CONTINUOUS POWER ALLOCATION STRATEGIES FOR SENSING-BASED MULTIBAND SPECTRUM SHARING

Zhong Chen†, Xiaodong Wang‡, Fellow, IEEE, Xianda Zhang†, Senior member, IEEE

†Department of Automation, Tsinghua University, Beijing, China, 100084
‡Electrical Engineering Department, Columbia University, New York, 10027

ABSTRACT

We propose continuous power allocation strategies for secondary users (SUs) based on sensing the primary user (PU) channels in a multiband cognitive radio (CR) network. Unlike the conventional sensing-based spectrum sharing, where there are two transmit power levels corresponding to whether the PU is sensed present or not, in the proposed strategy, the power levels are continuous functions of the sensing statistics, and optimized with respect to the achievable rate of the SU. The power control process consists of two phases: in the first phase, the SU listens to the multiple bands licensed to the PU and obtains the received signal energies on these bands; in the second phase, the SU adjusts its transmit power levels on these bands based on the sensing results. Simulation results demonstrate that the proposed strategies can significantly improve the achievable throughput of the SU compared to the conventional methods.

Index Terms— Cognitive radio (CR), power allocation, multiband spectrum sensing, underlay, opportunistic spectrum access.

1. INTRODUCTION

As a promising solution to the spectrum scarcity problem, the cognitive radio (CR) system has received much attention lately [1], where the secondary user (SU) can access the primary bands without degrading the quality of service (QoS) of the primary user (PU). Currently, there exist three main spectrum access approaches for CR: i) Underlay or the so-called spectrum sharing scheme, where the SU is allowed to coexist with the PU as long as the QoS of the PU is protected [2]; ii) Opportunistic spectrum access, where the SU can only access the primary bands that are detected to be idle [3]-[4]; and iii) sensing-based spectrum sharing, where the SU first senses the frequency spectrum to determine the status of the PU and then chooses its transmit power based on the decision [5]-[6].

According to both theoretical analysis and simulations, sensing-based spectrum sharing achieves the maximum secondary rate under a given interference constraint to the PU [7]. This approach consists of two phases: sensing and data transmission. During the sensing slot, the SU performs spectrum sensing and determines whether the PU is absent or not. During data transmission, the SU accesses the primary band with a high transmit power when the PU is determined to be absent and with a low transmit power otherwise, in order to control the interference caused to the PU. Clearly, all three spectrum access approaches essentially adopt either constant or binary power allocation. However, such strategies are by no means optimal, and by allowing the transmit power to be continuous, the achievable rate can be significantly increased.

In this paper, we propose a continuous power allocation framework based on the SU's sensing statistics in CR networks. The conventional constant or binary power allocations are special cases of the proposed strategy. The power allocation process is composed of a sensing slot and transmission slot. In the first slot, some sensing statistics about the PU are collected based on which the transmit power is determined; in the second slot, the SU transmits data using the power level obtained in the sensing slot. Under several possible combinations of the peak/average transmit power constraints at the SU and the peak/average interference power constraints at the PU, the optimal power allocation functions are derived to maximize the average achievable rate at the SU.

2. SYSTEM MODEL AND BACKGROUND

2.1. System Model

Consider a CR network with a pair of primary transmitter and receiver, and a pair of secondary transmitter and receiver as depicted in Fig.1. We assume that the number of multiband channels is M, and the channels are orthogonal narrowbands. Let $\gamma_{1,j}$, $\gamma_{2,j}$, h_j and g_j denote the instantaneous channel *power gains* (i.e., squared magnitudes of the complex channel gains) of channel j from the primary transmitter (PU-Tx) to the secondary transmitter (SU-Tx), from PU-Tx to the secondary receiver (SU-Rx), from SU-Tx to the primary receiver (PU-Rx) and from SU-Tx to SU-Rx, respectively.

The frame structure is based on the conventional two-phase model, namely sensing slot with duration τ and transmission slot with duration $T-\tau$, as shown in Fig.2. During the sensing slot, the SU-Tx listens to all the M narrowbands

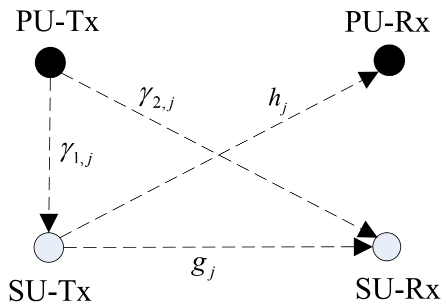

Fig. 1. System model under sensing-based spectrum access.

