CENTROID-BASED TEXTURE CLASSIFICATION USING THE GENERALIZED GAMMA DISTRIBUTION

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ABSTRACT

This paper introduces a centroid-based (CB) supervised classification algorithm of textured images. In the context of scale/orientation decomposition, we demonstrate the possibility to develop centroid approach based on a stochastic modeling. The aim of this paper is twofold. Firstly, we introduce the generalized Gamma distribution (GΓD) for the modeling of wavelet coefficients. A comparative goodness-of-fit study with various univariate models reveals the potential of the proposed model. Secondly, we propose an algorithm to estimate the centroid from the collection of GΓD parameters. To speed-up the convergence of the steepest descent, we propose to include the Fisher information matrix in the optimization step. Experiments from various conventional texture databases are conducted and demonstrate the interest of the proposed classification algorithm.

Index Terms— textured images, Jeffrey divergence, generalized Gamma distribution, centroid, supervised classification.

1. INTRODUCTION

Classification of textured images is used in a large field of applications ranging from the classification of orchards from remote sensing images, to quality check of manufactured pieces by comparison of internal structures. Among classification methods, clustering approaches have known an increased interest providing effective and tractable algorithms for various domains. Classification techniques based on clustering such as supervised centroid-based (CB) and unsupervised k-means methods assume that (i) textured images are sorted in k sub-collections of samples, i.e. the clusters, (ii) each cluster can be represented by the most centrally localized object, i.e. the barycenter or centroid. Evaluating a centroid implies to define an adapted measure of similarity/dissimilarity between a set of estimated parameters characterizing each sample in the cluster. For texture clustering, the main purpose is thus to define an effective set of parameters and a dissimilarity measure which can be minimized in order to estimate the centroid coordinates in the parameter space knowing the sub-set of samples associated to the cluster.

Over the last decade, numerous works devoted to texture analysis have shown the interest to use jointly scale-space decomposition and stochastic modeling for characterizing the textural content [1, 2, 3, 4, 5, 6]. The more recent works propose parametric probability density function (pdf), *i.e.* prior such as generalized Gaussian density (GGD) or Weibull density, to fit the empirical histogram of sub-band coefficients [2, 7]. Those works have further been extended by the use of the generalized Gamma distribution (GΓD) which generalizes both models [8, 9]. The distribution parameters of such models are then estimated and compose the signature of the texture while a probabilistic metric is used to measure similarity. Previous works [2, 7] show that the Jeffrey divergence (JD), increases significantly the classification rate in the framework of stochastic models. Thereby, the parametric space forms a smooth Riemannian manifold for which well-founded processing can be derived. In this way, Choy and Tong proposed in [3], for the GGD modeling, to compute a centroid from several instances of parameter vectors from each sub-band for a given class. However, they do not fully exploit the geometry of the stochastic model during this estimation step, which is one of the main contributions of the paper. The paper contribution is twofold. Firstly, we validate the benefit of the GΓD for the modeling of wavelet coefficients by a comparative study with state-of-the-art univariate models. Secondly, we propose a CB classification algorithm based on the GΓD model. To speed-up the convergence of the centroid computation, we propose to include the Fisher information matrix of the GΓD in the optimization step.

The paper is structured as follows. Section 2 introduces the GΓD for the modeling of wavelet coefficients. The benefit of such model is validated by a goodness-of-fit experiment on the VisTex database. In Section 3, we derive the proposed algorithm to compute the centroid. In Section 4, we present some classification results to evaluate the performance of the proposed CB classification algorithm on texture databases. We provide also some comparisons with classical univariate models. Conclusions and future works are finally reported in Section 5.

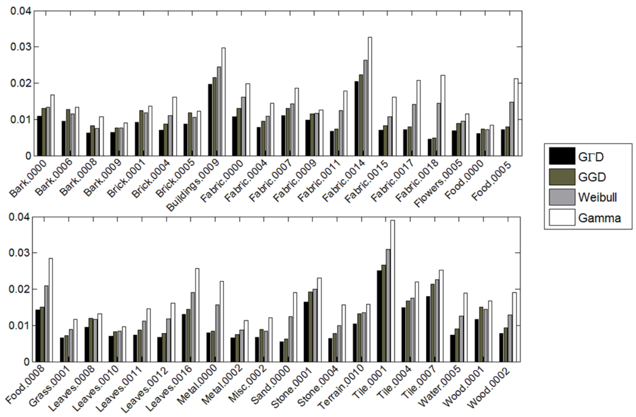

Fig. 1. Bar plots of the average Kolmogorov distance performed for 40 classes of the VisTex database, for various univariate models (GΓD, GGD, Weibull and Gamma).

