A SPACE-VARIANT CUBIC-SPLINE INTERPOLATION

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ABSTRACT

In this paper, a space-variant cubic-spline interpolation (CSI) scheme by the use of the warped distance is developed to improve the performance. Furthermore, a modified overlap-save sub-image method is introduced to solve the boundary condition problems that occur between two neighboring subimages in the actual image. Experimental results show that the proposed improved CSI scheme can actually achieve a better PSNR than the existing interpolation algorithms including the original CSI scheme.

***Index Terms*—**Space-variant, cubic-spline interpolation, warped distance

1. INTRODUCTION

Interpolation estimates the intermediate values of a set of discrete samples, which is used extensively in the image data compression. There have been some interpolation functions, such as linear interpolation, cubic convolution interpolation (CCI) [1], cubic B-spline interpolation [2], [3], and so on. The main disadvantage of these interpolation schemes is that they are not generally designed to minimize the error between the original image and its reconstructed image. In 1981, based on the least-squares method with a special linear interpolation function, Reed first developed a linear spline interpolation scheme for re-sampling discrete image data [4]. In 2000, using an extension of the ideas of Reed, a modified version of the linear spline interpolation algorithm was developed by Truong et al., called the cubic-spline interpolation (CSI) algorithm [5]. It is based on the least-squares method with the CCI function, which is superior in performance to the other existing interpolation schemes [1-4]. Recently, Y. Zhang et at. generated these ideas and proposed an interpolation-dependent image downsampling algorithm, where the downsampled image is obtained by means of the least-squares method [6].

The original CSI scheme is a space invariant technology, which is similar to the conventional space invariant method. In 1999, G. Ramponi changed the traditional space invariant interpolation operators into space-variant techniques by introducing the concept of the warped distance [7] and the results showed that it is able to reduce the interpolation error. In order to improve the performance, a space-variant CSI scheme by the use of warped distance is proposed in this paper. Furthermore, to solve the boundary condition problems that occur between two neighboring subimages during the implementation of the proposed space-variant CSI scheme, a modified overlap-save sub-image method [8] is introduced. Computer simulations on several standard images show that the proposed space-variant CSI scheme is far superior in performance to the existing interpolation algorithms.

The remainder of this paper is organized as follows: the original CSI algorithm is briefly reviewed in Section II. The proposed space-variant CSI scheme is presented in considerable detail in Section III. Section IV illustrates the experimental results and discussions. Finally, conclusions are given in the last section.

2. THE ORIGINAL CSI SCHEME

It is well-known that the philosophy of the CSI scheme is to recalculate the sampled values of the image data by means of the least-squares method with the CCI formula. In this

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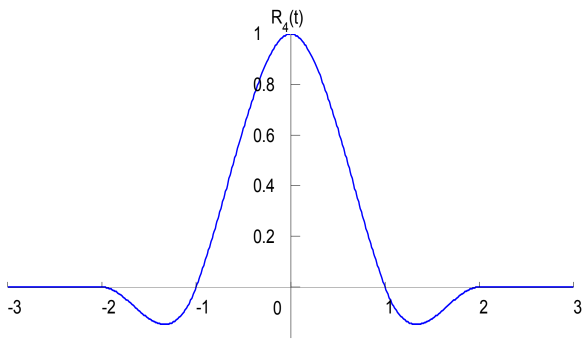

Fig.1. 1-D CCI function.

section, the original CSI scheme is briefly reviewed based on matrix form.

The constraint $\alpha = -1$ in CCI kernel function is superior to $\alpha = -0.5$ because it provides a better PSNR performance in the CSI scheme with the same arithmetic operations [9], so the parameter $\alpha = -1$ is utilized in this paper. Therefore, from [9], the 1-D CCI kernel function $R(t)$ as shown in Fig. 1 is given by

$$R(t) = \begin{cases} |t|^3 - 2|t|^2 + 1, & 0 \le |t| < 1 \\ -|t|^3 + 5|t|^2 - 8|t| + 4, & 1 \le |t| < 2 \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Let τ be a fixed, positive integer. Also let $\mathbf{Y}$ denote the data function with size $n\tau \times 1$, $\mathbf{X}$ the compressed values after downsampling to size $n \times 1$, where n is an integer. Our desire is to approximate $\mathbf{Y}$ by the values

$$\hat{\mathbf{Y}} = \mathbf{HX} \quad (2)$$

in a least-squares fashion, where $\mathbf{H}$ represents the interpolation matrix with size $n\tau \times n$, which consists of interpolation coefficients determined by the CCI function and τ.

