# TRAINING SYMBOL EXPLOITATION IN CP-OFDM FOR DOA ESTIMATION IN MULTIPATH CHANNELS

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### ABSTRACT

In this paper, the problem of line-of-sight (LOS) direction of arrival (DoA) estimation in multipath channels is addressed. The system that we propose is based on the well-known cyclic prefixed (CP) orthogonal frequency division multiplexing (OFDM) scheme. The algorithm searches the time instant corresponding to the first arrival path (FAP), i.e., the LOS component that brings the DoA information. A time domain channel estimator evaluates the complex amplitude of the FAP. Finally, the FAP complex amplitude of each antenna is used to estimate the DoA with a low complexity single source method. Several numerical results validate the robustness of this algorithm, also in the case when the multipath components are superimposed to the LOS component.

#### 1. INTRODUCTION

The knowledge of the direction of arrival (DoA) of a radio source is required by many engineering applications, including wireless communications, radar, navigation, object tracking, and rescue and other emergency assistant devices [1]. For this reason, the problem of DoA estimation is very important in the array signal processing literature, and many algorithms have been proposed in the last two decades, both for narrowband and wideband signals. For narrowband signals<sup>1</sup>, super resolution techniques, e.g., Multiple Signal Classification (MUSIC) [2] and Estimation of Signal Parameters Via Rotational Invariance Techniques (ESPRIT) [3],[4], are the most popular. Although these methods present higher resolution capabilities, they suffer from some drawbacks. Firstly, they can not resolve coherent waves directly, and this is a great limitation in the case of multipath channels since the multipath components (MPCs) can be coherent with the direct path. To overcome this difficulty, a preprocessing technique, called Spatial Smoothing Preprocessing (SSP), and other variants have been proposed [5],[6]. Another problem of the super resolution algorithms is that the total number of signals impinging on the array must be less than the number of sensors. Moreover, the SSP decreases the effective array size, and this further limits the total number of signals that can be distinguished. In this respect, the Joint Angle and Delay Estimation (JADE) algorithm has been presented in [7]. This solution, that aims at estimating DoAs and delays of the MPCs using a collection of space-time channel estimates, can work in cases where the total number of impinging signals exceeds the number of antennas. However, similarly to the traditional MUSIC and ESPRIT, it needs to compute an eigendecomposition that is computationally expensive. Other complex algorithms that do not rely on the eigendecomposition have been proposed to deal with multipath propagation [8], [9], [10].

Motivated by these considerations, in this paper we propose a line-of-sight (LOS) DoA estimation approach based on the first arrival path (FAP) identification. The proposed algorithm comprises the following steps: a) a coarse synchronization that identifies the start of the transmitted frame; b) a time domain channel estimation; c) a threshold based fine synchronization that accurately selects the FAP; d) a low complexity single source DoA estimation. The above steps are applied to a cyclic prefixed (CP) orthogonal frequency division multiplexing (OFDM) system. Cyclic prefixing avoids both intersymbol interference (ISI) and inter-carrier interference (ICI) if the synchronization error is within the ISI-free zone of the CP [11], and this allows the correct estimation of the channel impulse response (CIR). The coarse synchronization, the time domain channel estimation, and the fine synchronization are based on the algorithm proposed by Minn et al. in [11] that shows better performance than other synchronization procedures usually considered in the OFDM literature [12]. Finally, the low complexity single source DoA estimator is the one proposed in [13].

Our algorithm is capable to distinguish between the LOS DoA and a MPC spaced by one sample period. Further, we will show that the proposed technique guarantees good performance also when the LOS DoA is corrupted by a MPC with delay less than one sample period. Moreover, this method works regardless of the number of antenna (at least two) or the number of impinging signals. Finally, it does not involve an eigen-decomposition.

