THEORETICAL ANALYSIS OF RECONFIGURABLE ADAPTIVE ANTENNA ARRAY IN GNSS APPLICATIONS

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ABSTRACT

Traditional adaptive array processing techniques are based on a fixed array configuration. We propose a novel reconfigurable adaptive antenna array strategy in the context of Global Navigation Satellite Systems to add the array configuration as an extra degree of freedom. Specifically we formulate the problem as selecting K from N antennas that are then connected to the following K front-ends and array processing network via switches. The determination of optimal K-antenna subarray configuration can be expressed as a minimization of Spatial Correlation Coefficient (SCC). It denotes the spatial separation between the desired signal and interference. A lower bound of the SCC gives the best achievable performance of all K-antenna subarrays. Two relaxation methods are introduced to obtain the lower bound. The simulation results show that the lower bound formula is reliable in any scenario and the optimum SCC value of an array with more antennas is even larger than the small array under some scenarios. Thus the subarray can reduce both hardware and computational cost dramatically with preserved performance supposing it is optimally configured.

***Index Terms*—** Lagrange dual relaxation, Reconfigurable antenna array, SCC, SDP relaxation, SINR

1. INTRODUCTION

Radio Frequency Interference (RFI) is proved to be a significant obstacle to the successful operation of Global Navigation Satellite Systems (GNSS) receivers. The use of adaptive antenna arrays to mitigate the detrimental effects of RFI on GNSS receivers has gained significant interest in the signal processing community. Most of the work in the literature focuses on adaptive beamforming and filtering techniques under a predetermined array configuration [1, 2]. However, the array configuration plays a fundamental role in the performance of adaptive array processor and is therefore an important design problem [3, 4]. Fixing the array architecture can lead to significant inefficiencies and/or loss in performance of array signal processing under different scenarios. When the array dimensionality is large, adaptive array can become unnecessarily computationally expensive [5]. In order to reduce the cost (both hardware and software) while preserving the performance, we propose in this paper a reconfigurable adaptive antenna array strategy to "choose K from N antennas" that are then switched into the network and connected to the K following front-ends as shown in Fig. 1. Then the corresponding adaptive array weight vector is developed based on the chosen subarray to obtain the maximum interference suppression adaptively.

This paper focuses on the theoretical analysis of the reconfigurable adaptive antenna array scheme. In particular, we solve the problem of determining the number K of selected antennas with the aim of getting the best compromise between the performance and the cost. To this end, the spatial correlation coefficient (SCC), which characterises the spatial separation between the desired signal and interference is introduced and the lower bound of the optimal SCC is formulated using two convex relaxation methods. Since the detection performance of GNSS receivers is closely related to the output signal to interference plus noise ratio ($SINR_{out}$), we derive in closed form the relationship between the $SINR_{out}$ of the adaptive array processor and the SCC. This formula reveals that the $SINR_{out}$ is non-linear in the number K of selected antennas as the optimal SCC also depends on K. The trade-off curve of the $SINR_{out}$ and the computational cost with respect to K gives the most suitable value of K with the best compromise.

The paper is organised as follows. In Section II the parameter SCC is introduced, the relationship between the $SINR_{out}$ and SCC is described as well. The lower bound of optimal SCC is formulated with two relaxation methods in the Section III. In Section IV a set of representative numerical results are reported and discussed. Finally, some conclusions are drawn in Section V.

2. SPATIAL CORRELATION COEFFICIENT

In addition to Doppler separation, the spatial dimension of antenna array [6, 2] plays a vital part in quantifying the effect of interference on the desired signal. The spatial separation between the desired signal and interference with respect to the array is characterized by a single parameter, the SCC [3].