One wants to find the optimal downsampled values such that

$$\mathbf{L}(\mathbf{X}) = \hat{\mathbf{Y}} - \mathbf{Y} = \mathbf{HX} - \mathbf{Y} \quad (3)$$

is a minimum. To minimize (3), the partial differentiation of $\mathbf{L}(\mathbf{X})$ with respect to $\mathbf{X}$ yields the following equation:

$$\frac{\partial \mathbf{L}}{\partial \mathbf{X}} = 2\mathbf{H}^T(\mathbf{HX} - \mathbf{Y}) = 0, \quad (4)$$

where $\mathbf{H}^T$ denote the transpose of the matrix $\mathbf{H}$.

Thus, the optimal downsampled values can be obtained by

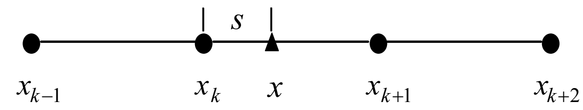

Fig. 2. Signal interpolation

$$\mathbf{X} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{Y}. \quad (5)$$

For the 2-D case, the CSI scheme based on matrix form can be accomplished by the use of 1-D CSI scheme with respect to each coordinate.

3. THE PROPOSED SPACE-VARIANT CSI SCHEME

For the original CSI scheme, the interpolation coefficients are constant during the process of interpolation. It is shown in [7] that the interpolation coefficients taking into account the local characteristics of the data to be interpolated can reduce the interpolation error. In what follows, a space-variant CSI algorithm is proposed in such a way that it can improve the performance.

3.1. The Proposed Algorithm

As shown in Fig. 2, x_{k-1}, x_k, x_{k+1} and x_{k+2} are downsampled image data, and x is the unknown pixel to be interpolated. Then, the distance between x_k and x is defined by

$$s = x - x_k. \quad (6)$$

For the traditional space invariant interpolation methods, s is set to be 1/2 for $\tau = 2$. In [7], the conventional space invariant methods can be changed into space-variant ones by the use of the warped distance, defined by

$$s' = s - \lambda As(s-1), \quad (7)$$

where λ is 1 or 2, A indicates the local properties of data by evaluating the dissymmetry of the data adjacent to pixel x defined by

$$A = \frac{|x_{k+1} - x_{k-1}| - |x_{k+2} - x_k|}{L-1} \quad (8)$$

with $L = 256$ for 8-bit luminance image.

The proposed algorithm is based on the least-squares method with space-variant CCI interpolation by the use of warped distance to achieve better quality of reconstructed

