A GENERALIZED NASH GAME FOR ENERGY EFFICIENCY OPTIMIZATION: A QUASI VARIATIONAL INEQUALITY APPROACH

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ABSTRACT

This paper presents the Nash equilibrium problem for a multipoint-to-multipoint network whose nodes aim at maximizing their energy efficiency. The original game is reformulated as a max-rate generalized Nash equilibrium problem with coupling among the strategy sets of the players, thus revealing the relationship between the spectral efficiency- and the energy efficiency-based equilibrium points in such a competitive scenario. This general game is then modeled resorting to the quasi-variational inequality framework. This provides us with all the mathematical tools necessary to analyze the features of the equilibrium points and devise novel distributed algorithms.

1. INTRODUCTION

Seamless and pervasive wireless communication systems has paved the way to an extraordinary proliferation of network infrastructures with the consequent dramatic escalation of energy demands. To cope with this challenge, green radio research direction, encompassing advanced physical layer techniques, new network architectures, as well as radio resource management (RRM) techniques, has become an important trend in both the academic and industrial worlds [1]. In particular, green radio concept calls for novel dynamic RRM strategies incorporating the cost of energy or power in the objective function. Towards this goal, the concept of link capacity per unit cost has been originally proposed by [2], and successively extended for different wireless systems targeting the maximization of the number of bits/sec reliably transmitted per energy unit (e.g. see [3], [4], [5]).

Moreover, in the past years, an intense research activity has been devoted to the design of wireless devices capable of self-enforcing the negotiated agreements on the resource usage. Not surprisingly, the natural theoretical tool for designing decentralized strategies in such scenarios has been identified in game theory. In 2008, the work of Scutari et al., [6], proposed an accurate analysis of the Nash equilibrium (NE) point for a group of wireless devices targeting the maximization of their individual spectral efficiency (SE) in parallel Gaussian multi access channels, or max-SE game for short. This approach seems to be well tailored to green small cell networks (SCNs) where low-power base stations are capable of self-adapting their transmission parameters in a decentralized manner, [7]. More recently, in [5], the authors investigated the same scenario and analyzed the NE problem for a group of players aiming at maximizing of their own energy efficiency (EE), which we refer to as max-EE game.

However, [5] do not provide any insights into the relationship between max-SE and max-EE equilibrium points and their possible trade-offs. The need of a deep understanding of the relationship between SE and EE in such a distributed scenario is what motivates our contribution. In particular, the main objective of this work is to demonstrate that the max-EE game and max-SE game are two particular cases of a generalized NE (GNE) problem whose utility functions are the users' rates. Under this perspective, we provide existence results for the GNE, as well as a distributed power allocation algorithm, which remain valid for all the possible instances of this general game. This includes the case of *application sensitive* behavior of the players, where each player can choose to maximize either the spectral efficiency, or the energy efficiency, according to the particular application associated to the codeword to be transmitted.

Toward this goal, we will resort to the quasi variational inequality (QVI) framework. QVIs were introduced by Bensoussan in [8] as a modeling tool capable of describing equilibrium situations in different fields such as generalized Nash games, economics, and biology (see [9] and reference therein). Unlike the traditional VI problems, which has an extensive literature also in the field of wireless communications (see [10] for example), contributions devoted to the numerical solution of QVIs are relatively recent [11].

2. GAME FORMULATION

In this section, we model the strategic interactions of a group of Q transmitter-receiver pairs aiming at maximizing the

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number of bits/sec that can be reliably transmitted per unit of energy as a generalized Nash equilibrium (GNE) problem.

