end{aligned} \quad (10)$$

The first term in (10) is a constant and independent of the precoder T. Therefore, the constant term is excluded from the optimization. Then the optimization problem is further reduced to

$$\min_{W_e, F} \quad \mathrm{Tr} \sum_{e=1}^{E} w_e W_e R_{ne} \quad (11)$$

subject to

$$W_e = (P_e H F H^T P_e^T + R_{ne})^{-1}, W_e \geq 0 \quad (12)$$
$$for \quad e = 1, 2, ..., E$$
$$\mathrm{Tr}(F) \leq P_{MAX}, \ \ \mathrm{rank}(F) = N, \ \ F \geq 0, F = F^T. \quad (13)$$

The precoder T in the original problem is substituted by the symmetric positive definite matrix F such that $F = TR_xT^T$. Since T is a rank N matrix, F should also have rank N. But having the rank constraint makes above problem non-convex and hence difficult to solve. Therefore the rank constraint is removed from the optimization to obtain a relaxed version of the problem. Also the set of inequalities for W_e is relaxed with $W_e \geq (P_e H F H^T P_e^T + R_{ne})^{-1}$ and converted to a set of LMIs using Schur complement [14]. The following relaxed optimization problem is obtained.

$$\min_{F, W_e} \quad \mathrm{Tr} \sum_{e=1}^{E} w_e W_e R_{ne} \quad (14)$$

subject to

$$\begin{bmatrix} W_e & I_{L_e} \\ I_{L_e} & P_e H F H^T P_e^T + R_{ne} \end{bmatrix} \geq 0, \ W_e \geq 0, \quad (15)$$
$$W_e = W_e^T \ \ for \quad e = 1, 2, ..., E$$
$$\mathrm{Tr}\, F \leq P_{MAX}, \ F \geq 0, F = F^T \quad (16)$$

The above problem is convex with respect to the variables F and W_e. Thus solution of the above problem yields a globally optimal solution for the matrix F and can be solved using the interior point method in MATLAB LMI and CVX toolboxes [15]. Since the rank constraint is dropped the optimal F is a full rank matrix. The next problem is to extract the rank reduced $M \times N$ precoder matrix T from the optimal F efficiently such that $TR_xT^T = F$ and $\mathrm{Tr}(TR_xT^T) = \mathrm{Tr}(F)$.

One method to find the low rank approximation is by ignoring the lowest singular values of the original matrix F [16]. Let the singular value decomposition for F, T and R_x be $F = U_F \Delta_F U_F^T$, $T = U_T \Delta_T V_T^T$ and $R_x = U_R \Delta_R U_R^T$, respectively, where U_F, U_T, V_T and U_R are orthogonal matrices with $U_F, U_T \in \mathbb{R}^{M\times M}$, $U_R, V_T \in \mathbb{R}^{N\times N}$, and Δ_F, Δ_R and Δ_T are diagonal matrices in the form $\Delta_F = \mathrm{diag}(f_1, f_2, \ldots, f_M)$, $\Delta_R = \mathrm{diag}(r_1, r_2, \ldots, r_N)$ with $f_1 > f_2 > \cdots > f_M$ and $r_1 > r_2 > \cdots > r_N$. Then from (8), we get

$$F = U_F \Delta_F U_F^T = U_T \Delta_T V_T^T U_R \Delta_R U_R^T U_T^T \Delta_T^T V_T. \quad (17)$$

Equating $U_T = U_F$ and $V_T = U_R$, (17) can be diagonalized to

$$\Delta_F = \Delta_T \Delta_R \Delta_T^T \quad (18)$$

with $\Delta_T = \left(\begin{smallmatrix} \tilde{\Delta}_{T_{N\times N}} \\ 0_{(M-N)\times N} \end{smallmatrix}\right)$, $\tilde{\Delta}_T = \operatorname{diag}(t_1, t_2, \ldots, t_N)$ and $t_1 > t_2 > \cdots > t_N$. Partitioning $\Delta_F = \left(\begin{smallmatrix} \Delta_{F1} & 0 \\ 0 & \Delta_{F2} \end{smallmatrix}\right)$ such that Δ_{F1} contains the maximum N eigenvalues in descending order, we can derive the low rank approximation for precoder T as

$$T = U_F \begin{bmatrix} \Delta_{F1}^{\frac{1}{2}} \times \Delta_R^{-\frac{1}{2}} \\ 0_{(M-N)\times N} \end{bmatrix} U_R^T \quad (19)$$

This method is optimal if the Δ_{F2} has significantly less energy compared to Δ_{F1}. As this is not the general case, we propose to distribute the energy in Δ_{F2} proportionately to diagonal components of the precoder such that

$$T = U_F \begin{bmatrix} \left(\left(\frac{Tr(\Delta_{F2})}{Tr(\Delta_{F1})} + 1 \right) \Delta_{F1} \right)^{\frac{1}{2}} \times \Delta_R^{-\frac{1}{2}} \\ 0_{(M-N)\times N} \end{bmatrix} U_R^T. \quad (20)$$

4. NUMERICAL EXAMPLES

This section illustrates the proposed method and presents two example. A 3×2 precoder design for 3×3 MIMO channel with all the possible erasure states and 4×3 precoder design for 4×4 MIMO channel with only six erasure states are considered.

