A HIDDEN MARKOV MODEL FOR INDOOR USER TRACKING BASED ON WIFI FINGERPRINTING AND STEP DETECTION

M.K. Hoang, J. Schmalenstroeer, C. Drueke, D.H. Tran Vu, R. Haeb-Umbach

Department of Communications Engineering, University of Paderborn, 33098 Paderborn, Germany {hoang, schmalen, tran, haeb}@nt.uni-paderborn.de

ABSTRACT

In this paper we present a modified hidden Markov model (HMM) for the fusion of received signal strength index (RSSI) information of WiFi access points and relative position information which is obtained from the inertial sensors of a smartphone for indoor positioning. Since the states of the HMM represent the potential user locations, their number determines the quantization error introduced by discretizing the allowable user positions through the use of the HMM. To reduce this quantization error we introduce "pseudo" states, whose emission probability, which models the RSSI measurements at this location, is synthesized from those of the neighboring states of which a Gaussian emission probability has been estimated during the training phase. The experimental results demonstrate the effectiveness of this approach. By introducing on average two pseudo states per original HMM state the positioning error could be significantly reduced without increasing the training effort.

Index Terms— Indoor positioning, fingerprint, pseudo node, step detection, RSSI measurement

1. INTRODUCTION

Accurate user positioning inside buildings in the absence of an extra infrastructure dedicated to localization remains to be an important research challenge, because the GPS signal, which is the key component of outdoor localization, is usually not available.

Two classes of approaches to estimate the position of user inside buildings are common. One of them, the fingerprinting method, is utilizing existing wireless communications infrastructure, such as WiFi, by measuring the RSSI and comparing it with a database of RSSI measurements generated in a training phase. Positioning is then formulated as a classification problem, where the measured RSSI value is classified to that position, which, according to the training data, is most probable [1, 2]. The classification can be conducted with different classifiers, such as support vector machines [1] or k-nearest neighbor (K-NN) [3, 4]. Prominent examples of systems falling in this category are the RADAR [5] and LOCATOR system [6]. The other class of approaches relies on inertial sensors to track a user starting from a known location [7]. As today's smartphones are equipped with a multitude of sensors, information of other sensors, such as a camera, may also be fused with the inertial sensor information to improve accuracy.

Various combinations of the aforementioned two classes of approaches have been proposed. Sensor fusion may be achieved by a Kalman filter [8] or with the use of a HMM. In the latter case the hidden states represent the possible user locations. In the HMM approach of [9] and [10] the RSSI measurements and the inertial sensor information are taken as observations attached to the hidden state, the user position, and the transition probabilities between the states are chosen to reflect which positions are accessible from a given state, i.e., user position, within one measurement interval. The user is then tracked by employing either the Viterbi, the Forward, or the Forward-Backward algorithm. The latter, however, is only of academic interest, because the induced latency is not acceptable.

The paper at hand also employs a HMM based sensor fusion of RSSI and inertial sensor information. The problem we are addressing here is the reduction of the quantization error introduced by the HMM. On the one hand the number of HMM states should be small in order to reduce the training effort, as for each position a multitude of RSSI measurements are to be recorded to learn a model (the expected signal strengths). On the other hand a dense grid of possible user locations is desirable to track the user closely. In this paper we propose an approach, which employs a fairly coarse grid of states for which RSSI measurements need to be taken during training, while at the same time achieving a low discretization error. This is achieved by introducing "pseudo" states in-between the regular HMM states. The emission probability density functions (PDF) of the pseudo states are computed from the models of the neighboring regular states. By doing so, the quantization error can be significantly reduced at no increase in training effort, resulting in an overall improved positioning accuracy.

The paper is organized as follows: In the next section we describe the HMM-based sensor fusion and show how the individual knowledge sources are combined to arrive at an estimate of the user location. In Section 3 we briefly review our approach to compute the likelihood of RSSI measurements in the presence of clipped observations [11]. Section 4 describes how the inertial sensor measurements are employed for step detection, from which a movement vector is obtained. The modified HMM incorporating the pseudo states is presented in Section 5. Section 6 presents experimental results, both on artificial and on field data, and the paper finishes off with some conclusions in Section 7.