We assume a distributed system, with neither a centralized control nor coordination among users, where each link is modeled as a frequency selective block channel $\mathbf{H}_{r,q} = \mathrm{Diag}([H_{r,q}^{(1)}, H_{r,q}^{(2)}, \cdots, H_{r,q}^{(N)}]^T)$, with $r, q \in \mathcal{Q}$, $\mathcal{Q} \triangleq \{1, 2, \cdots, Q\}$ being the set of users in the network. The baseband signal received by the qth terminal through N parallel interference channels is

$$\mathbf{y}_q = \mathbf{H}_{q,q}\mathbf{B}_q\mathbf{s}_q + \sum_{r\neq q}\mathbf{H}_{r,q}\mathbf{B}_r\mathbf{s}_r + \mathbf{n}_q, \tag{1}$$

where $\mathbf{s}_q$ is the signal vector, $\mathbf{n}_q$ is a zero-mean circularly symmetric complex Gaussian white noise with covariance $\sigma_q^2\mathbf{I}$, and $\mathbf{B}_q \triangleq \sqrt{\mathrm{Diag}(\mathbf{p_q})}$, $\mathbf{p_q} = [p_q^{(1)}, p_q^{(2)}, \cdots, p_q^{(N)}]^T$ representing the power allocation vector at the qth transmitter. Moreover, individual channel state information (CSI) is assumed to be available at both transmitter and receiver sides of each link. Under these assumptions, the achievable SE on the qth link is given by

$$R_q(\mathbf{p}_q, \mathbf{p}_{-q}) = \frac{1}{N}\log\left(\left|\mathbf{I} + \mathbf{H}_{q,q}^H\mathbf{B}_q^H\mathbf{\Omega}_q^{-1}\mathbf{B}_q\mathbf{H}_{q,q}\right|\right), \tag{2}$$

where $\mathbf{\Omega}_q = \sigma_q^2\mathbf{I} + \sum_{r\neq q}\mathbf{H}_{r,q}\mathbf{B}_r\mathbf{B}_r^H\mathbf{H}_{r,q}^H$, and $\mathbf{p_{-q}}$ is the collection of strategies played by all the users but the qth. Hence, the EE utility function is defined as

$$u_q(\mathbf{p}_q, \mathbf{p}_{-q}) \triangleq \frac{R_q(\mathbf{p}_q, \mathbf{p}_{-q})}{(\Psi_q + \mathbf{1}^T\mathbf{p}_q)/N}, \tag{3}$$

where Ψ_q represents the circuit power. Based on (3), we define the EE-based power allocation game for a group of rational selfish players whose strategies are limited by the available transmit power, i.e. $\mathbf{p}_q \in \mathcal{P}_q$, with $\mathcal{P}_q = \left\{\mathbf{p}_q \in \Re^N : \sum_{n=1}^N p_q^{(n)} \leq P_t\right\}$. Stated formally, the max-EE game is a triplet $\mathcal{G} =< \mathcal{Q}, \mathcal{P}, \mathcal{U} >$ where $\mathcal{Q}$ is the set of players, $\mathcal{P} \triangleq \mathcal{P}_1 \times \mathcal{P}_2 \times \cdots \times \mathcal{P}_Q$ represents the strategy set, and $\mathcal{U} \triangleq \{u_q(\cdot)\}_{q=1}^Q$ is the set of utility functions. The strategy profile corresponding to an NE of the game is the joint solution of the optimization problems

$$(\mathcal{G}1): \quad \begin{array}{cl} \max\limits_{\mathbf{p}_q} & u_q(\mathbf{p}_q, \mathbf{p}_{-q}) \\ \text{s.t.} & \mathbf{p}_q \in \mathcal{P}_q \end{array} \qquad \forall q \in \mathcal{Q}.$$

It is worth noting that each user has to maximize an objective function in the form

$$u_q(\mathbf{p}_q, \mathbf{p}_{-q}) = \frac{f_c(\mathbf{p}_q, \mathbf{p}_{-q})}{f_l(\mathbf{p}_q)}, \tag{4}$$

where $f_c(\cdot)$ is concave in the first argument $\mathbf{p}_q$ and $f_l(\cdot)$ is a linear function. According to the *parameter free* convex program approach, originally proposed in [12], let us define

$$\mathbf{y}_q \triangleq \frac{1}{f_l(\mathbf{p_q})}\mathbf{p}_q, \qquad t_q \triangleq \frac{1}{f_l(\mathbf{p_q})}, \tag{5}$$

and write down the following equivalent problem

$$(\mathcal{P}1): \quad \begin{array}{cl} \max\limits_{\mathbf{y}_q/t_q} & f_c(\mathbf{y}_q/t_q, \mathbf{p}_{-q}), \\ \text{s.t.} & t_q\left(\sum_{n=1}^N y_q^{(n)}/t_q - P_t\right) \leq 0, \\ & t_q\left(\Psi_q + \sum_{n=1}^N y_q^{(n)}/t_q\right) \leq 1. \end{array}$$