Frame m		Frame m+1	
τ	$T-\tau$	τ	$T-\tau$
Sensing	Data Transmission	Sensing	Data Transmission

Fig. 2. Frame structure of the cognitive radio network.

and obtains their accumulated energies. When transmitting, the SU-Tx accesses the primary multiband with the optimal powers decided by the accumulated energies in order to meet the interference constraint at the PU-Rx.

The ith received signal sample at channel j, $r_{i,j}$, is modeled as

$$r_{i,j} = \begin{cases} n_{i,j}, & H_{0,j}, \\ \sqrt{\gamma_{1,j}} s_{i,j} + n_{i,j}, & H_{1,j}, \end{cases} \tag{1}$$

where the hypothesis $H_{0,j}$ and $H_{1,j}$ corresponds to idle and busy channel j respectively; $s_{i,j}$ is the ith symbol transmitted from PU-Tx in channel j which is assumed to follow a circularly symmetric complex Gaussian distribution with zero mean and variance $P_{s,j}$, i.e., $s_{i,j} \backsim \mathcal{N}_c(0, P_{s,j})$; and $n_{i,j} \backsim \mathcal{N}_c(0, N_0)$ is the additive noise.

The detection statistic x_j for channel j based on the accumulated received signal energy can be written as

$$x_j = \sum_{i=1}^{\tau f_s} |r_{i,j}|^2, \tag{2}$$

where f_s is the sampling frequency at SU-Tx. Then the probability density functions (pdf) of x_j conditioned on $H_{0,j}$ and $H_{1,j}$ are given by [8]

$$f_0(x_j) \triangleq f(x_j|H_{0,j}) = \frac{x_j^{\tau f_s - 1} e^{-\frac{x_j}{N_0}}}{\Gamma(\tau f_s)(N_0)^{\tau f_s}}, \tag{3}$$

and

$$f_1(x_j) \triangleq f(x_j|H_{1,j}) = \frac{x_j^{\tau f_s - 1} e^{-\frac{x_j}{N_0 + \gamma_{1,j} P_{s,j}}}}{\Gamma(\tau f_s)(N_0 + \gamma_{1,j} P_{s,j})^{\tau f_s}}, \tag{4}$$

respectively, where $\Gamma(.)$ denotes the gamma function.

2.2. Conventional Sensing-based Power Allocation Strategies

In the conventional sensing-based power allocation schemes, using energy detection as the decision rule, the SU-Tx compares x_j to a threshold θ_j to decide whether channel j is occupied by the PU or not, i.e., $x_j \underset{\mathcal{H}_{0,j}}{\overset{\mathcal{H}_{1,j}}{\gtrless}} \theta_j$.

Sensing-based spectrum sharing: The SU-Tx adapts its transmit power based on the decision made during the sensing slot. If channel j is detected to be absent, the SU-Tx will transmit with high power P_j^0, otherwise, with low power P_j^1.

Opportunistic spectrum access: Different from the sensing-based spectrum sharing approach, when transmitting, if channel j is detected to be busy, the SU-Tx will not use this channel ($P_j^1 = 0$). A pre-defined threshold on the detection probability $q_{d,j}^{th}$ is chosen to protect the PU.

Underlay: The SU can coexist with the PU under the condition of meeting the QoS of the PU without sensing.

3. SENSING-BASED CONTINUOUS POWER ALLOCATION

3.1. Proposed Continuous Power Allocation Strategy

We propose to adapt the transmit power continuously with respect to the spectrum sensing variable x_j in (2). Define the transmit power for channel j as a function of the received signal energy x_j, i.e., $P(x_j)$, and obviously it satisfies the non-negative constraint as

$$P(x_j) \geq 0, \forall x_j, j. \tag{5}$$

Then the conventional power allocation rules become the special cases. For the sensing-based spectrum sharing or opportunistic spectrum access, it has the following form

$$P(x_j) = \begin{cases} P_j^0, & x_j < \theta_j, \\ 0 \text{ or } P_j^1, & x_j \geq \theta_j, \end{cases} \tag{6}$$

whereas for the underlay approach, $P(x_j)$ is a constant.