2. HMM-BASED SENSOR FUSION

In [9, 10] a hidden Markov model (HMM) is used to combine RSSI measurements and step detection for position estimation. The hidden states comprise the possible user positions, of which reference RSSI fingerprints have been recorded in the training phase. The estimation of the user position can then be carried out either by the Forward algorithm or by the Viterbi algorithm. While the former computes the probability of being in a certain state by gathering the probabilities over all possible predecessor states, the latter considers only the most probable predecessor. In the following we consider the Forward algorithm.

Let s_t denote the value the hidden state variable takes at time t, which we identify with the position of the user at time t: if $s_t = j$ then the user is at the location ℓ_j at time t, where ℓ_j is a two-dimensional vector containing x and y coordinates of the user's location. Further, let $o_{1:t} = [o_1, \ldots o_t]$ be the sequence of RSSI measurements up to time t. The step detection information is gathered in the sequence $v_{1:t} =$ $[v_1, \ldots v_t]$, where the two-dimensional movement vector v_t denotes the traversed route from the user position at time t-1 to time t. Our goal is to compute $P(s_t=j|v_{1:t}, o_{1:t})$, i.e., the probability of being in state j for all possible user positions ℓ_j , $j = 1, \ldots, J$, given all RSSI values and step detection vectors measured sofar. Using Bayes' rule, the probability can be expressed as follows:

$$P(s_t=j|\boldsymbol{v}_{1:t}, \boldsymbol{o}_{1:t}) = \frac{P(s_t=j, \boldsymbol{v}_{1:t}, \boldsymbol{o}_{1:t})}{p(\boldsymbol{v}_{1:t}, \boldsymbol{o}_{1:t})} \\ \propto p(s_t=j, \boldsymbol{v}_{1:t}, \boldsymbol{o}_{1:t}) =: \alpha_t(j), \quad (1)$$

where the so-called Forward variable $\alpha_t(j)$ is the probability of being at time t in state j, while having observed the sequence of $o_{1:t}$ and $v_{1:t}$.

$$\alpha_{t}(j) = \sum_{i} p(s_{t}=j, s_{t-1}=i, \boldsymbol{v}_{1:t}, \boldsymbol{o}_{1:t})$$

$$= \sum_{i} p(\boldsymbol{v}_{t}|s_{t}=j, s_{t-1}=i, \boldsymbol{v}_{1:t-1}, \boldsymbol{o}_{1:t-1}, \boldsymbol{o}_{t})$$

$$\cdot p(\boldsymbol{o}_{t}|s_{t}=j, s_{t-1}=i, \boldsymbol{v}_{1:t-1}, \boldsymbol{o}_{1:t-1})$$

$$\cdot P(s_{t}=j|s_{t-1}=i, \boldsymbol{v}_{1:t-1}, \boldsymbol{o}_{1:t-1})$$

$$\cdot P(s_{t-1}=i, \boldsymbol{v}_{1:t-1}, \boldsymbol{o}_{1:t-1}). \qquad (2)$$

Applying the properties of the HMM, which are depicted in the graphical model of Fig. 1, and assuming the step detection and RSSI information to be statistically independent of each other given the user location, we arrive at

$$\alpha_{t}(j) = \sum_{i} p(\mathbf{v}_{t}|s_{t}=j, s_{t-1}=i) \cdot p(\mathbf{o}_{t}|s_{t}=j)$$
$$\cdot P(s_{t}=j|s_{t-1}=i) \cdot \underbrace{P(s_{t-1}=i, \mathbf{v}_{1:t-1}, \mathbf{o}_{1:t-1})}_{=\alpha_{t-1}(i)} \quad (3)$$

which is a recursion of the forward variable.

Ê

The final location estimate $\hat{\ell}$ is obtained by the weighted average over the set \mathcal{P} of the *M* most likely positions:

$$= \frac{1}{\sum_{k \in \mathcal{P}} \alpha_t(k)} \sum_{k \in \mathcal{P}} \alpha_t(k) \cdot \boldsymbol{\ell}_k.$$
(4)

Equation (3) shows how the different knowledge sources are combined. The transition probabilities $P(s_t=j|s_{t-1}=i)$ are nonzero only for those locations ℓ_j that can be reached from position ℓ_i within one time step. The choice of the transition probabilities thus encodes our knowledge about the floor plan. The term $p(o_t|s_t=j)$ is the likelihood of the RSSI measurement o_t , assuming the user's position is ℓ_j . Its computation is described in the next section. The movement information gathered from the step detection is captured by the term $p(v_t|s_t=j, s_{t-1}=i)$, i.e., the likelihood of observing the movement vector v_t when moving from position ℓ_i to ℓ_j . Its computation is detailed in Section 4.