Clearly, the original fractional program is solved at the condition that the parameter free convex program $(\mathcal{P}1)$ is solved too. Following the same procedure as in [4], we get that $({\mathbf{y}^*}_q, t_q^*)$ is a solution for $(\mathcal{P}1)$ if

$${y^*}_q^{(n)}/t_q^* = \left[\frac{1}{\lambda_q} - \frac{\sigma_q^2 + \sum_{r\neq q}\left|H_{r,q}^{(n)}\right|^2 p_r^{(n)}}{\left|H_{q,q}^{(n)}\right|^2}\right]^+, \tag{6}$$

where, by definition, $t_q^* \in (0, 1/\Phi_q]$. Hence, defining the variable $\mathbf{z}_q = {\mathbf{y}^*}_q/t_q^*$, $\mathbf{z}_q$ must satisfy the waterfilling equation (6). Moreover, due to the complementary slackness, $\lambda_q^* \neq 0$ if and only if

$$\sum_{n=1}^N z_q^{(n)} = \tilde{P}_q, \tag{7}$$

with

$$\tilde{P}_q \triangleq \min\left\{P_t, \frac{1}{t_q^*} - \Psi_q\right\}. \tag{8}$$

According to [6, Lemma1], we can conclude that for every possible value of t_q^*, the slack variable $\mathbf{z}_q$ can be interpreted as the Euclidean projection of the vector

$$\Theta_q^{(n)} \triangleq -\frac{\sigma_q^2 + \sum_{r\neq q}\left|H_{r,q}^{(n)}\right|^2 p_r^{(n)}}{\left|H_{q,q}^{(n)}\right|^2}, \tag{9}$$

onto the simplex

$$\mathcal{S}_q = \left\{\mathbf{z}_q \in \Re^N : \sum_{n=1}^N z_q^{(n)} = \tilde{P}_q\right\}. \tag{10}$$

Few remarks are now in order: *i)* the max-EE player q's best response to $\mathbf{p}_{-q}$, denoted with $\mathbf{p}_q^*$, is an instance of $\mathbf{z_q}$. It follows that $\mathbf{p}_q^*$ is the Euclidean projection of Θ_q onto the set $\mathcal{S}_q$; *ii)* the only difference between the max-SE player q's best response analyzed in [6] and $\mathbf{p}_q^*$ is that, in the latter case, $\mathcal{S}_q$ is not determined by the available transmit power P_t as in the former case, but it depends on t_q^*, which is an outcome of $(\mathcal{P}1)$. Hence, since t_q^* depends on the other players' strategies, from now on, the set $\mathcal{S}_q$ will be denoted as $\mathcal{S}_q(\mathbf{p}_{-q})$; *iii)* the interpretation of $\mathbf{p}_q^*$ as a projection is independent on how the parameter t_q^* is obtained. The problem of evaluating t_q^* is addressed in Section 4.1.

Taking advantage of the above results, we model the max-EE game as stated in the following proposition.