3.2. Achievable Rate, Power Constraints and Problem Formulation

The instantaneous rates of the SU given x_j for the cases of idle and busy channel j, are given by

$$R_0(x_j) = \log_2\left(1 + \frac{P(x_j) g_j}{N_0}\right), \tag{7}$$

and

$$R_1(x_j) = \log_2\left(1 + \frac{P(x_j) g_j}{N_0 + \gamma_{2,j} P_{s,j}}\right), \tag{8}$$

respectively. Then the average throughput of the SU can be written as

$$R = \frac{T-\tau}{T} \cdot \sum_{j=1}^{M} \int_0^{\infty} [p_{0,j} R_0(x_j) f_0(x_j) + p_{1,j} R_1(x_j) f_1(x_j)] dx_j,$$

where $p_{0,j}$ and $p_{1,j} = 1 - p_{0,j}$ are the idle and busy probabilities of channel j respectively.

Due to the nonlinearity of the power amplifiers, the peak transmit power has to be constrained. Let $\hat{P}_j$ be the maximum peak transmit power for channel j. Then we have

$$P(x_j) \leq \hat{P}_j, \forall x_j, j. \tag{9}$$

In order to meet the long-term power budget of the SU, an average transmit power constraint should also be considered, which can be written as

$$\frac{T-\tau}{T} \sum_{j=1}^{M} \int_0^{\infty} P(x_j) \left[p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)\right] dx_j \leq \bar{P}. \tag{10}$$

Furthermore, to protect the instantaneous QoS of the PU, peak interference power has to be constrained as

$$h_j P(x_j) \leq \hat{I}_j, \forall x_j, j, \tag{11}$$

where $\hat{I}_j$ is the peak interference power level in channel j that is tolerable by the PU.

To protect the long-term QoS of the PU, an average interference power constraint should be imposed. Setting the maximum average interference power as $\bar{I}_j$, then the average interference power constraint is

$$\frac{T-\tau}{T} \int_0^{\infty} p_{1,j} h_j P(x_j) f_1(x_j) dx_j \leq \bar{I}_j, \ \forall j. \tag{12}$$

Finally, the problem of maximizing the average achievable rate of the SU-Tx under the power constraints can be formulated as

$$\begin{aligned} &\max_{\tau, \{P(x_j)\} \in F'} \quad R \\ &\text{s.t.} \quad (5),\ 0 \leq \tau \leq T, \end{aligned} \tag{13}$$

where $F^{'}$ is the feasible set specified by a particular combination of the power constraints (9)-(12). The following lemma is instrumental to solving (13).

Lemma 1: Problem (13) is concave with respect to the transmit power $P(x_j)$ under any combination of constraints of (9)-(12) .

Proof: Limited to the page size, all of the proofs are ignored. However, we will give them in the further version.

Note that in general the rate R in (13) is a highly nonlinear and non-convex function of τ and therefore there is no efficient way of optimizing over τ. Following [7], we will simply use a one-dimensional exhaustive search within the interval $[0, T]$ to find the optimal τ.

3.3. Average Transmit Power and Average Interference Power Constraints

Consider the constraints of average transmit power at SU and average interference power at PU, i.e., (10) and (12).

First we write the Lagrangian $\mathcal{L}(P(x_j), \lambda, \boldsymbol{\mu})$ for problem (13) under the constraints (10) and (12) as

$$\begin{aligned} &\mathcal{L}(P(x_j), \lambda, \boldsymbol{\mu}) = R \\ &+ \lambda \left(\bar{P} - \frac{T-\tau}{T} \sum_{j=1}^{M} \int_0^{\infty} P(x_j) \left[p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)\right] dx_j \right) \\ &+ \sum_{j=1}^{M} \mu_j \left(\bar{I}_j - \frac{T-\tau}{T} \int_0^{\infty} p_{1,j} h_j P(x_j) f_1(x_j) dx_j \right). \end{aligned} \tag{14}$$

Define the Lagrange dual function $g(\lambda, \boldsymbol{\mu})$ corresponding to problem (13). Then we can build the dual optimization problem as

$$\min_{\lambda \geq 0,\ \boldsymbol{\mu} \geq \mathbf{0}} \ g(\lambda, \boldsymbol{\mu}) \triangleq \sup_{P(x_j) \geq 0} \ \mathcal{L}(P(x_j), \lambda, \boldsymbol{\mu}). \tag{15}$$