Note that this derivation assumes that RSSI measurements are obtained at the same rate as the step detection information. In Section 5 we show how a dense grid of possible user locations can be obtained despite coarse grid RSSI measurements.



Fig. 1. HMM for position estimation based on the fusion of RSSI and movement vector observations o and v, respectively.

3. TREATMENT OF CENSORED DATA

The WiFi receivers used in smartphones or other mobile devices have a finite sensitivity, which is limited to, e.g., the range of $-30 \,\mathrm{dBm}$ to $-100 \,\mathrm{dBm}$. Thus, if for example the signal strength of a far away access point falls below $-100 \,\mathrm{dBm}$ the value is clipped and a RSSI value of $-100 \,\mathrm{dBm}$ will be output.

In [11] we proposed an Expectation Maximization (EM) algorithm to estimate the parameters of a Gaussian from clipped observations. Let \tilde{o}_i , i = 1, ..., N be draws from

 $\mathcal{N}(\mu, \sigma^2)$. These draws are censored according to

$$o_i = \max(\tilde{o}_i, c) = \begin{cases} \tilde{o}_i & z_i = 0\\ c & z_i = 1 \end{cases}$$
 (5)

Here, c is the clipping threshold, and the binary variable z_i indicates whether an observation is clipped ($z_i = 1$) or not ($z_i = 0$). For simplicity of exposition we consider only one-sided clipping. In [11] we have shown that the following iterative algorithm

$$\mu^{(\kappa+1)} = \frac{1}{N} \sum_{i=1}^{N} (1-z_i) o_i + \frac{1}{N} \frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} \sum_{i=1}^{N} z_i$$
(6)
$$(\sigma^2)^{(\kappa+1)} = \frac{1}{N} \sum_{i=1}^{N} (1-z_i) \left(o_i - \mu^{(\kappa)} \right)^2 + \left[\frac{I_2(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} - 2\mu^{(\kappa)} \frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} + (\mu^2)^{(\kappa)} \right] \frac{1}{N} \sum_{i=1}^{N} z_i.$$
(7)

delivers unbiased and efficient parameter estimates. Here, κ is the iteration index of the EM algorithm and $\theta = (\mu, \sigma^2)$ denotes the parameters to be estimated. Further

$$I_j(\theta^{(\kappa)}) = \int_{-\infty}^c y^j \mathcal{N}\left(y; \theta^{(\kappa)}\right) dy.$$
(8)

During classification we need to compute the likelihood of an observation vector $\boldsymbol{o} = (o_1 \cdots o_{N_{\text{AP}}})^T$, comprising the RSSI values of N_{AP} WiFi access points (APs), for an hypothesized user location ℓ_k . To account for censored data this is carried out as follows

$$p(\boldsymbol{o}|\boldsymbol{\ell}_k) = \prod_{i=1}^{N_{AP}} p(o_i|\boldsymbol{\ell}_k)$$
(9)

where

$$p(o_i|\boldsymbol{\ell}_k) = \begin{cases} \mathcal{N}(o_i; \hat{\boldsymbol{\mu}}_{\boldsymbol{\ell}_k, i}, \hat{\sigma}^2_{\boldsymbol{\ell}_k, i}), \text{if } o_i > c\\ I_0(\hat{\boldsymbol{\mu}}_{\boldsymbol{\ell}_k, i}, \hat{\sigma}^2_{\boldsymbol{\ell}_k, i}), & \text{if } o_i = c \end{cases}$$
(10)

Here, $(\hat{\mu}_{\ell_k,i}, \hat{\sigma}_{\ell_k,i}^2)$ are the estimated parameters of the *i*-th AP at location ℓ_k . In case all observations of the *i*-th AP at fingerprint location ℓ_k are clipped, the mean estimate is set to a small value $\hat{\mu}_{\ell_k,i} \ll c$ and $\hat{\sigma}_{\ell_k,i}^2$ is set to an average value.