Proposition 1. Let $\mathcal{K}$ be a point-to-set map given by

$$\mathcal{K}(\mathbf{p}) = \prod_{q=0}^{Q-1} \mathcal{K}_q(\mathbf{p}_{-q}), \quad (11)$$

where

$$\mathcal{K}_q(\mathbf{p}_{-q}) \triangleq \left\{\mathbf{p}_q \in \Re^N : w_q(\mathbf{p}_q, \mathbf{p}_{-q}) \leq 0, \quad v_q(\mathbf{p}_q) \leq 0\right\}. \quad (12)$$

If $w_q(\mathbf{p}_q, \mathbf{p}_{-q})$ and $v_q(\mathbf{p}_q)$ are two affine functions of $\mathbf{p}_q$ defined as

$$w_q(\mathbf{p}_q, \mathbf{p}_{-q}) \triangleq \sum_{n=0}^{N-1} p_q^{(n)} - \frac{1}{t_q^*} + \Psi_q, \quad (13)$$

and

$$v_q(\mathbf{p}_q) \triangleq \sum_{n=0}^{N-1} p_q^{(n)} - P_t, \quad (14)$$

then the NE of the max-EE game corresponds to the solution of the QVI problem denoted by QVI($\mathcal{K}, F$), which is to find a vector $\mathbf{p}^*$ such that

$$(\mathbf{p} - \mathbf{p}^*)^T \, F(\mathbf{p}^*) \geq 0 \quad \forall \mathbf{p} \in \mathcal{K}(\mathbf{p}^*), \quad (15)$$

where $\mathbf{p} \triangleq [\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_Q]$, and $F : \Re^{NQ} \to \Re^{NQ}$ is obtained by collecting the mappings $F_q(\mathbf{p}_q, \mathbf{p}_{-q}) = \nabla_{\mathbf{p}_q} R_q(\mathbf{p}_q, \mathbf{p}_{-q})$.

Proof. Since $w(\mathbf{p}_q, \mathbf{p}_{-q})$ and $v(\mathbf{p}_q)$ are convex and continuously differentiable with respect to $\mathbf{p}_q$, (15) has solution if and only if there exist some multipliers $\boldsymbol{\lambda} \triangleq [\lambda_1, \lambda_2, \cdots, \lambda_Q]^{\mathrm{T}}$ and $\boldsymbol{\mu} \triangleq [\mu_1, \mu_2, \cdots, \mu_Q]^{\mathrm{T}}$ such that the KKT systems

$$F_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*) - \lambda_q \nabla_{\mathbf{p}_q} w_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*) - \mu_q \nabla_{\mathbf{p}_q} v_q(\mathbf{p}_q^*) = 0$$

$$(S1): \qquad \begin{array}{ll} \lambda_q \geq 0, \;\; \mu_q \geq 0 & (S1.\mathrm{a}) \\ \mu_q v_q(\mathbf{p}_q^*) = 0 & (S1.\mathrm{b}) \\ \lambda_q w_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*) = 0 & (S1.\mathrm{c}) \end{array}$$

hold $\forall q \in \mathcal{Q}$. From now on, for the sake of briefness we shall omit trivial mathematical details, just providing a global picture of the proof. First of all, recalling (9), (13), and (14), let us rewrite the first equation of (S1) in the form

$$p^{*(n)}_q - \Theta_q^{(n)} - \nu_q = 0 \quad \forall q \in \mathcal{Q}, \quad \forall 1 \leq n \leq N, \quad (16)$$

where ν_q is a strictly positive scalar. Moreover, from (S1.a)-(S1.c), we can derive that we have a solution if and only if

$$\nu_q \; g_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*) = 0, \quad (17)$$

where $g_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*) \triangleq \max\left(v_j(\mathbf{p}_q^*), w_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*)\right)$. Putting together equations (16) and (17) we obtain the dual representation of the following optimization problems

$$(\mathcal{P}2): \quad \begin{array}{ll} \min\limits_{\mathbf{p}_q} & ||\mathbf{p}_q - \Theta_q||_2^2, \\ \text{s.t.} & g(\mathbf{p}_q, \mathbf{p}_{-q}^*) = 0. \end{array} \quad \forall q \in \mathcal{Q}$$

whose solution is the NE of the max-EE game.