The rest of the paper is organized as follows. Section

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<sup>&</sup>lt;sup>1</sup>The signal will be narrowband if  $\frac{BW}{f_c} \ll \frac{1}{D}$ , where BW is the signal bandwidth,  $f_c$  is the carrier frequency, and D denotes the array aperture in wavelengths [1].



Fig. 1. Single-input multiple-output (SIMO) scenario, with linearly equispaced (LES) antenna array with M elements spaced by  $\lambda/2$ .

2 presents the single-input multiple-output (SIMO) system model, including the CP-OFDM transmission scheme. Section 3 summarizes the coarse and the fine synchronization procedures, the time-domain channel estimation, and the DoA estimation. Performance evaluation and simulation results are described in Section 4. Finally, the conclusions follow.

# 2. SYSTEM MODEL DESCRIPTION

Let us assume a SIMO system model as in Fig. 1, where the multiple antenna receiver is equipped with a linearly equispaced antenna array with M elements spaced by  $\lambda/2$ , where  $\lambda$  is the wavelength. If the propagation were ideal, the *i*-th antenna received signal would be expressed as

$$r^{(i)}(t) = \alpha_0^{(i)} s(t - \tau_0) + n^{(i)}(t), \ i \in \{1, .., M\}, \quad (1)$$

where s(t) is the transmitted signal,  $\tau_0$  is the propagation delay,  $\alpha_0^{(i)} = \rho_0 e^{-j\pi(i-1)\cos(\phi_0)}$  is a complex channel coefficient that comprises the attenuation  $\rho_0$  (that can be complex due to an arbitrary phase rotation, common to all the elements, introduced by the propagation) and the phase contribution due to the DoA  $\phi_0$ , while  $n^{(i)}(t)$  is the receiver noise. In this case, the DoA  $\phi_0$  can be easily estimated by applying the low complexity DoA estimator

$$\tilde{\phi_0} = \arccos\left(\frac{\angle \Lambda}{\pi}\right) \tag{2}$$

with

$$\Lambda = \frac{1}{N(M-1)} \sum_{n=0}^{N-1} \sum_{i=1}^{M-1} r^{(i)}(n) r^{(i+1)^*}(n), \qquad (3)$$

where  $r^{(i)}(n), n \in \{0, .., N-1\}$  are N samples of  $r^{(i)}(t)$ .

Let us assume the presence of a multipath fading channel whose impulse response can be expressed as

$$h^{(i)}(t) = \sum_{l=0}^{L-1} \alpha_l^{(i)} \delta(t - \tau_l), \ i \in \{1, .., M\},$$
(4)

where  $\alpha_l^{(i)}$ ,  $l \in \{0, ..., L-1\}$  are the complex channel coefficients of the *i*-th antenna channel, and  $\tau_l$ ,  $l \in \{0, ..., L-1\}$  are the corresponding path delays. The signal at the input of the *i*-th receiver (in the absence of carrier frequency offset and sampling clock errors) can be written as

$$r^{(i)}(t) = \sum_{l=0}^{L-1} \alpha_l^{(i)} s(t-\tau_l) + n^{(i)}(t), \ i \in \{1, .., M\}.$$
 (5)

To proceed, it is necessary to estimate the complex channel coefficients  $\alpha_0^{(i)}$ ,  $i \in \{1, ..., M\}$  in (5). Then, it will be possible to estimate the DoA as in (2) with

$$\Lambda = \frac{1}{M-1} \sum_{i=1}^{M-1} \tilde{\alpha}_0^{(i)} \tilde{\alpha}_0^{(i+1)^*}, \tag{6}$$

where  $\tilde{\alpha}_{0}^{(i)},\;i\in\{1,..,M\}$  is the estimated channel coefficient.

To do so, we propose to perform the following steps: a) a coarse synchronization, in order to identify the beginning of the signal s(t); b) a time domain channel estimation; c) a threshold-based fine synchronization, to estimate the position of the first arrival path (FAP).