Now let the directions of arrival (DOA) of the desired signal and interference be given by (θ_s, ϕ_s) and (θ_j, ϕ_j) respectively. Then the **u**-space DOA parameters are

$$\mathbf{u}_i = [\cos\theta_i \cos\phi_i \quad \cos\theta_i \sin\phi_i]^T \quad \text{for } i = s, j \tag{1}$$

where ^{T} is the transpose. The steering vectors of the desired signal and interference are

$$\mathbf{v}_s = e^{jk_0 \mathbf{P}\mathbf{u}_s}, \quad \mathbf{v}_j = e^{jk_0 \mathbf{P}\mathbf{u}_j} \tag{2}$$

where the matrix $\mathbf{P}$ contains coordinates of antenna elements (see Eq. (7)). Under the assumption that the noise and interference are uncorrelated, their covariance matrix is given by

$$\mathbf{R}_n = \sigma^2 \mathbf{I} + P_j \mathbf{v}_j \mathbf{v}_j^H, \tag{3}$$

where σ^2 is the thermal noise power, P_j is the interference power and the superscript ^{H} denotes the conjugate transpose operation. Following the application of adaptive array filter based on maximum signal to interference ratio, the output $SINR_{out}$ becomes [7],

$$SINR_{out} = P_s \mathbf{v}_s^H \mathbf{R}_n^{-1} \mathbf{v}_s, \tag{4}$$

where P_s denotes the signal power. Assuming the thermal noise power is much smaller than the interference, and using the Sherman-Morrison-Woodbury formula, Eqs. (3) and (4) can be combined to give

$$SINR_{out} \approx \frac{NP_s}{\sigma^2}(1 - |\alpha_{js}|^2). \tag{5}$$

In this expression, the SCC, α_{js} is given by

$$\alpha_{js} = \frac{\rho_{js}}{\sqrt{\rho_{jj}}\sqrt{\rho_{ss}}} = \frac{\rho_{js}}{\|\mathbf{v}_j\|\|\mathbf{v}_s\|} = \frac{\rho_{js}}{N}. \tag{6}$$

where $\rho_{js} = \mathbf{v}_j^H \mathbf{v}_s$, $\rho_{ss} = \mathbf{v}_s^H \mathbf{v}_s$ and $\rho_{jj} = \mathbf{v}_j^H \mathbf{v}_j$ and we have assumed, without loss of generality, $\|\mathbf{v}_j\| = \|\mathbf{v}_s\| = \sqrt{N}$. The absolute value of the SCC is bounded between zero and one and can be interpreted as the angle between the desired signal and interference. A smaller absolute SCC indicates that the more separable the desired signal and interference are spatially with respect to the particular array. Eq. (5) shows that a reduction in $|\alpha_{js}|$ or increase in the number, N, of antenna elements will improve the array performance.

3. LOWER BOUND OF OPTIMAL SCC

Consider a $M \times M$ uniform antenna array with half wavelength inter-element spacing placed on the x-y plane as shown in Fig. 1. The position coordinate of the $(i,j)_{th}$ antenna element of this array is expressed as

$$p_{i,j} = [(i-1)d \quad (j-1)d] \text{ for } i,j = 1...M \tag{7}$$

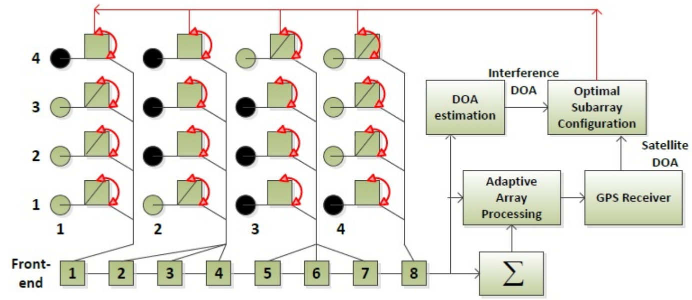

Fig. 1. Block diagram of reconfigurable adaptive array strategy: choose 8 from 4×4 planar array

where d is the inter-element spacing. Next we stack the position coordinate of every antenna element $p_{i,j}$ of the whole array into a $N \times 2$ matrix $\mathbf{P}$ (where $N = M^2$). Thus the antenna selection problem becomes choosing K from N antennas to compose a subarray in order to minimize SCC value.