According to Proposition 1, it is worth remarking that the equilibrium point of the max-EE game can be seen as a generalization of the traditional max-SE game as defined in [6], which is obtained as the solution of the VI problem

$$(\mathbf{p} - \mathbf{p}^*)^T \, F(\mathbf{p}^*) \geq 0 \quad \forall \mathbf{p} \in \mathcal{K}, \quad (18)$$

where $\mathcal{K} = \prod_{q=1}^{Q} \mathcal{K}_q$, and $\mathcal{K}_q \triangleq \left\{\mathbf{p}_q \in \Re^N : v_q(\mathbf{p}_q) \leq 0\right\}$. An important consequence of proposition 1 stems from the connection between QVIs and generalized NE (GNE) revealed in [8] by Bensoussan. Differently from a traditional NE problem, in GNE problems, the strategy set depends on the particular strategy profile and each player must solve a mathematical program with equilibrium constraints. From (15) and (18), it can be argued that the max-EE game and the max-SE game are two particular cases of the same GNE problem which can include a number of different scenarios. As an example, we just mention the case of application sensitive behavior of the players, wherein each player can maximize his/her SE or his/her EE according to the particular application associated to the codeword to be transmitted. In the next subsection, we will provide sufficient conditions for such a GNE problem to have a solution.

3. EXISTENCE OF THE EQUILIBRIUM POINT

In the previous section, the original max-EE game is reinterpreted as a QVI problem thus revealing the existent connections between the SE-based and the EE-based Nash equilibria in the proposed distributed scenario. Notably, we have shown that the max-EE game and the max-SE game are two instances of a broader game whose utility functions are the users rates. Under this perspective, though the existence result for the max-EE game can be obtained by exploiting the strictly quasiconcavity of the EE function, [5], we pursue an alternative route that can be easily extended to other scenarios in which the proposed GNE problem could be applied.

In order to assess the existence of the GNE, let $\Phi_q(\mathbf{p}_{-q})$ be the set of possible solutions for player q parametrized with respect to $\mathbf{p}_{-q}$ and define the sequentially bounded constraints qualification (SBCQ) assumption as follows.

Definition 1 (SBCQ). *for any bounded sequence of vectors* $\{\mathbf{p}_q(k)\}$ *whose elements belong to* $\Phi_q(\mathbf{p}_{-q}(k))$ $\forall k$, *there exist some bounded sequences* $\{\lambda_q(k)\}$ *and* $\{\mu_q(k)\}$ *of Lagrange multipliers satisfying the qth KKT system (S1).*

According to [11], since affine constraint functions always satisfy the SBCQ assumptions, we can assess the the existence of the QVI solution resorting to [11, *Th.2*].

Theorem 1 (Solution Existence). Let $F : \Re^{NQ} \to \Re^{NQ}$ be a point-to-point map and let $\mathcal{K}$ be a point-to-set map such that

- *a)* for each allowed value of $\mathbf{p}_{-q}$ the set $\mathcal{K}_q(\mathbf{p}_{-q})$ is nonempty, $\forall q \in \mathcal{Q}$ (feasibility assumption),

- *b)* $\mathcal{K}_q(\mathbf{p}_{-q})$ is composed of affine functions such that the SBCQ assumption holds,
- *c)* the set $\mathcal{P}_q$ is nonempty and bounded, $\forall q \in \mathcal{Q}$ (compactness assumption),

then the QVI$(\mathcal{K}, F)$ has at least one solution.
In our case, assumptions *b)* and *c)* can be verified by simply looking at the definition of the constraints functions (13) and (14). Moreover, since $t_q^* \neq 0$ if the norm $||\mathbf{H}_{q,q}||_\infty$ is finite, also assumption *a)* holds, and the GNE exists.

It is worth pointing out that QVIs are inclined to present manifolds of solutions. The assessment of the uniqueness conditions is beyond the scope of this paper and we do not go into details on this here. However, let us note that if $t_q^* \leq 1/(P_t + \Psi_q)$, $\forall q \in \mathcal{Q}$, then $g_q(\mathbf{p}_q^*, \mathbf{p}_{-q}^*) = v_q(\mathbf{p}_q^*)$ and the uniqueness is guaranteed at the condition that $F(\mathbf{p})$ is strongly monotone [9], i.e. the game would have a unique equilibrium point if the matrix Ξ, defined as (see [10])