### 2.1. CP-OFDM Transmission Scheme

In a SIMO system, the transmitter is shared, while the receiver antenna array is equipped with a bank of receivers each dealing with its own channel, as in (5). We consider a CP-OFDM system since it is one of the most used transmission techniques.

In the discrete time domain, the samples of the transmitted OFDM symbol at the output of the parallel-to-serial (P/S) block, with ideal Nyquist pulse shaping, can be written as

$$s(k) = \frac{N}{N+\mu} \sum_{n=0}^{N-1} c_n e^{j2\pi \frac{kn}{N}}, \ k \in \{-\mu, ..., N-1\}, \quad (7)$$

where  $c_n$ ,  $n \in \{0, ..., N-1\}$  are the subchannel symbols, N is the number of subchannels, while  $\mu$  is the CP length.

These samples are convolved with a specific CIR, and the result is corrupted by the additive noise. At the receiver, a synchronization block allows recovering the OFDM symbol start instant, in order to neglect the CP and to successively apply the fast Fourier transform (FFT.) If the coarse synchronization

The path delays  $\tau_l$ ,  $l \in \{0, ..., L-1\}$  do not depend on the antenna index *i* under the narrowband signal assumption.

introduces a timing offset  $\epsilon \in \{-\mu + \tau_L, -\mu + \tau_L + 1, ..., 0\}$ (the so called ISI-free zone) the orthogonality among the subcarriers will not be destroyed by the introduction of ISI and ICI [14]. Finally, as suggested in [11], channel estimation can be performed in the time domain on the received samples.

### 3. SYNCHRONIZATION, CHANNEL ESTIMATION, AND DOA ESTIMATION

The timing synchronizer has to determine the starting point of the FFT window, that corresponds to the FAP delay  $\tau_0$  (plus the CP length  $\mu$ ). Minn et al. [11] have proposed to transmit an OFDM training symbol with Q = 4 (or another power of two) identical portions each comprising  $N_q = N/Q$  symbols. Each portion corresponds to the FFT of a quarter length Golay complementary sequence. We propose to extend the Minn's timing metric to the case of multiple receivers, by simply averaging the correlation sequences  $P^{(i)}(m)$ ,  $i \in \{1, ..., M\}$ and the symbol energies  $R^{(i)}(m)$ ,  $i \in \{1, ..., M\}$ . After the analog-to-digital conversion, the timing metric to be maximized can be expressed as

$$\Lambda_c(m) = \left(\frac{Q}{Q-1} \frac{|\bar{P}(m)|}{\bar{R}(m)}\right)^2,\tag{8}$$

with  $\bar{P}(m)=\frac{1}{M}\sum_{i=1}^M P^{(i)}(m)$  and  $\bar{R}(m)=\frac{1}{M}\sum_{i=1}^M R^{(i)}(m),$  while

$$P^{(i)}(m) = \sum_{q=0}^{Q-2} p(q)p(q+1) \sum_{n=0}^{N_q-1} r^{(i)^*}(m+qN_q+n)$$
  

$$\cdot r^{(i)}(m+(q+1)N_q+n),$$
  

$$R^{(i)}(m) = \sum_{q=0}^{Q-1} \sum_{n=0}^{N_q-1} |r^{(i)}(m+n+qN_q)|^2,$$
(9)

where  $p(q), q \in \{0, ..., Q - 1\}$ , denotes the sign of the repeated part of the training symbol. According to [11], we have adopted Q = 4 and p = [- + --].

#### 3.1. Coarse Synchronization

The coarse timing estimate can be obtained as

$$\tau_c = \arg\max_{m \in \mathbb{Z}} \left\{ \Lambda_c(m) \right\} - \lambda_c, \tag{10}$$

where  $\lambda_c$  is a pre-shift that should be chosen higher than the (designed) mean shift caused by the channel dispersion. In this way, the coarse timing estimate will be in the ISI-free part of the cyclic prefix.