2. THE UNIVARIATE GENERALIZED GAMMA DISTRIBUTION

The probability density function of a generalized Gamma distribution (GΓD) is [10]:

$$p_X(x;\alpha,\beta,\lambda) = \frac{\beta x^{\beta\lambda-1}}{\alpha^{\beta\lambda}\Gamma(\lambda)} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}, \quad (1)$$

where $x \in \mathbb{R}^+$. α, β and λ are respectively the scale, power and shape parameters. Note that the GΓD admits the Gamma, Weibull and generalized Gaussian distributions (GGD) as special cases.

To evaluate the benefit of the GΓD model to characterize the textural content, we compute the empirical histogram of the wavelet coefficients for each sub-band of textured images from the VisTex database. This histogram is then modeled by four univariate distributions *i.e.* GΓD, GGD, Gamma and Weibull. The Kolmogorov distance, denoted d_K, is next used to evaluate the goodness-of-fit. The Kolmogorov distance is defined as:

$$d_K = \sup_{\tau} |F(\tau) - F_N(\tau)|. \quad (2)$$

where $F_N(\cdot)$ is the empirical cumulative distribution function (cdf) and $F(\cdot)$ is the theoretical (hypothesized) cdf. Fig. 1 draws some bar plots of the average Kolmogorov distance per class on the VisTex database for the four considered stochastic models. In this experiment, the stationary wavelet transform with Daubechies' db4 filter has been considered. As observed, the GΓD model has the lowest Kolmogorov distance, exhibiting its interest for the modeling of wavelet coefficients.

To confirm the potential of the GΓD modeling, a texture retrieval experiment has been conducted on the VisTex, OuTex and VisTex Complete (VisTexC) databases [2]. Retrieval rates are displayed in Table. 1. As observed, the GΓD exhibits

Table 1. Indexing retrieval rate on the VisTex, OuTex and VisTexC databases.

	GΓD	GGD	Weibull	Gamma
VisTex	**76.73**	76.31	76.13	75.23
OuTex	**49.29**	48.98	48.50	47.35
VisTexC	**47.38**	46.95	46.72	46.20

the highest retrieval rate. This model will hence be considered for modeling wavelet coefficients. In the following, a supervised classification algorithm based on this model is introduced. To this aim, the central element (barycenter) from a collection of GΓD parameters should be computed, which is the purpose of the next section.

3. BARYCENTRIC REPRESENTATION

Let I be a texture image. Let N_o and N_s be respectively the number of orientation and scale of a multi-scale decomposition. I is hence decomposed into $N_o \times N_s$ sub-bands. Let us consider the parametric vector $\Theta_{s,o}$ of the pdf associated to each sub-band. The collection T_I of those parametric vectors will represent the texture image I.

$$T_I = \{\Theta_{s,o} | s = 1, \ldots, N_s, o = 1, \ldots, N_o\}. \quad (3)$$

The components $\Theta_{s,o}$ of the vector T_I form a parametric Riemannian manifold. In the sequel of the paper, we call $\mathcal{M}$ the corresponding manifold.

3.1. Computing a centroid

Let $(T_{c,n})_{n=1}^{N_{Tr}}$ be N_{Tr} training samples from the same class c. In [3], Choy and Tong have introduced an iterative algorithm to estimate the barycentric sample $\bar{T}_c$ (also called centroid) from this collection of samples. Let $l_c(T)$ be the cost function defined by:

$$l_c(T) = \frac{1}{N_{Tr}} \sum_{n=1}^{N_{Tr}} m(T \| T_{c,n}), \quad (4)$$

the centroid is obtained as the solution of the following optimization problem:

$$\bar{T}_c = \arg\min_{T \in \mathcal{M}} l_c(T). \quad (5)$$

The dissimilarity measure m between two instances of T is computed as the sum of the dissimilarity measures SIM between all sub-band distributions at each scale and orientation:

$$m(T_{c,n} \| T_{c',n'}) = \sum_{s=1}^{N_s} \sum_{o=1}^{N_o} \mathrm{SIM}(p(x;\Theta_{c,n,s,o}) \| p(x';\Theta_{c',n',s,o})); \quad (6)$$

where $p(x;\Theta_{c,n,s,o})$ is the probabilistic distribution which models the sub-band coefficients x at scale s and orientation o.