$$[\Xi]_{r,q} \triangleq \begin{cases} 1 & \text{if } q = r \\ -\max\limits_{1\leq n \leq N} \left\{ \frac{|H_{q,r}^{(n)}|^2}{|H_{r,r}^{(n)}|^2} \varsigma_{q,r}^{(n)} \right\} & \text{if } q \neq r \end{cases} \quad (19)$$

with

$$\varsigma_{q,r}^{(n)} \triangleq -\frac{\sigma_r^2 + \sum_{r'} \left| H_{r,r'}^{(n)} \right|^2 P_t}{\sigma_q^2} \quad (20)$$

is positive definite.

4. A SEQUENTIAL PENALTY APPROACH TO THE EE EQUILIBRIUM PROBLEM

In this section we propose a solution method which relies on a sequential penalty approach. The key idea is to achieve the GNE through the solution of a sequence of penalized VIs, that is to say a sequence of equilibrium problems on the set $\mathcal{P}$ where mappings $F_q(\mathbf{p})$ are decreased by a penalty term.

In particular, as advocated by Pang and Fukushima in [11], a viable route to cope with the non-standard constraint $w_q(\mathbf{p}_q, \mathbf{p}_{-q})$ is to reduce the utility function of each player via a smooth quadratic term. Stated formally, let $\{\rho_q(k)\}$ be a sequence of positive scalars satisfying $\rho_q(k) < \rho_q(k+1)$ and tending to ∞. Given a sequence $\{\alpha_q(k)\}$ whose elements are

$$\begin{aligned} \alpha_q(k) \triangleq \max\{0, \alpha_q(k-1) + \rho_q(k-1) \\ \cdot w(\mathbf{p}_q(k-1), \mathbf{p}_{-q}(k-1))\}, \end{aligned} \quad (21)$$

we generate the sequence of iterates $\{\mathbf{p}(k)\}$ as the joint solution of the following optimization problems

$$(\mathcal{G}2): \quad \begin{array}{ll} \max\limits_{\mathbf{p}_q(k)} & R_q(\mathbf{p}_q(k), \mathbf{p}_{-q}(k)) - C_q(\mathbf{p}_q(k), \mathbf{p}_{-q}(k)) \\ \text{s.t.} & \mathbf{p}_q(k) \in \mathcal{P}_q \end{array}$$

$\forall q \in \mathcal{Q}$, where

$$\begin{aligned} C_q(\mathbf{p}_q(k), \mathbf{p}_{-q}(k)) \triangleq \tfrac{1}{2\rho_q(k)}[\max\{0, \alpha_q(k) \\ +\rho_q(k) w(\mathbf{p}_q(k), \mathbf{p}_{-q}(k))\}]^2 \end{aligned} \quad (22)$$

represents the penalty function. Since the squared max function $C_q(\mathbf{p}_q(k), \mathbf{p}_{-q}(k))$ is once continuously differentiable, the KKT systems for the NE problem $(\mathcal{G}2)$ are:

$$(S2): \quad \begin{array}{l} F_q(\mathbf{p}_q^*(k), \mathbf{p}_{-q}(k)) - \alpha_q(k+1)\nabla_{\mathbf{p}_q} w_q(\mathbf{p}_q^*(k), \mathbf{p}_{-q}(k)) \\ \qquad -\mu_q(k)\nabla v_q(\mathbf{p}_q^*(k)) = 0, \\ \mu_q(k) \geq 0, \qquad (S2.\text{a}) \\ \mu_q(k) v_q(\mathbf{p}_q(k)) = 0. \quad (S2.\text{b}) \end{array}$$

For the sake of brevity, we must omit the proof of convergence for the proposed method. Nevertheless, this can be easily verified by proving that (S2) is a particular case of the more general problem tackled in [11], for which convergence is assessed through [11, *Th.*3]. Accordingly, $\mathbf{p}(\infty)$ solves the original QVI problem thus being a GNE of the game.