Let us define the correlation steering vector between the desired signal and interference to be

$$\mathbf{v}_{js} = e^{jk_0 \mathbf{P}(\mathbf{u}_s^H - \mathbf{u}_j^H)} \tag{8}$$

Let $\mathbf{x}$ be a selection vector with N elements whose value can only be 0 (not selected) or 1 (selected), thus the SCC expression based on the selected subarray with K antenna elements can be expressed as

$$\alpha_{js} = \frac{\mathbf{x}^H \mathbf{v}_{js}}{K}. \tag{9}$$

Put $\mathbf{W} = \mathbf{v}_{js}\mathbf{v}_{js}^H$. Clearly $\mathbf{W}$ is Hermitian. Also, let us define $\mathbf{W}_r = real(\mathbf{W})$. The antenna selection problem can then be cast as a two-way partitioning model [8] as follows:

$$\min |\alpha_{js}|^2 = \frac{\mathbf{x}^H \mathbf{W}_r \mathbf{x}}{K^2} \tag{10a}$$

$$\text{s.j.t } x_i(x_i - 1) = 0 \quad i = 1...N, \tag{10b}$$

$$\text{and } \mathbf{x}^H \mathbf{x} = K. \tag{10c}$$

Due to the existence of the quadratic equality constraints in Eq. (10b) and Eq. (10c), the primal problem above is not a convex optimization. Thus, we resort to relaxation methods to get the lower bound of the optimal value. There are two kinds of commonly used relaxation methods: Lagrange Dual Relaxation and Direct Semidefinite Programming (SDP) relaxation. The Lagrange Dual Relaxation utilises weak duality and the convexity of duals to get the lower bound, while the Direct SDP Relaxation delete the rank-one matrix constraint to obtain a bound. Next the two methods will be formulated respectively and the relationship between them described.

3.1. Lagrange Dual Relaxation Method

Now proceeding with the analysis, the Lagrangian is

$$L(\mathbf{x},\boldsymbol{\mu},\upsilon) = \mathbf{x}^H(\frac{1}{K^2}\mathbf{W}_r + diag(\boldsymbol{\mu}) + \upsilon\mathbf{I})\mathbf{x} - \boldsymbol{\mu}^H\mathbf{x} - K\upsilon. \tag{11}$$

The Lagrange Dual function for the minimization of $L(\mathbf{x},\boldsymbol{\mu},\upsilon)$ over $\mathbf{x}$ becomes

$$\begin{aligned} g(\boldsymbol{\mu},\upsilon) &= \inf_{\mathbf{x}} \{L(\mathbf{x},\boldsymbol{\mu},\upsilon)\} \qquad (12) \\ &= \begin{cases} -\frac{1}{4}\boldsymbol{\mu}^H(\frac{1}{K^2}\mathbf{W}_r + diag(\boldsymbol{\mu}) + \upsilon\mathbf{I})^{-1}\boldsymbol{\mu} - K\upsilon \\ if\ \frac{1}{K^2}\mathbf{W}_r + diag(\boldsymbol{\mu}) + \upsilon\mathbf{I} \geq 0 \\ -\infty \qquad otherwise. \end{cases} \end{aligned}$$

Now it is evident that the Lagrange dual function in Eq. (12) is a concave function. Using the Schur complement, we can express Eq. (12) as a linear matrix inequality (LMI),

$$\begin{aligned} &\max \quad \lambda \qquad (13) \\ &\mathrm{s.j.t} \quad \begin{bmatrix} \frac{1}{K^2}\mathbf{W}_r + diag(\boldsymbol{\mu}) + \upsilon\mathbf{I} & -\frac{1}{2}\boldsymbol{\mu} \\ -\frac{1}{2}\boldsymbol{\mu}^H & -K\upsilon - \lambda \end{bmatrix} \succeq 0, \end{aligned}$$

The dual problem (13) is a Semidefinite Programming (SDP) with three variables λ, υ and $\boldsymbol{\mu}$ and can be effectively solved using the embedded Matlab software CVX [9]. Therefore, after calculating the maximum value of $g_{max}(\tilde{\boldsymbol{\mu}},\tilde{\upsilon})$, the lower bound of optimal SCC is

$$|\alpha_{js}|^2_{min} \geq g_{max}(\tilde{\boldsymbol{\mu}},\tilde{\upsilon}) \tag{14}$$

where $\tilde{\upsilon}$ and $\tilde{\boldsymbol{\mu}}$ are dual optimal solutions.