4. STEP DETECTION AND MOVEMENT VECTOR ESTIMATION

The step detection system proposed in [10] is depicted in Fig. 2. The step detection uses the 3-dimentional acceleration vec-



Fig. 2. Step detection and position estimation system overview

tor a from the accelerometer of the smartphone. Its absolute value ||a|| is calculated and the gravity constant G is sub-

tracted from it. A lowpass filter is then applied to reduce the influence of sensor errors. Step detection is carried out on the output of the lowpass filter by counting the number of times the signal exceeds a certain threshold. This avoids spurious step counts caused by noise which may occur in the peak detection method of [12] and the zero crossing counter of [13].

The heading estimation uses the magnetometer data mand the gravity information g to calculate the rotation matrix R and the magnetometer yaw angle ψ . The matrix R is required for projecting the gyroscope data gyr into the world coordinate system and subsequently estimating the yaw angle velocity $d\psi$. A Kalman filter is used for fusing the two information sources to obtain a yaw angle estimate $\hat{\psi}$.

The movement vector v_t , which is the difference of the user position at time t and t - 1, is then calculated by combining the estimate of the number of detected steps with the movement heading estimate. To this end, an average step length has to be estimated in advance.

The PDF of the movement vector is assumed to follow a Gaussian distribution with a mean vector $\mu_{i,j} = \ell_j - \ell_i$ and a predefined diagonal covariance matrix Σ_v :

$$p(\boldsymbol{v}_t|s_t=j, s_{t-1}=i) = \frac{e^{\left(-\frac{1}{2}(\boldsymbol{v}_t-\boldsymbol{\mu}_{i,j})^T \boldsymbol{\Sigma}_v^{-1}(\boldsymbol{v}_t-\boldsymbol{\mu}_{i,j})\right)}}{\sqrt{(2\pi)^2 |\boldsymbol{\Sigma}_v|}} \quad (11)$$

5. INTRODUCTION OF PSEUDO-STATES

While the desire to limit the training effort asks for a small number of HMM states and a correspondingly large average distance between potential user positions, a large number of HMM states is preferable to have a small position quantization error and thus improved positioning accuracy.

In order to improve this tradeoff we suggest to introduce pseudo states in-between two regular HMM states of which RSSI measurements have been taken during training. By this the average distance between two allowable user positions can be reduced. These pseudo states, however, lack a model of the RSSI measurements at these positions. The emission PDF is then computed as the product of the emission PDFs of the two closest neighboring regular states, where each PDF is raised to a power which decreases linearly from one to zero as a function of the distance of the pseudo node from the regular node. More clearly, the parameters $(\mu_{\ell_j}, \sigma_{\ell_j}^2)$ of the Gaussian emission PDF of a pseudo state $s_t = j$ are computed by a weighted multiplication of those of the two neigboring states *i* and *k*, and results in

$$\mu_{\boldsymbol{\ell}_j} = \frac{\sigma_{\boldsymbol{\ell}_k}^2 d(\boldsymbol{\ell}_k, \boldsymbol{\ell}_j) \mu_{\boldsymbol{\ell}_i} + \sigma_{\boldsymbol{\ell}_i}^2 d(\boldsymbol{\ell}_j, \boldsymbol{\ell}_i) \mu_{\boldsymbol{\ell}_k}}{\sigma_{\boldsymbol{\ell}_i}^2 d(\boldsymbol{\ell}_j, \boldsymbol{\ell}_i) + \sigma_{\boldsymbol{\ell}_k}^2 d(\boldsymbol{\ell}_k, \boldsymbol{\ell}_j)}.$$
 (12)

$$\sigma_{\boldsymbol{\ell}_j}^2 = \frac{\sigma_{\boldsymbol{\ell}_k}^2 \sigma_{\boldsymbol{\ell}_i}^2 d(\boldsymbol{\ell}_i, \boldsymbol{\ell}_k)}{\sigma_{\boldsymbol{\ell}_i}^2 d(\boldsymbol{\ell}_j, \boldsymbol{\ell}_i) + \sigma_{\boldsymbol{\ell}_k}^2 d(\boldsymbol{\ell}_k, \boldsymbol{\ell}_j)}.$$
(13)

Here, $d(\ell_i, \ell_j)$ denotes the Euclidian distance between ℓ_i and ℓ_j .

Note that during testing RSSI measurements are taken at

regular time instances, thus, the position at which the RSSI is measured may not coincide with a regular HMM state.