4.1. Distributed Algorithm and Numerical Results

Capitalizing on the proposed penalty method, we propose the following distributed power allocation algorithm. Define with $\mathrm{T}_q : \Re^{NQ-1} \rightarrow \Re^N$ the operator solving the system (S2), such that $\mathbf{p}_q^* = \mathrm{T}_q(\mathbf{p}_{-q})$. At the generic iteration k, each player computes $\mathbf{p}_q^*(k)$ as $\mathbf{p}_q^*(k) = \mathrm{T}_q(\mathbf{p}_{-q}(k-1))$ following this procedure: *i)* Compute the parameter $t_q^*(k)$ via the Dinkelbach algorithm depicted in Table 1 where ϵ represents the tolerance of the algorithm; *ii)* given $t_q^*(k)$, built the function $w_q(\mathbf{p}_q(k), \mathbf{p}_{-q}(k-1))$ and solve (S2); *iii)* repeat *i)* and *ii)* until the maximum number of iterations is reached, i.e. $k = N_{\text{it}}$.

Fig.1 shows the EE performance of the proposed algorithm (solid lines) versus the number of iterations, for a multipoint-to-multipoint channel composed of $Q = 8$ active users and $N = 16$ parallel subchannels. We assumed $P_t = N = 16$, $\Psi_q = 1$ and $\sigma_q = 1$,$\forall q \in \mathcal{Q}$. The average signal-to-interference ratio (SIR) at all receivers is equal to 8 dB, while the per sub-channel average SNR is set to 0 dB. For the sake of comparison, we also report the energy efficiency achieved with the traditional max-rate distributed strategy (dotted lines) proposed in [6]. Fig.2 illustrates the total power consumption of the proposed max-EE algorithm versus the number of iterations, showing a great saving of power when compared to the max-rate strategy which always transmits at the maximum power ($P_t + \Psi_q = 17$).

5. CONCLUDING REMARKS

In this paper, we considered the competitive maximization of the EE for a multipoint-to-multipoint transmission. We first characterized the proposed game as a QVI which has been associated to an equivalent GNE problem. After assessing the existence of the GNE, we pointed out that the solution of EE equilibrium problem can be seen as a generalization of the maximum rate strategy. Eventually, we proposed a sequential

Initialize $j = 0$, $\lambda_q = \lambda_0$,

Set $\Theta_q^{(n)} = -\frac{\sigma_q^2 + \sum_{r \neq q} |H_{r,q}^{(n)}|^2 p_r^{(n)}(k-1)}{|H_{q,q}^{(n)}|^2}$, $\forall 1 \leq n \leq N$

Do

  Set $j \leftarrow j + 1$,

  Calculate $z_q^{(n)}(j) = [1/\lambda_q + \Theta_q^{(n)}]^+, \forall 1 \leq n \leq N$

  Set $\lambda_q = \frac{R_q(\mathbf{z}_q(j), \mathbf{p}_{-q}(k-1))}{\Psi_q + \mathbf{1}^T \mathbf{z}_q(j)/N}$

While $||R_q(\mathbf{z}_q(j), \mathbf{p}_{-q}(k-1)) - \lambda_q(\Psi_q + \mathbf{1}^T \mathbf{z}_q(j)/N)|| > \epsilon$

Output $t_q^*(k) = 1/(\Psi_q + \sum_{n=1}^N z_q^{(n)}(j))$

Table 1. Dinkelbach method for max-EE game

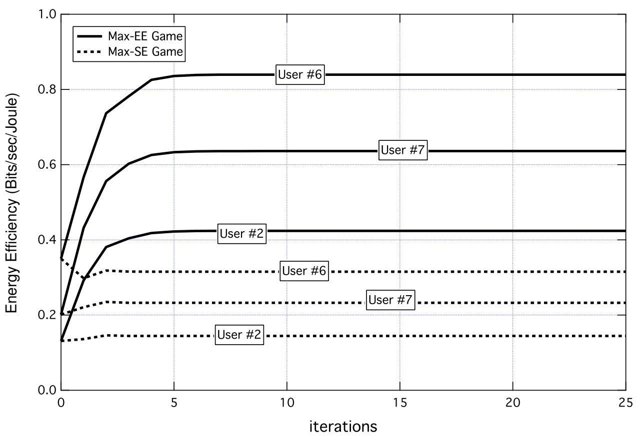

Fig. 1. Max-EE Vs Max-Rate. Energy Efficiency.

penalty approach to reach the GNE and, based on this, we proposed a distributed power allocation algorithm.