This paper introduces the natural gradient algorithm [11, 12] to solve the optimization problem defined in (5). Let $\bar{T}_c$ be the solution of (5), *i.e.* the minimizer of the cost function $l_c(T)$. To speed-up the convergence, the Fisher information matrix $G(T)$ is included with the gradient $\nabla l_c(T)$ of the cost function in the optimization step. Then, the sequence $\left(\bar{T}_{c,i}\right)_{i=1}^{\infty}$ defined by:

$$\bar{T}_{c,i+1} = Proj_{\mathcal{M}}\left(\bar{T}_{c,i} - G^{-1}(\bar{T}_{c,i})\nabla l_c(\bar{T}_{c,i})\right), \quad (7)$$

converges to the centroid $\bar{T}_c$. The operator $Proj_{\mathcal{M}}$ representing the projection on the manifold $\mathcal{M}$ assures that $\bar{T}_c$ belongs to the manifold $\mathcal{M}$. Practically, on the VisTex database, the projected gradient descent algorithm of [3] converges in 170 iterations whereas the proposed projected natural gradient converges in only 9 iterations to the same solution.

In the following, the computation of the centroid will be applied to the GΓD since this model has been successfully validated for the modeling of wavelet coefficients of texture images [8]. Note that this methodology can be generalized to any other stochastic models provided that a closed-form expression of the similarity measure m and the Fisher information matrix exist.

3.2. Application to the generalized Gamma distribution

The parameter space of a GΓD for one sub-band is represented by $\Theta = \{\alpha, \beta, \lambda\}$. The Jeffrey divergence (i.e. the double sided Kullback-Leibler divergence) is considered as a dissimilarity measure SIM between two GΓDs, its expression is given in [8]:

$$\text{JD}(p(x;\Theta)\|p(x;\Theta')) = -\lambda - \lambda' + A + A' + BC, \quad (8)$$

where

$$A = \left(\frac{\alpha}{\alpha'}\right)^{\beta'} \frac{\Gamma(\lambda + \beta'/\beta)}{\Gamma(\lambda)}, A' = \left(\frac{\alpha'}{\alpha}\right)^{\beta} \frac{\Gamma(\lambda' + \beta/\beta')}{\Gamma(\lambda')}$$
$$B = \left(\ln\left(\frac{\alpha}{\alpha'}\right) + \frac{\Psi(\lambda)}{\beta} - \frac{\Psi(\lambda')}{\beta'}\right), C = \beta\lambda - \beta'\lambda',$$

where $\Psi(z)$ is the digamma function. Next, by combining (6) and (8), one obtains the dissimilarity measure m between two samples. Then, to derive the cost function and to obtain its gradient $\nabla l_c(T)$, the partial derivatives of the JD are computed:

$$\begin{aligned}
\frac{\partial \text{JD}}{\partial \alpha}(\Theta, \Theta') &= \frac{C}{\alpha} + \frac{\beta'}{\alpha}A - \frac{\beta}{\alpha}A', \\
\frac{\partial \text{JD}}{\partial \beta}(\Theta, \Theta') &= \lambda B - C\frac{\Psi(\lambda)}{\beta^2} - \frac{\beta'}{\beta^2}\Psi(\lambda + \beta'/\beta)A \\
&+ A'\left[\ln\left(\frac{\alpha'}{\alpha}\right) + \frac{\Psi(\lambda' + \beta/\beta')}{\beta'}\right], \quad (9) \\
\frac{\partial \text{JD}}{\partial \lambda}(\Theta, \Theta') &= -1 + \beta B + C\frac{\Psi(1,\lambda)}{\beta} \\
&+ A\left[\Psi(\lambda + \beta'/\beta) - \Psi(\lambda)\right].
\end{aligned}$$

To compute the natural gradient, the 3×3 Fisher information matrix of a GΓD is also required. Its components are:

$$\begin{aligned}
g_{\alpha\alpha}(\Theta) &= \frac{\lambda\beta^2}{\alpha^2}, \quad g_{\alpha\beta}(\Theta) = -\frac{1}{\alpha}\left(\lambda\Psi\left(\lambda\right) + 1\right), \\
g_{\alpha,\lambda}(\Theta) &= \frac{\beta}{\alpha}, \\
g_{\beta\beta}(\Theta) &= \frac{1}{\beta^2}\left[1 + \lambda\Psi(1,\lambda) + \lambda\Psi(\lambda)^2 + 2\Psi(\lambda)\right], \quad (10) \\
g_{\beta\lambda}(\Theta) &= -\frac{\Psi(\lambda)}{\beta}, \quad g_{\lambda\lambda}(\Theta) = \Psi(1,\lambda).
\end{aligned}$$

where $\Psi(1, z)$ is the trigamma function. Finally, by injecting (9) and (10) in (7), one can iteratively estimate the centroid for generalized Gamma distributed sub-bands.