3.2. Direct SDP Relaxation Method

In addition to the above Lagrange Dual Relaxation method, there is another commonly used direct SDP Relaxation method that relaxes the original problem by deleting the rank-one constraint as follows:

$$\min \frac{1}{K^2}\mathrm{tr}(\mathbf{X}\mathbf{W}_r) \tag{15a}$$

$$\mathrm{s.j.t} \begin{bmatrix} \mathbf{X} & \mathbf{x} \\ \mathbf{x}^T & 1 \end{bmatrix} \geq 0 \tag{15b}$$

$$\mathrm{tr}(\mathbf{X}\mathbf{E}_i) - \mathbf{e}_i^H\mathbf{x} = 0,\ i = 1,2...N; \tag{15c}$$

$$\mathrm{tr}(\mathbf{X}) = K; \tag{15d}$$

where the optimization variables are $\mathbf{X} \in R^{N\times N}$ and $\mathbf{x} \in R^N$, vector $\mathbf{e}_i \in R^N$ is i_{th} unit vector and matrix $\mathbf{E}_i = \mathbf{e}_i\mathbf{e}_i^T$. The function tr($\mathbf{M}$) is the trace of the matrix $\mathbf{M}$. The rank-one equality constraint $\mathbf{X} = \mathbf{x}\mathbf{x}^T$ is replaced by an looser constraint $\mathbf{X} \geq \mathbf{x}\mathbf{x}^T$. Finally the inequality constraint can be expressed as a LMI by using the Schur complement, which gives Eq. (15b) [8].

Now we turn our attention to the relationship between the Lagrange Dual Relaxation Eq. (13) and the Direct SDP Relaxation Eq. (15). According to [10], they are duals of each other, and therefore the two bounds are identical provided that Slater's condition is satisfied. In our specific problem, there is no duality gap between the pair of dual problems Eq. (13) and Eq. (15). Next we summarize the difference between the two relaxation methods briefly as follows:

1. Eq. (10)→ Eq. (13): dualizes $N+1$ constraints, producing a dual problem in R^{N+1};
2. Eq. (10)→ Eq. (15): linearises $N+1$ constraints, producing Eq. (15) with extra N^2 variables.

Usually the Lagrangian Duality Relaxation method is preferred due to its simplicity.

4. SIMULATION RESULTS

In this section some simulation results are presented to validate the theoretical results obtained above.

4.1. Lower Bound Of Optimal SCC Under Different Scenarios

In the first example we consider a 4×4 square array shown in Fig. 1 and proceed to choose a subarray with $K = 8$ antennas in order to minimize the SCC value. The desired signal is assumed to be coming from azimuth and elevation $\phi_s = 0.2\pi$ and $\theta_s = 0.1\pi$ respectively. The azimuth angle of the interference is set to $\phi_j = 0.25\pi$ and the elevation angle, θ_j is varied from $\theta_j = 0$ to 0.5π. The comparison among the true minimum SCC and lower bounds obtained from the two methods is shown in the Fig. 2, which clearly demonstrates that the calculated lower bound of the optimal SCC value coincides with the true minimum in any scenario. Also observe that the two lower bounds are exactly same.

In order to show the lower bound of optimal SCC representing the best achievable performance, the $SINR_{out}$ of all 8-antenna subarrays is plotted in the Fig. 3 by taking the $SINR_{out}$ value calculated from the lower bound of SCC as a reference. The DOA of the satellite signal and the azimuth angle of the interference are fixed to the value used above and the elevation angle of the interference is set to be 0.2π. We can see that none of these subarrays can achieve better performance than the upper bound calculated from lower bound of SCC.

4.2. Lower Bound Of Optimal SCC with Different Number of Selected Antennas

In the second example we investigate the dependence of the lower bound of optimal SCC on the number K of selected antennas. We still use the 4×4 planar array and the number K is changing from 2 to 16. The arrival direction of desired