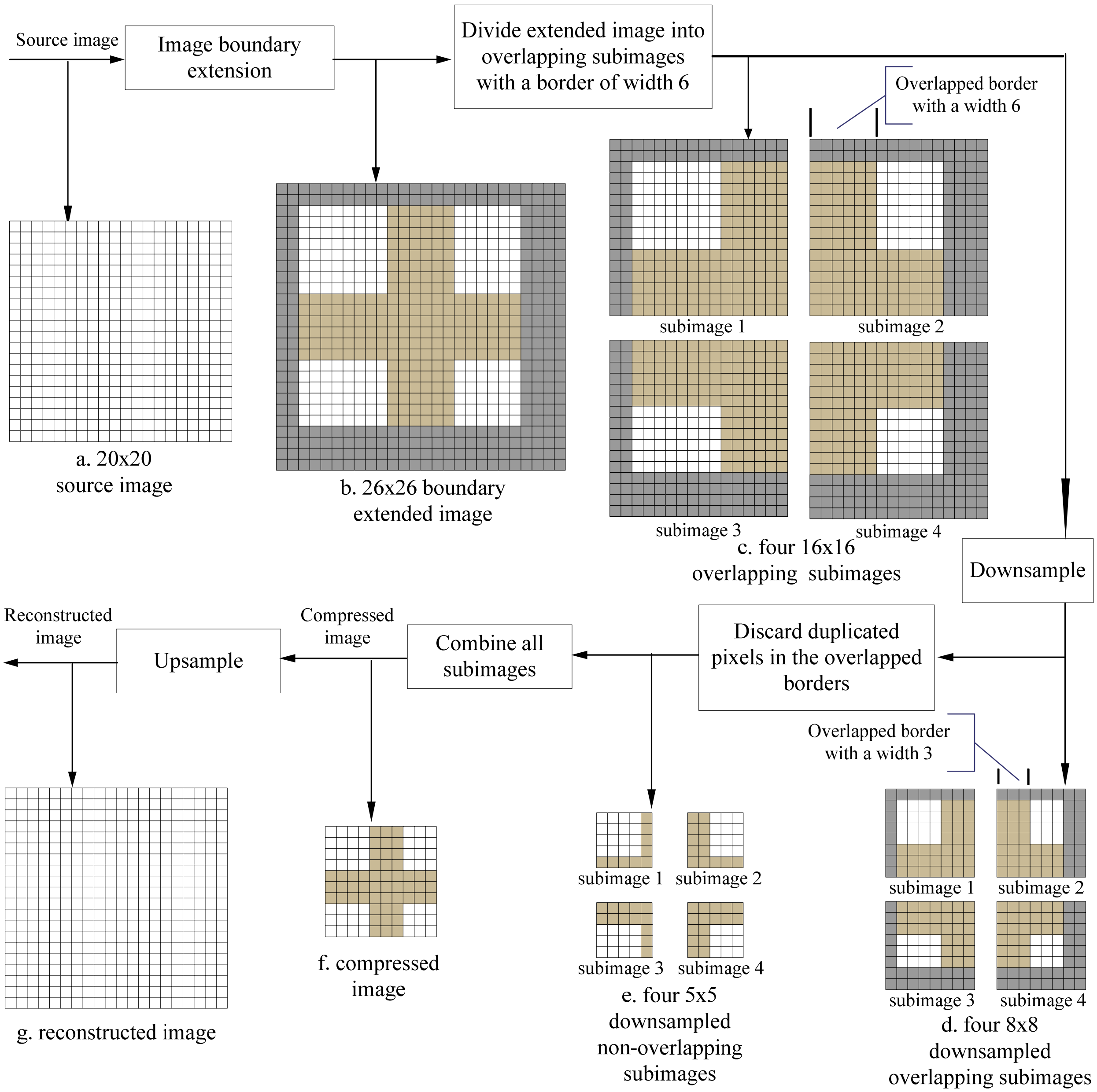

Fig. 3. Illustration of the modified overlap-save subimage method

image. Unfortunately, the interpolation matrix $\mathbf{H}$, which consists of interpolation coefficients calculated according to the local properties of the sampled data, is unknown before downsampling. To solve this problem, the content-dependent interpolation algorithm proposed in [6], whose iteration number is set to 2, is introduced in this paper.

Let $\mathbf{H}^0$ and $\mathbf{X}^0$ be comprised of space invariant CCI coefficients and the result generated by the direct downsampling, respectively. The details of the proposed space-variant CSI scheme are described as follows:

1) Initialize $\mathbf{H}^0$ and $\mathbf{X}^0$;
2) Compute $\mathbf{H}$ based on $\mathbf{X}^0$ according to (7);
3) Compute $\mathbf{X}$ according to (5).

Similarly, the proposed 2-D space-variant CSI scheme is derived in the same manner as the 1-D scheme with respect to each coordinate.