#### 3.2. Channel Estimation

The *i*-th antenna maximum likelihood channel response estimate can be obtained as

$$\tilde{\mathbf{h}}^{(i)} = \left(\mathbf{S}^H \mathbf{S}\right)^{-1} \mathbf{S}^H \cdot \mathbf{r}^{(i)}(\tau_c), \ i \in \{1, .., M\},$$
(11)

where  $\tilde{\mathbf{h}}^{(i)}$  is the  $\mu$ -length column vector that contains the complex path gains of the channel, while

$$\mathbf{S} = \begin{bmatrix} s(0) & s(-1) & \dots & s(-\mu+1) \\ s(1) & s(0) & \dots & s(-\mu+2) \\ \dots & & & \\ s(N-1) & s(N-2) & \dots & s(N-\mu) \end{bmatrix}$$
(12)

is the matrix associated to the transmitted training symbol (including the CP), and

$$\mathbf{r}^{(i)}(\tau_c) = [r^{(i)}(\tau_c + \mu) \ r^{(i)}(\tau_c + \mu + 1) \dots \ r^{(i)}(\tau_c + \mu + N - 1)]^T$$
(13)

is the column vector of the received samples (excluding the cyclic prefix) in the window from  $\tau_c + \mu$  to  $\tau_c + \mu + N - 1$ .

# 3.3. Fine Synchronization

The FAP delay can be found as follows. Firstly, we denote with  $\chi$  the channel response vector obtained by averaging each entry of the vector  $\tilde{\mathbf{h}}^{(i)}$ , in absolute value, over the antenna elements, i.e.,  $\chi(m) = \frac{1}{M} \sum_{i=1}^{M} |\tilde{h}^{(i)}(m)|$ . Secondly, the strongest tap gain  $\chi_{max}$  is found as

$$\chi_{max} = \arg \max_{m \in \{0, \dots \mu - 1\}} \{\chi(m)\}.$$
 (14)

Then, the fine timing  $\tau_f$  is given by

$$\tau_f = \arg \max_{m \in \{0, \dots, \mu - K\}} \{E_h(m)\},$$
 (15)

where

$$E_h(m) = \begin{cases} \sum_{k=0}^{\mu-1} \chi(m+k)^2, & \text{if } \chi(m) > \eta \cdot \chi_{max}, \\ 0, & \text{otherwise} \end{cases}$$
(16)

will be the channel energy estimate contained in a window of length  $\mu$  starting from the m - th tap, if the channel energy estimate of the m - th tap is greater than the threshold  $\eta$ .

At this point, the FAP delay estimation  $\tilde{\tau_0}$  can be found as

$$\tilde{\tau_0} = \tau_c + \tau_f. \tag{17}$$

It should be noted that in [11] the FAP delay estimation is further pre-shifted but we have found that, with our parameter setup, (17) minimizes the timing error.

#### 3.4. DoA Estimation

The LOS DoA can be found by applying (2) with

$$\Lambda = \frac{1}{M-1} \sum_{i=1}^{M-1} \tilde{h}^{(i)}(\tau_f) \tilde{h}^{(i+1)^*}(\tau_f).$$
(18)

It should be noted that the DoA estimate can be improved by using  $L_{tr}$  CIR estimates, obtained by transmitting  $L_{tr}$ OFDM training symbols, and averaging  $\Lambda$  as it is done in (3).

### 4. NUMERICAL RESULTS

The performance of the proposed algorithm has been investigated by simulations in terms of root mean-squared error (RMSE) obtained by the DoA estimator, defined as

$$\mathbf{RMSE} = \sqrt{E\left\{\left(\phi_0 - \tilde{\phi_0}\right)^2\right\}}.$$
 (19)

The used OFDM parameters are: N = 64, CP length  $\mu = 16$ ,  $L_{tr} = 1$ . The synchronization parameters are:  $\lambda_c = 10$ ,  $\eta = 0.5$ . The number of antennas is M = 4.