6. EXPERIMENTAL RESULTS

In the following we are going to evaluate the effectiveness of the proposed modified HMM for an indoor localization problem. We will first consider artificially generated data and then present results on real field data.

6.1. Artificial Data

Fig. 3 shows the floor plan of our department at the University of Paderborn consisting of 10 office rooms, a long aisle and a staircase area having an overall size of 34 m by 23 m (a), with the locations of the regular and pseudo states (b) marked with red and green circles, respectively.



Fig. 3. Floor plan displaying the HMM states, i.e., the allowable user positions, and the transitions between them. Red and green circles indicate regular and pseudo states, respectively.

The RSSI measurements of 15 randomly placed APs for training the Gaussian emission PDF for the regular HMM states are generated artificially as follows: The signal strength follows a large-scale log-normal fading model with an additional zero mean Gaussian random variable with standard deviation $\sigma_L = 5$ to model small-scale fading. At each position, we collected a set of 300 RSSI measurements as the training data, then estimated the parameters of RSSI distribution of each AP at each position as described in Section 3.

The step detection information is modeled as a bivariate Gaussian with the mean $\mu_{i,j} = \ell_j - \ell_i$ and a diagonal covariance matrix Σ_v with entries $\Sigma_v[1,1] = \Sigma_v[2,2] = 0.25 \,\mathrm{m}^2$. This value has been determined in offline experiments.

In our experiment, we introduced pseudo states such that the Euclidean distance between neighboring states is not more than at most $0.75 \,\mathrm{m}$ (which is close to the measured average step length within the experimental data), while the distance between two neighboring regular states is about 3-5 m except for some special areas such as the stairs. The total number of pseudo states is 125, which has to be compared to the number of 81 regular states.

In the experiments we assume that a user cannot move faster than 3 m/s. User movements are simulated by a ran-

Table 1. Mean positioning error on artificial data

Method	Mean error [m]
RSSI only [11]	1.74
RSSI + step det. [10]	1.37
RSSI + step det. + pseudo states (here)	1.02

dom walk on the HMM graph. Since movement vectors and RSSI measurements are generated every 1.5 s, only a limited number U of HMM states can be reached from any given state $s_{t-1}=i$ (on the average U=15 states) and the corresponding transition probability $P(s_t=j|s_{t-1}=i)$ is set to $\frac{1}{U}$. For all states outside this neighborhood the corresponding transition probability is set to zero.

Table 1 presents the mean positioning error in meters, averaged over 100 experiments, where each experiment corresponds to a different random walk on the HMM grid of length of about 200 m. We compare the performance of the proposed algorithm with our earlier work of [10], which also fused RSSI and step detection information, however without the introduction of pseudo states. The use of pseudo states improves the mean positioning error from $1.37 \,\mathrm{m}$ to $1.02 \,\mathrm{m}$. If step detection information is neglected and user positioning relies only on RSSI information, a mean positioning error of $1.74\,\mathrm{m}$ is obtained.

6.2. Field Data

We have tested our approach with the field data recorded in the office building depicted in Fig. 3. In the training phase, we collected 100 RSSI measurements per position and estimated the RSSI distribution as described in Section 3. In the online testing phase, the smartphone user randomly went through the whole floor area to collect the test data. Two different trajectories were recorded, each trajectory consists of approximately 140 test potitions. The position estimate was performed every 1.5 s using the proposed approach. The step detection information is modeled in the same fashion as in the experiments using artificial data.

From our experimental results, we realized that the step detection information is much more reliable than the RSSI information, so we decided to introduce a heuristic weighting factor γ in the calculation of the forward variable as follows

$$\alpha_{t}(j) = [p(\boldsymbol{o}_{t}|s_{t}=j)]^{\gamma} \\ \cdot \sum_{i} [p(\boldsymbol{v}_{t}|s_{t}=j, s_{t-1}=i)]^{(1-\gamma)} \cdot P(s_{t}=j|s_{t-1}=i) \cdot \alpha_{t-1}(i).$$
(14)

For the determination of γ a jackknife procedure was employed: The data of one out of the 2 trajectories was used for the estimation of γ , whereas tests were conducted on the held-out data. This was repeated 2 times every time one trajectory was used to estimate γ . In our case, the estimated value was always $\gamma \approx 0.003$.