It follows from Lemma 1 that, the optimal value of problem (15) is equal to that of problem (13). Thus we can solve the dual optimization problem (15) instead of solving (13). In (15), we have to obtain the supremum of $\mathcal{L}(P(x_j), \lambda, \boldsymbol{\mu})$. To find the optimal $P(x_j)$, we take the derivative of $\mathcal{L}(P(x_j), \lambda, \boldsymbol{\mu})$ with respect to $P(x_j)$, which can be obtained as

$$\begin{aligned} &\frac{\partial \mathcal{L}(P(x_j), \lambda, \boldsymbol{\mu})}{\partial P(x_j)} = \\ &\frac{T-\tau}{T} \left\{ \frac{\log_2(e) p_{0,j} f_0(x_j)}{P(x_j) + N_0/g_j} + \frac{\log_2(e) p_{1,j} f_1(x_j)}{P(x_j) + (N_0 + \gamma_{2,j} P_{s,j})/g_j} \right. \\ &\left. - \lambda \left[p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)\right] - \mu_j p_{1,j} h_j f_1(x_j) \right\}. \end{aligned} \tag{16}$$

By applying the Karush-Kuhn-Tucker (KKT) conditions, the optimal power allocation $P(x_j)$ for the given Lagrange multipliers λ and $\boldsymbol{\mu}$ is given by

$$P(x_j) = \left[\frac{A_j + \sqrt{\triangle_j}}{2} \right]^+, \tag{17}$$

where $[x]^+ \triangleq \max(0, x)$, and

$$\begin{aligned} A_j &= \frac{\log_2(e) \left[p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)\right]}{\lambda \left[p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)\right] + \mu_j p_{1,j} h_j f_1(x_j)} \\ &\quad - \frac{2N_0 + \gamma_{2,j} P_{s,j}}{g_j}, \\ \triangle_j &= \left\{ \frac{\log_2(e) [p_{0,j} f_0(x_j)(N_0 + \gamma_{2,j} P_{s,j}) + p_{1,j} f_1(x_j) N_0]}{\lambda \left[p_{0,j} f_0(x_j) + p_{1,j} f_1(x_j)\right] + \mu_j p_{1,j} h_j f_1(x_j)} \right. \\ &\quad \left. - \frac{N_0(N_0 + \gamma_{2,j} P_{s,j})}{g_j} \right\} \frac{4}{g_j} + A_j^2. \end{aligned} \tag{18}$$

Proposition 1: $P(x_j)$ is a non-increasing function with respect to x_j.

Remark: An interpretation of Proposition 1 is that, with a smaller x_j, the probability that channel j is busy is smaller, thus the SU can transmit at higher power to effectively use the primary band. On the other hand, with a larger x_j, lower transmit power should be used to avoid harmful interference to the PU.

Subgradient based methods are used here to find the optimal Lagrange multipliers λ and $\boldsymbol{\mu}$, e.g., the ellipsoid method [10] or the gradient descent method[11]. In particular, for the gradient descent method, the Lagrange multipliers are updated according to the following

$$\begin{aligned}\lambda_{\text{new}} &= \lambda_{\text{old}} + t_1 c, \\ \boldsymbol{\mu}_{\text{new}} &= \boldsymbol{\mu}_{\text{old}} + t_2 \boldsymbol{v},\end{aligned} \tag{19}$$

where $t_1 > 0$ and $t_2 > 0$ are step-size parameters and the subgradients c and $\boldsymbol{v}$ are given by the following proposition.

Proposition 2: The subgradient of the Lagrange dual function $g(\lambda, \boldsymbol{\mu})$ is $[c, \boldsymbol{v}]$, where

$$\begin{aligned}c &= \bar{P} - \frac{T-\tau}{T}\sum_{j=1}^{M}\int_0^{\infty} P^{'}(x_j)\left[p_{0,j}f_0(x_j) + p_{1,j}f_1(x_j)\right]dx_j, \\ v_j &= \bar{I}_j - \frac{T-\tau}{T}\int_0^{\infty} p_{1,j}h_j P^{'}(x_j)f_1(x_j)dx_j,\end{aligned} \tag{20}$$

and $P^{'}(x_j)$ is the optimal power allocation for fixed λ and $\boldsymbol{\mu}$ given by (18).

Finally, in Table 1, we summarize the algorithm that computes the optimal sensing time and continuous power allocation function for sensing-based multiband spectrum sharing.

Table 1.