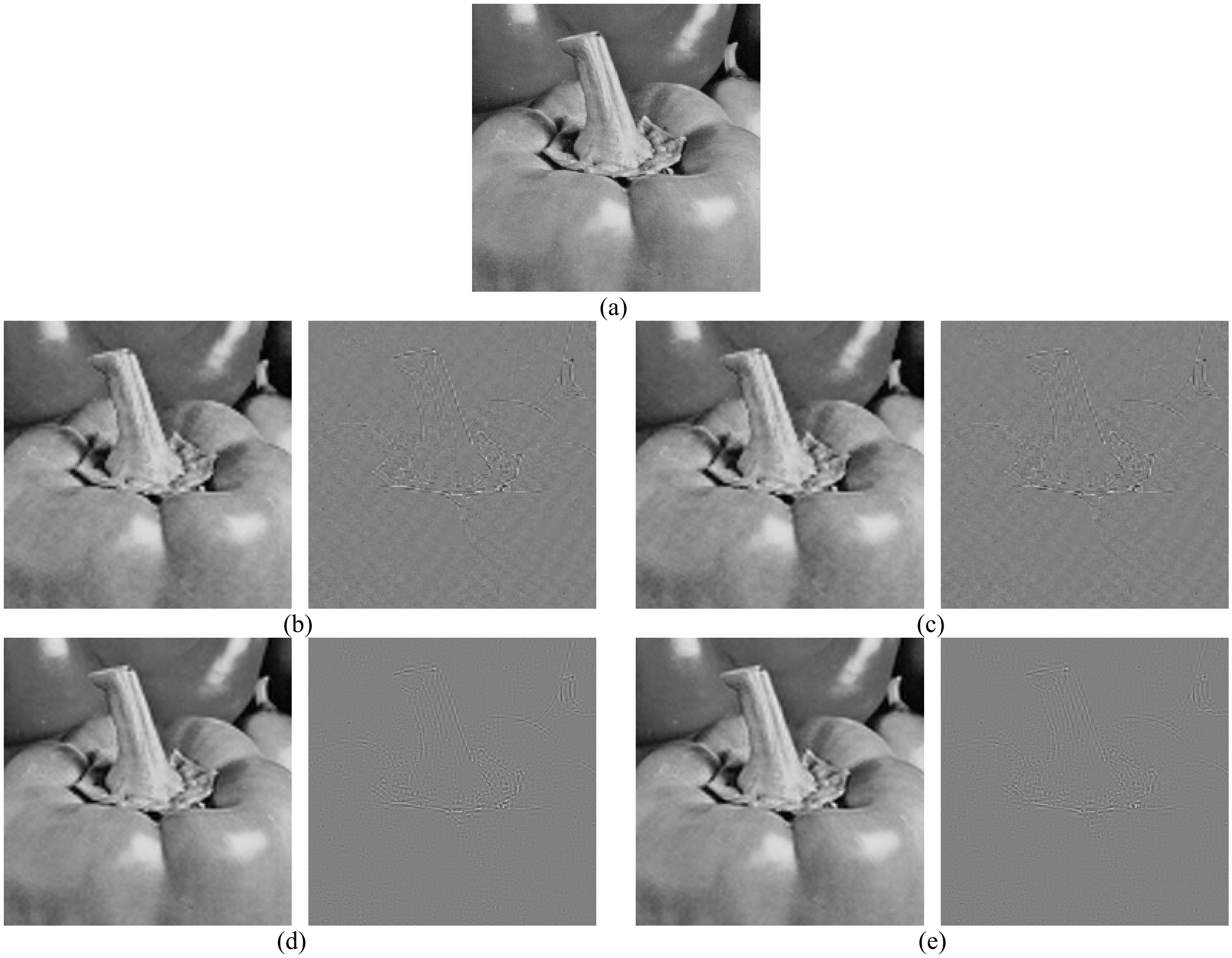

Fig. 4. Reconstructed and residual images of Peppers (zoom in) with a compression ratio 4:1. (a) Original Peppers image. (b) Reconstructed image by space-invariant CCI (PSNR=31.59dB). (c) Reconstructed image by space-variant CCI (PSNR=31.73dB). (d) Reconstructed image by space-invariant CSI (PSNR=32.62dB). (e) Reconstructed image by the proposed algorithm (PSNR=33.14dB).