Example 1: A random Gaussian source with a know covariance matrix of $\left(\begin{smallmatrix} 3.7525 & 1.4053 \\ 1.4053 & 2.1466 \end{smallmatrix}\right)$ is considered. The source is transmitted through a known MIMO channel of 3 transmitters and 3 receivers with additive Gaussian noise of covariance $0.1 I_3$. We derived the optimal 3×2 precoder that minimizes the weighted sum of distortion for all channel erasure states, $P_1 = [1\ 0\ 0]$, $P_2 = [0\ 1\ 0]$ and $P_3 = [0\ 0\ 1]$, $P_4 = \left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{smallmatrix}\right]$, $P_5 = \left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$, $P_6 = \left[\begin{smallmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$, $P_7 = \left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$, with 10 $watts$ total output power constraint by solving (14) and obtaining the low rank approximation.

$$T_{3\times 2} = \begin{bmatrix} 0.8366 & 0.1271 & 1.0166 \\ 0.7281 & 0.1048 & -1.4945 \end{bmatrix}^T .$$

Example 2: Here the optimal 4×3 precoder that minimizes the weighted sum of distortion for channel erasure sates $P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$, $P_6 = \left[\begin{smallmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{smallmatrix}\right]$, $P_{10} = \left[\begin{smallmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{smallmatrix}\right]$, $P_{12} = \left[\begin{smallmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{smallmatrix}\right]$ and $P_{14} = \left[\begin{smallmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{smallmatrix}\right]$ was derived for 4×4 MIMO channel. The total output power was constrained to 10 $watts$.

$$T_{4\times 3} = \begin{bmatrix} 0.0102 & -2.0284 & 1.5733 & -0.4047 \\ 0.8689 & 0.7443 & 1.1442 & 1.2834 \\ 0.6529 & -0.1855 & -2.1024 & 1.3685 \end{bmatrix}^T .$$

The expected value of distortion was calculated for a range of erasure probabilities. The figure 2:(a)-(b) compares the resultant precoder with the precoder designed without considering channel noise [10] and the precoder designed considering only channel noise but not erasures. Also the performance of precoder in Example 1 was compared with frame $T_{mb} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^T$ and with a cascaded precoder [4] for 4×3 case in example 2. The figure 2:(c)-(d) presents the performance of the side distortions for each precoder.

5. CONCLUSION

This paper has proposed a novel method for jointly designing optimal precoder and decoder for MIMO channel subject to erasures and channel noise. The proposed method is in the form of convex optimization subject to LMI constraints, which can be easily solved for globally optimal solution using standard semi-definite programming software. The numerical examples shows the effectiveness of the optimal precoder against channel losses and noise. In contrast to noiseless case, the precoding does not generate balanced descriptions. When more channels are received the channel noise is the major contributor to the distortion and the precoder is more effective in such situations. Nevertheless the weighted average distortion is still minimal for the designed precoder. However the optimization problem in (14)-(16) gives an suboptimal solution due to (6) and rank relaxation.

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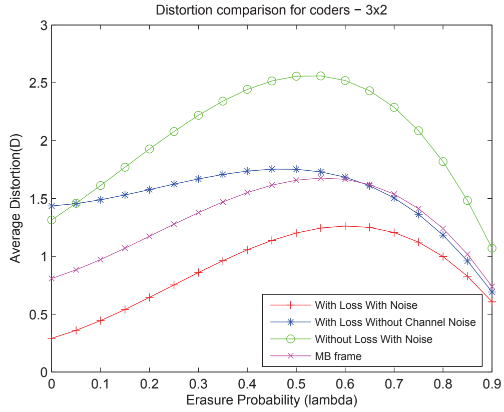
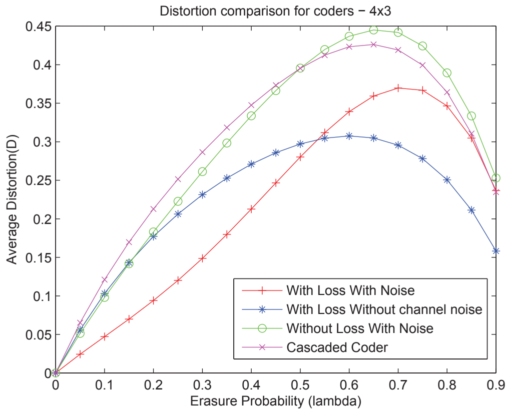
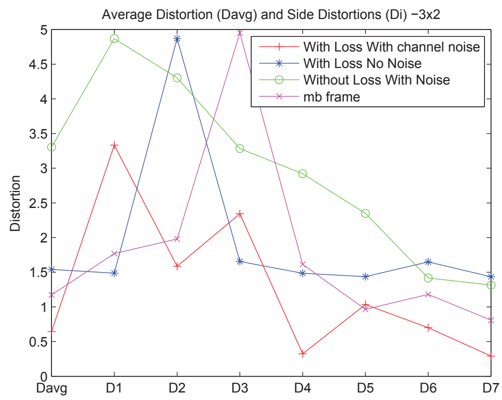


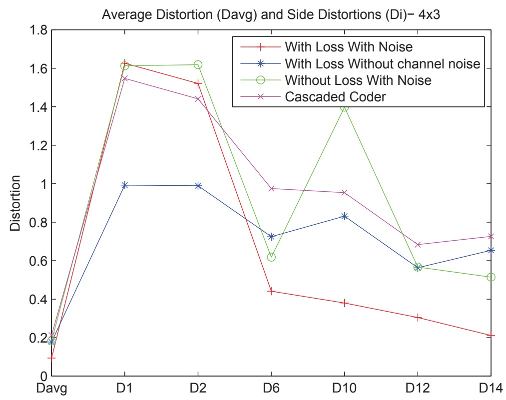
(a) 3x2 precoder



(b) 4x3 precoder



(c) 3x2 precoder



(d) 4x3 precoder

Fig. 2. (a)-(b): The average distortion variation across erasure probabilities. (c)-(d): Comparison of side distortion