- ► For each τ in $[0, T]$, do.
  - 1) Initialize λ and $\boldsymbol{\mu}$.
  - 2) Repeat:
    - - For each channel j, compute $P(x_j)$ using (18).
    - - Update λ and $\boldsymbol{\mu}$ using (19).
  - 3) Until λ and $\boldsymbol{\mu}$ converge.
- ► End for.
- ► Optimal sensing time and power allocation function are $\tau^* = \arg\max_{\tau} R(\tau, P(x_j))$, $P^*(x_j) = P(x_j)|_{\tau=\tau^*}$.

3.4. Other Power Constraints

For problems under other power constraints, first, we can build the similar Lagrangian function as (14) and dual optimization problem as (15). $P(x_j)$ for fixed Lagrange multipliers can be solved by Lagrangian method and Lagrange multipliers can be solved by the subgradient based methods described in Section 3.3. The details are not given here for brevity.

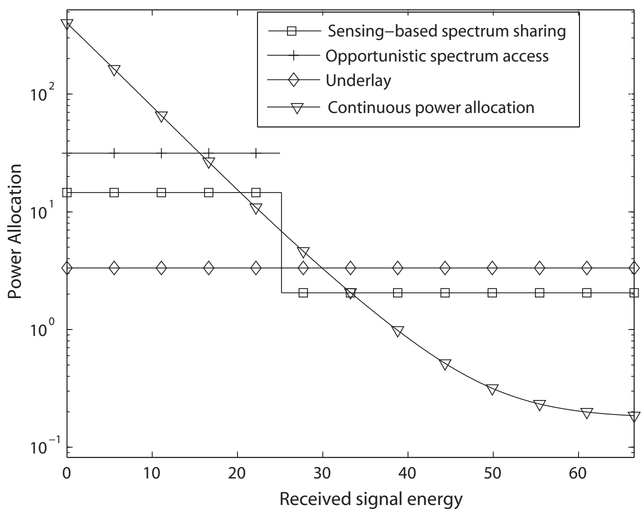

Fig. 3. Power allocation functions under the average transmit and average interference power constraints.

4. SIMULATION RESULTS

Three narrowband channels ($M = 3$) are assumed, each of 1 MHz bandwidth, and the frame duration is fixed as $T =$ 100 ms and the sampling frequency $f_s = 1$ MHz. We set $\gamma_{1,j} = N_0 = 0$ dB, $p_{0,j} = 0.7$, $P_{s,j} = 3$ dB, $\bar{I}_j = 0$ dB, $\bar{P} = 10$ dB, $\gamma_{2,j}$, h_j and g_j as Rayleigh distribution with mean 0 dB, and unless otherwise mentioned.

Fig. 3 compares the power allocation functions under the conventional strategies and the proposed continuous one with perfect CSI for fixed $h_j = g_j = \gamma_{2,j} = 0$ dB. From the figure we can see that, the function $P(x_j)$ for the proposed strategy is a non-increasing function of the received signal energy which corroborates Proposition 1. When x_j is small, the proposed scheme allocates more power than the conventional ones, and when x_j is large, it allocates less power than the conventional schemes.

Fig. 4 shows the average secondary achievable rate under average transmit and average interference power constraints for $\bar{I}_j$ = 0 dB. In the low $\bar{P}$ region, the proposed schemes and the conventional ones have the same rates. However, when $\bar{P}$ is high, the proposed continuous power allocation schemes achieve significantly higher rates. The rates of all schemes flatten out when $\bar{P}$ is sufficiently large since the rate is decided by $\bar{I}_j$ under this condition.

Fig. 5 shows the average secondary achievable rate under peak transmit and average interference power constraints for $\hat{P}_j$ = 3dB. In the low $\bar{I}_j$ region, the power allocation is decided by $\bar{I}_j$, the rates of the proposed schemes are better than the conventional ones. When $\bar{I}_j$ becomes larger, the rates tend to be equal where the power is decided by $\hat{P}_j$ and when $\bar{I}_j = +\infty$, $P(x_j) = \hat{P}_j$, the proposed schemes become the same the conventional ones.