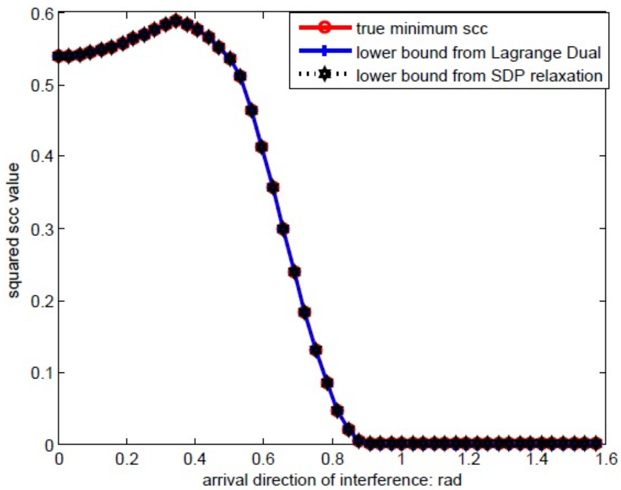

Fig. 2. Comparison between the true minimum SCC and lower bounds in different scenarios

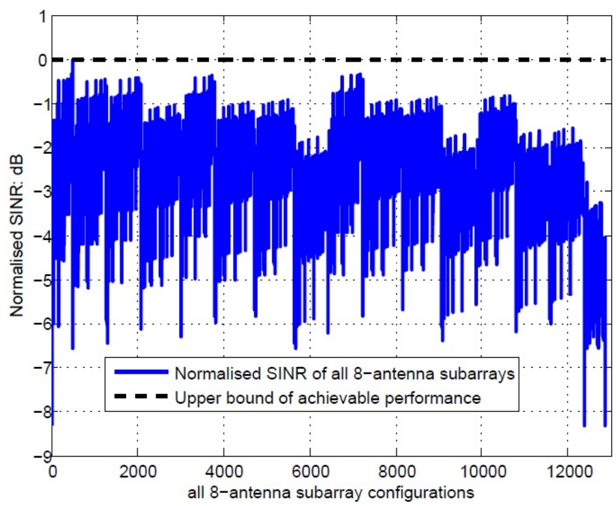

Fig. 3. $SINR_{out}$ of all 8-antenna subarrays and the best performance obtained from lower bound of SCC

signal is set at $\theta_s = 0.2\pi rad, \phi_s = 0.2\pi rad$ and the DOAs of interference under four different scenarios are as follows:

1. the elevation angle is 0.25π, the azimuth angle is 0.25π;
2. the elevation angle is 0.25π, the azimuth angle is 0.3π;
3. the elevation angle is 0.3π, the azimuth angle is 0.3π;
4. the elevation angle is 0.35π, the azimuth angle is 0.4π.

The simulation results, shown in Fig. 4, demonstrate that the achievable minimum SCC value is nearly zero, no matter how many antennas are selected, when the desired signal and interference are separated apart in space; whereas when the two are close spatially, the minimum SCC is increasing with the number of selected antennas. Mathematically we can interpret the SCC expression in Eq. (10a) as a sinc function with shape determined by the $\mathbf{u}_s - \mathbf{u}_j$ value and the number of selected antennas, K. When the value of $\mathbf{u}_s - \mathbf{u}_j$ is small, the satellite signal and interference may locate in the same lobe due to the low resolution. Then it is very possible that increasing K can move the interference from zero to non-zero position of the sinc lobe due to the shrinkage caused by higher resolution and thus increase the optimum SCC value.

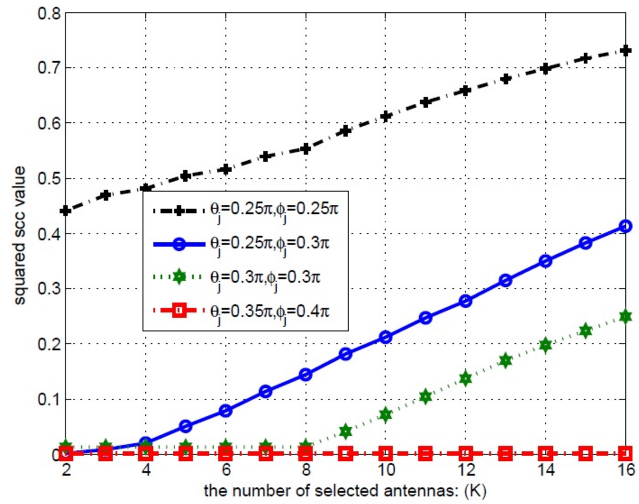

Fig. 4. Lower bound with different number of selected antennas in a 4×4 planar array