We compared the proposed approach to our earlier work

of [10] and [11]. Note that the weighting was also applied when implementing [10]. There, the weight was approximately 0.003. For both trajectories the experimental results showed that our new approach outperforms the others, especially for the 90% error quantile, in terms of the cumulative distribution function (CDF) of the positioning error which is defined as the probability that the positioning error ϵ is lower than a certain distance d:

$$CDF_{\epsilon}(d) = P(\epsilon \le d) \quad d \ge 0.$$
 (15)

Although the test area is limited, the experimental results in Fig. 4 indicate that the proposed approach is significantly better than the other approaches.



Fig. 4. CDF of the positioning error for different systems. Average over 2 test trajectories.

7. CONCLUSIONS

In this paper we have presented a modified HMM and Forward algorithm for indoor positioning to fuse the RSSI and step detection information. Through the introduction of pseudo states of which the emission probabilities are synthesized from those of the neighboring regular states, the quantization error introduced by discretizing to possible user locations to the positions corresponding to the HMM states could be significantly reduced. The performance of the approach was first validated on artificially generated data and subsequently tested on real field data of an experimental indoor positioning system. Improved positioning accuracy was observed, compared to our earlier work.

8. REFERENCES

- C. Figuera, J.L. Rojo-Álvarez, M. Wilby, I. Mora-Jiménez, and A.J. Caamaño, "Advanced support vector machines for 802.11 indoor location," *Signal Processing*, vol. 92, no. 9, pp. 2126–2136, September 2012.
- [2] E. Martin, O. Vinyals, G. Friedland, and R. Bajcsy, "Precise indoor localization using smart phones," in

Proc. of the Int. Conf. on Multimedia, Florence, Italy, 2010, ACM.

- [3] A. Rozyyev, H. Hasbullah, and F. Subhan, "Combined K-Nearest Neighbors and Fuzzy Logic Indoor Localization Technique for Wireless Sensor Network," *Research Journal of Information Technology*, vol. 4, no. 4, pp. 155–165, 2012.
- [4] P. Bahl, V.N. Padmanabhan, and A. Balachandran, "Enhancements to the RADAR User Location and Tracking System," Tech. Rep., Microsoft Research, 2000.
- [5] P. Bahl and V.N. Padmanabhan, "RADAR: An In-Building RF-Based User Location and Tracking System," in *INFOCOM 2000*, Tel Aviv, Israel, March 2000, IEEE.
- [6] A. Agiwal, P. Khandpur, and H. Saran, "Locator: location estimation system for wireless LANs," in *Proc. of the 2nd ACM Int. Workshop on Wireless mobile applications and services on WLAN hostpots*, Philadelphia, USA, September 2004, ACM.
- [7] K. Altun and B. Barshan, "Pedestrian dead reckoning employing simultaneous activity recognition cues," *Measurement Science and Technology*, vol. 23, no. 2, January 2012.
- [8] H. Leppäkoski and J. Collin and J. Takala, "Pedestrian Navigation based on Inertial Sensors, Indoor Map, and WLAN Signals," in *Proc. ICASSP*, Kyoto, March 2012, IEEE.
- [9] J. Seitz and T. Vaupel and S. Meyer and J. G. Boronat and J. Thielecke, "A Hidden Markov Model for pedestrian navigation.," in *Proc. Workshop on Positioning*, *Navigation and Communication*, Dresden, March 2010.
- [10] K. Hoang, S. Schmitz, C. Drueker, H. Tran, J. Schmalenstroeer, and R. Haeb-Umbach, "Server based Indoor Navigation using RSSI and Inertial Sensor Information," in *Proc. Workshop on Positioning, Navi*gation and Communication, Dresden, March 2013.
- [11] K. Hoang and R. Haeb-Umbach, "Parameter Estimation and Classification of Censored Gaussian Data with Application to WiFi Indoor Positioning," in *Proc. ICASSP*, Vancouver, May 2013, IEEE.
- [12] M. Mladenov and M. Mock, "A step counter service for Java-enabled devices using a built-in accelerometer," in Proc. of the 1st Int. Workshop on Context-Aware Middleware and Services, Dublin, June 2009, ACM.
- [13] W. Kang, S. Nam, and Y. Han, "Improved Heading Estimation for Smartphone-based Indoor Positioning Systems," in *Proc. IEEE Int. PIMRC*, Sydney, Australia, September 2012.