3.3. Centroid representation

In this section, some experiments are conducted on real texture images to evaluate the potential of the centroid definition. To represent the similarity of the texture images, a dimension reduction algorithm is necessary since the manifold $\mathcal{M}$ lives in an high dimensional space. Here, we have considered an isometric feature mapping (isomap) algorithm [13]. This algorithm operates in three steps. First, a pairwise dissimilarity matrix D_1 is computed on the database using the JD:

$$D_1(i,j) = \text{JD}(T_i, T_j), \qquad \forall i,j \in [1, \ldots, N_{Tr}]. \quad (11)$$

Since the JD does not satisfy the triangular inequality, a shortest path algorithm, for instance Dijkstra, is applied to find the shortest distance between two texture images. Hence, isomap estimates the geodesic distances $D_2(i, j)$ between all pairs of images on the manifold $\mathcal{M}$. Next, this matrix is transformed into a covariance-like matrix using a Gaussian kernel, *i.e.*:

$$W = \exp\left\{-\frac{D_2^2}{2 \cdot \sigma^2}\right\}, \quad (12)$$

Finally, a principal component analysis (PCA) is applied on W. The top first eigenvectors (principal component) associated to the highest eigenvalues of W allows an embedding of texture images in a low dimensional space.

To evaluate the potential of a barycentric approach, 15 texture images issued from three different classes of the VisTex database are extracted. For a concise presentation, only three texture classes have been considered in this experiment. For each texture class, one centroid has been estimated. Three centroids are hence computed, they are represented by the red, blue and green squares in Fig. 2. Fig. 2.(a) and Fig. 2.(b) draws a scatterplot of texture images in a subspace of dimension 2 composed by respectively the first two eigenvectors (F_1 and F_2) and by the 2nd and 3rd eigenvectors (F_2 and F_3). As observed the estimated centroids represent well the cluster of each class, validating the interest of the centroid based representation.

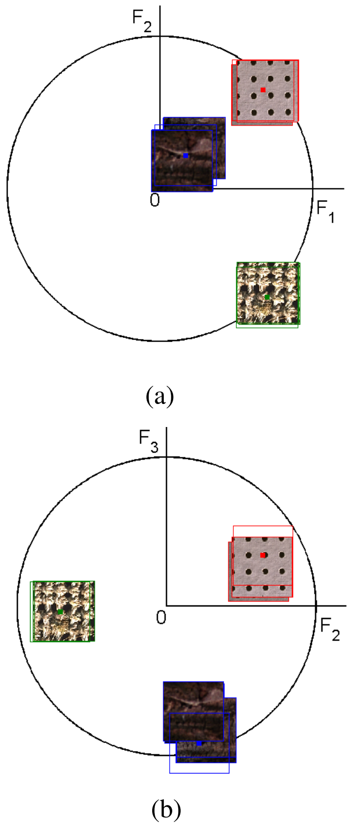

Fig. 2. Embedding of texture images and centroid representation in a subspace of dimension 2 characterized by: (a) the first two eigenvectors of W and (b) the 2nd and 3rd eigenvectors of W.

4. RESULTS

4.1. Context

Let N_s be the number of sub-bands of a multi-scale decomposition. Let us consider the parametric vector Θ_s of the pdf associated to each sub-band. The collection $T = (\Theta_s)_{s=1}^{N_s}$ of those parametric vectors will represent the textured image. Let $(T_{c,n})_{n=1}^{N_{Tr}}$ be N_{Tr} training samples from the same class c. Then, the centroid of this collection of sample is defined as $\bar{T} = (\bar{\Theta}_s)_{s=1}^{N_s}$, where $\bar{\Theta}_s$ is the centroid computed as the solution of (7) at sub-band s. For each texture class $c = 1, \ldots, N_{cl}$, one centroid $\bar{T}_c$ is computed according to the proposed algorithm.