4.3. The Relationship between $SINR_{out}$ and the Number of Selected Antennas

As we have mentioned above, and is implied by Eq. (5), the relationship between the $SINR_{out}$ and the number of selected antennas is non-linear. The reason is that $|\alpha_{js}|^2$ also depends on the number K like shown in Fig. 4. Now in order to derive the final relationship between $SINR_{out}$ and K, we proceed to substitute Eq. (14) into (5). Thus, $SINR_{out}$ after adaptive array processing becomes

$$SINR_{out} \doteq \frac{KP_s}{\sigma^2}(1 - g_{max}(\tilde{\boldsymbol{\mu}}, \tilde{\upsilon}). \tag{16}$$

Eq. (16) gives the final relationship between maximum $SINR_{out}$ and the number K. Here we still use the 4×4 planar array for simulation results in Fig. 5. In this simulation, the arrival direction of the desired signal is $\theta_s = 0.2\pi, \phi_s = 0.2\pi$ and the two DOAs of interference are $\theta_j = 0.22\pi, \phi_j = 0.22\pi$ and $\theta_j = 0.4\pi, \phi_j = 0.4\pi$ for the first and second scenarios respectively. When the desired signal and interference are separated apart in space, since the achievable minimum SCC value is constant (nearly zero), the $SINR_{out}$ exhibits a linear relationship with K as shown by the cross curve (as the scale is in dB, doubling K results in 3dB SINR gain). However, when the desired signal and interference are close in space, this linear relationship does not apply any more due

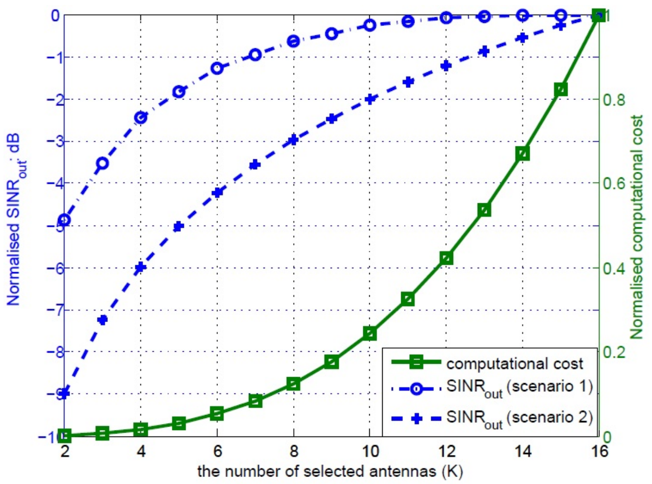

Fig. 5. Trade-off curve between the performance and the cost

to the increasing SCC for larger K. The $SINR_{out}$ curve with circle marker flattens out as K increases to the value 8.

The most computationally expensive step for adaptive array processing lies in the matrix inverse which needs the order K^3 complex operations [7], where K is the dimension of the covariance matrix. Thus, to appreciate the trade-off between the performance and the computational cost, we calculate the normalised $SINR_{out}$ gain and computational cost by taking the whole array as a reference. We can observe from Fig. 5 that using an 8-antenna subarray saves nearly 90% of computational cost with almost $0.5dB$ degradation of $SINR_{out}$ in scenario 1 and $3dB$ in scenario 2. Hence, both computational load and hardware cost are substantially reduced by using 8-antenna subarray instead of the whole array with small degradation in performance when the desired signal and interference are close in space.

5. CONCLUSION

In this paper, we have investigated the problem of composing a subarray for adaptive array processing in the context of GNSS in order to get the best compromise between the performance and the cost. We have proposed the Spatial Correlation Coefficient parameter and derived the lower bound of the optimal SCC using two relaxation methods. This lower bound gives the best achievable performance of all K-antenna subarrays. We then use this lower bound formula to obtain the final relationship between $SINR_{out}$ and the number K of selected antennas. We have shown though analysis and simulations that the novel reconfigurable adaptive antenna array scheme using switches to "choose K from N antennas" can reduce both hardware and software cost, while sacrificing very little in performance if the subarray size K is suitably chosen. The optimal number K can be obtained through the lower bound formula by varying different trade-off between the performance and the cost.

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