# 4.1. Channel Model

We assume a single cluster channel model as in (4), where the FAP always comprises the LOS component. The total number of MPCs, L - 1, is Poisson distributed. Each MPCs is associated to a specific DoA  $\phi_l$  as proposed in [15], so that the complex channel coefficient  $\alpha_l^{(i)}$ , in the case of a  $\lambda/2$ spaced LES array, can be written as

$$\alpha_l^{(i)} = \rho_l e^{-j\pi(i-1)\cos(\phi_l)}, \ l \in \{0, .., L-1\}, \ i \in \{1, .., M\}.$$
(20)

The amplitude of the *l*-th MPC,  $|\rho_l|$ , is modeled as a Rayleigh distributed random variable with power that follows an exponential decay profile, i.e.,  $\Omega_l \sim e^{-\tau_l/\Gamma}$ , with  $\Gamma$  the power-delay time constant, while its phase shift,  $\angle \rho_l$ , is uniformly distributed. The channel has unit average power. Furthermore, the DoAs  $\phi_l$ , l > 0 are Laplace distributed, with mean  $\phi_0$  and standard deviation AS (in the following we refer to it as angular spread). Specifically, we have assumed  $\phi_0 = 30$  deg, and  $\Gamma = 4$ .

The inter-arrival times  $\tau_l - \tau_0$  are exponentially distributed with parameter  $\Lambda$ . We then normalize (round) the path delays w.r.t. the sample period. So that the FAP delay  $\tau_0$  is uniformly distributed within the range [0, N - 1], while the L' MPCs may have the same delay of the LOS component. The ratio K between the LOS power  $\Omega_0$  and the total power of these NLOS components is defined as  $K = \Omega_0 / \sum_{l=1}^{L'} \Omega_l$ .

### 4.2. Scenario with $K = \infty$

In Fig. 2 we show the performance as function of the SNR and of  $\Lambda$ , with  $K = \infty$ , in the case of ideal synchronization and by applying the synchronization herein proposed. It should be noted that, in the case of ideal channel ( $\Lambda = 0$ ), the ideal synchronization curve and the one obtained by applying the proposed algorithm practically overlap, except for low SNRs. If we consider the ideal synchronization, we can observe a slight decrease of performance with the increase of  $\Lambda$ . This is mostly due to the decreasing total power associated to the LOS path when the number of the MPCs increases. When the synchronization algorithm is performed, the performance curves reach an error floor due to the synchronization error



Fig. 2. RMSE as function of SNR with AS = 5 deg.



**Fig. 3.** RMSE as function of SNR, with  $\Lambda = 4$  and  $K = \infty$ .

that increases with  $\Lambda$ . However, in the worst case, the proposed method shows an error floor lower than 1 deg.

In Fig. 3, the performance of the algorithm is tested for different angular spread AS, with  $\Lambda = 4$  and  $K = \infty$ . It can be observed that the performance decreases with the increase of the angular spread. However, with AS = 20 deg, the error floor at the high SNRs is approximately 1 deg.



Fig. 4. RMSE as function of SNR, with  $\Lambda = 4$ , AS = 5 deg.

# 4.3. Scenario with NLOS overlapping LOS component

In Fig. 4, we show the performance of the system with different values of K, i.e., when NLOS paths may overlap with the LOS. The performance degrades with the decrease of K. However, we have observed acceptable error floors (below 1 deg) for values of K higher than 5 dB. It should be noted that the factor K depends on the scenario. Indoor channels manifest lower K, while outdoor channels have higher K.

# 5. CONCLUSIONS

A DoA estimation algorithm for frequency selective channels has been proposed. This method exploits the CP-OFDM transmission scheme and it performs a coarse synchronization to locate the OFDM symbol start point, a channel estimation, and a threshold-based fine synchronization that finely locates the first arrival path (FAP). Once we have located the FAP component, a single source DoA estimator algorithm can be used. Several numerical results have confirmed the robustness of the method, also when the LOS and NLOS paths overlap.

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