Let T_t be a test sample. This sample is labeled to the class $\hat{c}$, corresponding to the class of the closest centroid, *i.e.*

$$\hat{c} = \arg\min_{c} \mathrm{JD}(T_t \| \bar{T}_c), \tag{13}$$

where the dissimilarity measure JD between two instances of T is computed as the sum of the dissimilarity measures JD between all sub-band distributions at each scale and orientation.

Table 2. Average kappa index on three texture databases for the GΓD, GGD, Weibull and Gamma models.

	GΓD	GGD	Weibull	Gamma
VisTex	**85.2%** $\pm$ **1.4**	81% $\pm$ 1.4	84.4% $\pm$ 1.4	83.8% $\pm$ 1.5
OuTex	**56.7%** $\pm$ **1.4**	**56.7%** $\pm$ **1.5**	55.3% $\pm$ 1.5	54.5% $\pm$ 1.5
VisTexC	**55.3%** $\pm$ **0.9**	50.7% $\pm$ 1.0	54.5% $\pm$ 1.0	54.5% $\pm$ 1.0

To evaluate the performance of the supervised classification algorithms, the database is split into a training database and a disjoint testing database. Practically, N_{Tr} training samples are randomly selected for each texture class, the remaining samples are used as testing samples. In this experiment, three databases are considered: VisTex [14] with 40 classes and $N_{sa} = 16$ images per class (128×128 pixels), OuTex (TC 13) [15], 68 classes, 20 samples per class and VisTex complete (VisTexC), 167 classes, 16 samples per class. In the following, 100 Monte Carlo runs have been used to evaluate the performance of the different classifiers (kappa index). The kappa index refers to the proportion of consistent classifications obtained beyond that expected by chance alone [16, 17].

4.2. Results and discussion

In this experiment, the stationary wavelet decomposition (with 2 scales) with Daubechies' filter db4 is considered. Table 2 displays the average kappa index for the four considered stochastic models (GΓD, GGD, Weibull and Gamma) on the VisTex, OuTex and VisTexC databases. Here, half of the samples are used for training. As observed on the three databases, the proposed supervised classification algorithm based on the GΓD allows a gain in terms of kappa index compared to other conventional univariate modeling.

Fig. 3 draws the evolution of the kappa index as a function of the number of training samples on the VisTex database for the nearest neighbor classifier with GΓD model (1-NN GΓD, solid line in magenta) and for the one centroid classifiers (1-CB) with the GΓD (solid line in blue), GGD (dot-dashed line in green), Weibull (dotted line in red) and Gamma (dashed line in cyan) model assumptions.

As observed, a gain of 4 to 7 points is observed when a centroid based classifier is considered compared to the nearest neighbor classifier. Note also that since the GΓD generalizes the GGD, Weibull and Gamma models, the CB classification algorithm based on the GΓD for the modeling of wavelet coefficient outperforms the one based on more conventional univariate models. A gain of 4%, 1% and 1.5% are respectively observed compared to the GGD, Weibull and Gamma models, illustrating the benefit of the GΓD in a texture retrieval experiment.

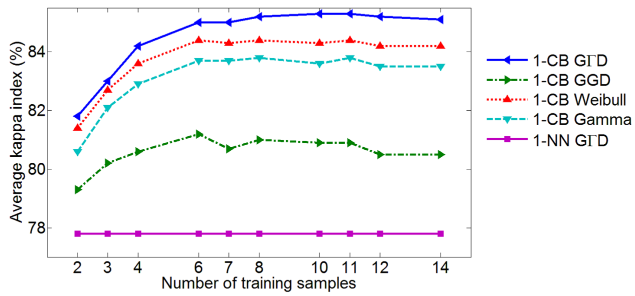

Fig. 3. Evolution of the average kappa index as a function of the number of training samples on the VisTex database for the GΓD, GGD, Weibull and Gamma models.

5. CONCLUSION

In this paper, a centroid-based supervised classification has been introduced to classify texture images. Based on the generalized Gamma distribution (GΓD) for the modeling of wavelet coefficients, we have derived a new algorithm to compute the centroid from a collection of GΓD parameters. Supervised classification results on various texture databases have shown a gain compared to other conventional models.

Further works will concerns the extension of the proposed work to a multi-barycentric classification algorithm in order to handle the intra-class diversity of natural texture images.

6. REFERENCES

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