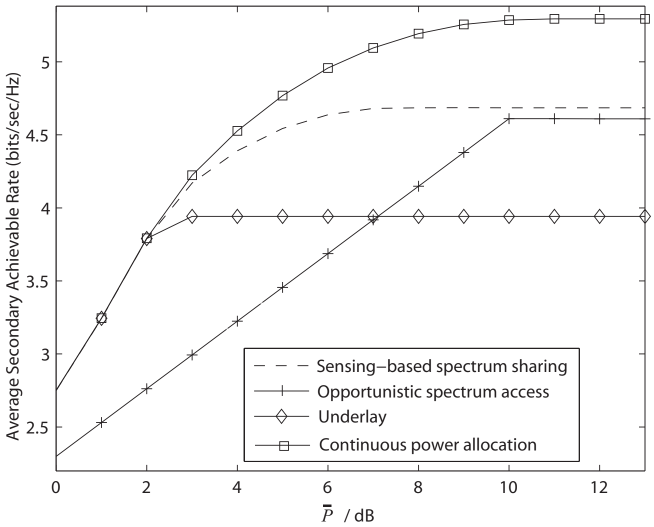

Fig. 4. Secondary achievable rate vs. $\bar{P}$ under the average transmit and average interference power constraints.

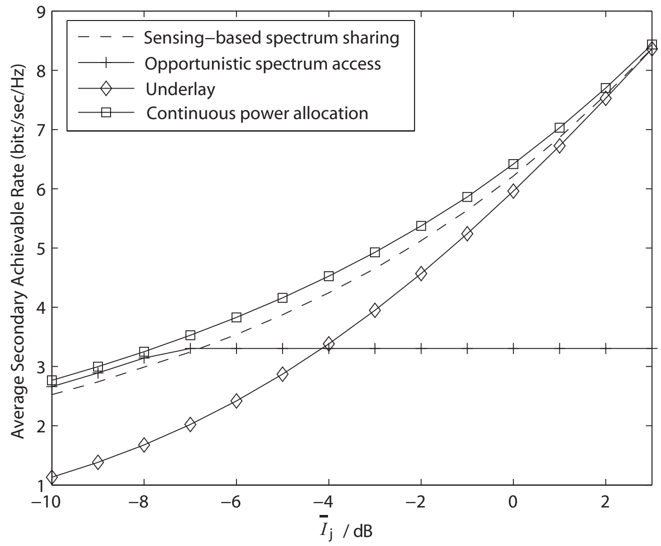

Fig. 5. Secondary achievable rate vs. $\bar{I}_j$ under the peak transmit and average interference power constraints.

5. CONCLUSIONS

We have proposed sensing-based continuous power allocation strategies for SUs in a multi-band cognitive radio system. The power allocation is a function of the received signal energy by the secondary user. For the different possible combinations of constraints on the peak/average transmit power at SU and the peak/average interference power at PU, the power allocation functions are obtained to maximize the average secondary achievable rate. Compared with the state-of-the-art sensing-based spectrum access which employs a binary power allocation strategy, the proposed schemes offer significant rate improvement for the SUs.

6. REFERENCES

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, 2005.

[2] X. W. Gong, S. A. Vorobyov, and C. Tellambura, "Optimal bandwidth and power allocation for sum ergodic capacity under fading channels in cognitive radio networks," *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1814-1826, 2011.

[3] X. Zhou, G. Y. Li, D. Li, D. Wang, and A. C. K. Soong, "Probabilistic resource allocation for opportunistic spectrum access," *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2870-2879, 2010.

[4] Y. Y. Pei, Y. C. Liang, Y. C. Teh, and K. H. Li, "How much time is needed for wideband spectrum sensing?" *IEEE Trans. Wireless Commun.*, vol. 8, no. 11, pp. 5466-5471, 2009.

[5] H. J. Lim, D. Y. Seol, G. H. Im, "Joint sensing adaptation and resource allocation for cognitive radio with imperfect sensing," *IEEE Trans. Commun.*, vol. 60, no. 4, pp. 1091-1100, 2012.

[6] Y. C. Liang, Y. H. Zeng, E. C. Y. Peh, A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326-1337, 2008.

[7] S. Stotas and A. Nallanathan, "Optimal sensing time and power allocation in multiband cognitive radio networks," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 226-235, 2011.

[8] J. Ma, G. D. Zhao, Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4502-4507, 2008.

[9] V. Lau, Y. J. Liu, T. A. Chen, "On the design of MIMO blockfading hannels with feedback-link capacity constraint," *IEEE Trans. Wireless Commun.*, vol. 52, no. 1, pp. 62-70, 2004.

[10] A. Ben-Tal and A. Nemirovski, Lectures on modern convex optimization: analysis, algorithms, and engineering applications, Philadelphia, PA: Society for Industrial and Applied Mathematics, 2001.

[11] S. Boyd and L. Vandenberghe, Convex optimization, Cambridge, UK: Cambridge University Press, 2005.