3.2. Block Implementation Utilizing Overlap-Save Sub-image Method

It is observed that the dimension of matrix **H** is $n\tau \times n$, which demands considerable storage requirement and high computational complexity. To tackle this problem, a modified overlap-save sub-image method illustrated in Fig.3 is proposed in the following. Consider the source image data to be of size 20×20 pixels, as shown in Fig. 3(a). The symmetric extension scheme as given in [10] is utilized to solve the boundary conditions to obtain the extended image with size 24×24 pixels, as depicted in Fig. 3(b). Next, this extended image is divided into four overlapping 16×16 subimages, as shown in Fig. 3(c). Note that each subimage of size 16×16 pixels overlaps each adjacent subimage with a border of width 6. It is shown experimentally that this border can be used to solve the boundary condition between the two neighboring subimages when using the CSI scheme. Next, these overlapping subimages are decimated by the proposed algorithm to be four downsampled overlapping subimages, as shown in Fig. 3(d). Then, the duplicated pixels in the four overlapping 8×8 subimages are deleted to become the four non-overlapping 5×5 subimages. To illustrate this, in Fig. 3(d), each subimage has an overlapping border of width 3. The first and last two columns of all the subimages are the duplicated columns in the overlapping border to be removed. Then, the above overlap-save method in the row direction is similarly accomplished. Fig. 3(e) illustrates the remaining samples of each subimage obtained by the use of this overlap-save

TABLE I PSNR (dB) COMPARISONS OF DIFFERENT METHODS

Image	Space-invariant CCI	Space-variant CCI	Space-invariant CSI	Proposed
Peppers	31.59	31.73	32.62	33.14
Lake	29.37	29.42	30.64	31.01
Couple	29.17	29.12	30.50	30.83
Crowd	32.93	33.16	33.83	34.60
Lena	34.03	34.18	35.04	35.73
Airplane	30.49	30.64	31.92	33.14
Boat	29.27	29.23	30.82	31.16
Bridge	25.71	25.68	27.18	27.46

method. A combination of these four non-overlapping subimages yields the entire compressed image, shown in Fig. 3(f).

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

In this section, based on the ideas of Section III, the proposed space-variant CSI scheme can be shown to achieve a better PSNR performance. For some 2-D grey images with size 512×512, the experimental results with a compression ratio 4:1 ($\tau = 2$) that use the space invariant CCI, space-variant CCI, space invariant CSI and proposed space-variant CSI schemes are shown in Table I. In the experiment, each original image is downsampled to obtain data samples with a compression ratio of 4:1 ($\tau = 2$). Moreover, the reconstructed values between the samples are applied to obtain the reconstructed image with a ratio of 4:1 ($\tau = 2$), thereby yielding the corresponding PSNR values. Upon the inspection of Table I, the proposed space-variant CSI scheme achieves the best PSNR in comparison with other three methods. For the gray 512×512 Lena image at the same compression ratio 4:1 ($\tau = 2$), the PSNR value obtained by the proposed space-variant CSI scheme is higher by 1.7dB, 1.55dB and 0.69dB than the space invariant CCI, space-variant CCI, and space invariant CSI schemes, respectively.

Fig. 4 shows the reconstructed and residual images of Peppers at the same compression ratio of 4:1 ($\tau = 2$) using the space invariant CCI, space-variant CCI, space invariant CSI, and the proposed space-variant CSI schemes. It can be found that the Peppers image reconstructed by the proposed algorithm has a better subjective quality in comparison with the other three interpolation schemes.

5. CONCLUSIONS

In this paper, a space-variant CSI scheme that combines the least-squares with a space-variant CCI kernel is developed in order to improve the original CSI scheme for image data compression. Simulation results show that this proposed space-variant CSI scheme yields a better subjective quality and PSNR performance than existing interpolation methods for the reconstructed image.

6. REFERENCES

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