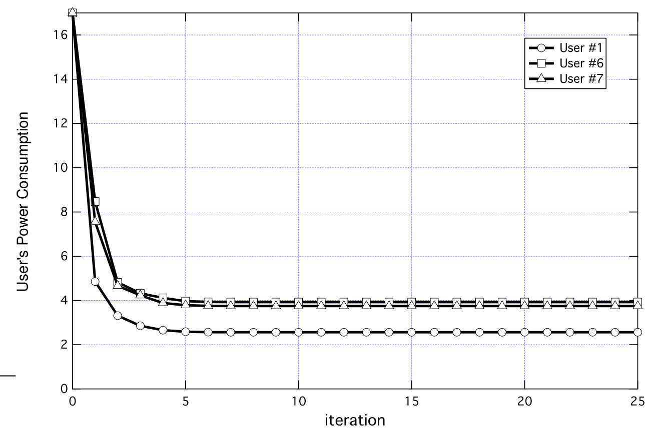

Fig. 2. Max-EE. Total Power Consumption.

6. REFERENCES

[1] D. Feng, C. Jiang, G. Lim, L.J. Cimini Jr., G. Feng, and G.Y. Li, "A survey of energy-efficient wireless communications," *IEEE Communications Surveys & Tutorials*, vol. 15, pp. 167–178, First Quarter 2013.

[2] S. Verdu, "On channel capacity per unit cost," *IEEE Trans. on Inf. Theory*, vol. 36, pp. 1019–1030, Sept. 1990.

[3] E.V. Belmega and S. Lasaulce, "Energy-efficient precoding for multiple-antenna terminals," *IEEE Trans. on Signal Proc.*, vol. 59, no. 1, pp. 329–340, 2011.

[4] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, "Framework for link-level energy efficiency optimization with informed transmitter," *IEEE Trans. on Wireless Commun.*, vol. 11, pp. 2946–2957, Aug. 2012.

[5] G. Miao, N. Himayat, G. Y. Li, and S. Talwar, "Distributed interference-aware energy-efficient power optimization," *IEEE Trans. on Wireless Commun.*, vol. 10, pp. 1323–1333, April 2011.

[6] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband noncooperative systems based on game theorypart ii: Algorithms," *IEEE Trans. on Signal Proc.*, vol. 56, pp. 1250–1267, March 2008.

[7] J. Hoydis, M. Kobayashi, and M. Debbah, "Green small networks: A cost- and energy-efficient way of meeting the future traffic demands," *IEEE Vehic. Technol. Mag.*, vol. 6, pp. 37–43, March 2011.

[8] A. Bensoussan, "Points de nash dans le cas de fonctionnelles quadratiques et jeux differentiels lineaires a n personnes," *SIAM J. Control*, vol. 12, pp. 460–499, 1974.

[9] F. Facchinei, C. Kanzow, and S. Sagratella, "Solving quasi-variational in- equalities via their kkt-conditions," *Institute of Mathematics, University of Wurzburg*, Jan. 2012.

[10] J.S. Pang, G. Scutari, D.P. Palomar, and F. Facchinei, "Design of cognitive radio systems under temperature-interference constraints: A variational inequality approach," *IEEE Trans. on Signal Proc.*, vol. 58, pp. 3251–3271, June 2010.

[11] J.S. Pang and M. Fukushima, "Quasi-variational inequalities, generalized nash equilibria, and multi-leader-follower games," *Computational Management Science*, vol. 2, pp. 21–56, 2005.

[12] S. Schaible, "Parameter-free convex equivalent and dual programs of fractional programming problems," *Zeitschrift fr Operations Research*, vol. 18, no. 5, pp. 187–196, 1974.