VIEW DEPENDENT 3-D MESH CODING BY FAST, VIEW IMAGE QUALITY OPTIMIZED, REGION BASED RATE ALLOCATION

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ABSTRACT

In this paper, the problem of rate allocation to the surface regions of 3-D meshes for view dependent compression of their geometries is addressed. The view image MSE based distortion measure is first experimentally shown to yield higher decoded view image Peak-Signal-To-Noise-Ratio (PSNR) than the distortion measures previously proposed in the literature when used in the framework for optimal rate allocation to regions. In place of the optimal rate allocation method, a logarithmic search rate allocation method is also proposed for use with the view image MSE based measure that requires far fewer view image renderings and thereby offers lower complexity in return for a small drop in coding performance.

Index Terms— Mesh, coding, view, rate allocation

1. INTRODUCTION

With increasing computational resources for 3-D graphics, even handheld devices are capable of processing and displaying detailed virtual 3D environments today. This is made possible with concurrent advances in 3-D graphics compression that reduce storage and transmission costs.

3-D mesh is the predominant surface representation tool for the visualization of 3-D objects. Compression of 3-D mesh geometry (vertex coordinates) is treated in this paper since geometry can be used by itself, as in a point cloud, to represent the surface of the 3-D object and requires more bits than mesh connectivity for its adequate description.

In view dependent mesh coding, the coding domain is restricted to only meet the user demands. Only the parts of a mesh visible from a viewpoint are coded along with a specification of lighting conditions. The view of the mesh can be rendered under the same conditions once these parts are decoded. High resolution views with sharp edge details can be transmitted with much fewer bits by view dependent coding than by direct coding of the view image.

A class of transform domain based mesh geometry compression methods [1–3] partition a mesh into regions, transform each region independently, quantize the coefficients and code the resulting levels. The rate for each region or subband is optimally allocated to minimize a distortion measure.

The rate allocation problem initially studied for general purpose mesh coding [4, 5] was extended to view dependent mesh coding in [6–8]. Unlike general purpose mesh coding, it is complicated by the fact that the distortion in the view image is not directly related to the distortion in the 3-D coordinates of the vertices or their wavelet coefficients. [6–8] propose distortion measures to capture this relation without actually rendering the view images and employ rate-distortion optimized rate allocation by using these measures.

The major deficiency of [6–8] is that the expected quality of the reconstructed view image is not measured by an objective measure like the PSNR during coding or rate allocation. In this work, an image rendering based distortion measure (reconstructed view image MSE) is proposed for use in rate allocation. It is experimentally verified that the objective coding performance with this measure bounds from above the objective performances with the measures of [6–8].

In the optimal rate allocation method, rendering of the reconstructed view image for each rate-distortion operating point of a dense set covering the entire rate interval of each region is computationally costly. As the main contribution, a fast rate allocation method, that requires the rendering of far fewer view images, is proposed. It makes an initial rate allocation estimate on a sparse set of rate-distortion points and iteratively refines it with rate transfers between region pairs.

Section 2 covers the view dependent coding system employed for assessing the coding performances with various distortion measures used in rate allocation. Section 3 reviews the method of optimal rate allocation to the visible geometry regions. Section 4 presents the image rendering based distortion measure and other distortion measures in the literature. Section 5 details the proposed logarithmic search rate allocation method. Section 6 covers implementation features and experimental results. Section 7 presents conclusions.

2. THE VIEW-DEPENDENT CODING SYSTEM

The viewpoint coordinates and the lighting conditions are assumed to be communicated to the encoder through a re-

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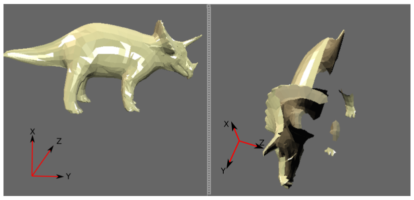

Fig. 1. Parts of *triceratops* visible from +z direction

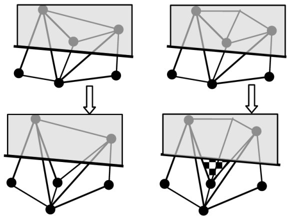

Fig. 2. Black circles show coded vertices. Left: No gap artifacts. Right: Gap artifact (checkerboard). The vertex that crosses the visible region boundary has an uncoded neighbor.

verse channel. As shown in Figure 1, the coding domain of the mesh(es) is limited to the visible parts by applying face culling [7] to conserve rate. First, vertices outside the view frustum pyramid are declared as invisible by using four plane decisions. The back faces are then determined by comparing each face normal against the view vector from any vertex of a face. Finally, a vertex is declared as invisible if it is not incident to the closest face that intersects (determined by the method in [9]) a ray from the viewpoint to the vertex. All vertices of front faces with at least one visible vertex are coded.

The distortion in the vertex coordinates may make an invisible vertex of an occluded face visible. If all the neighbor vertices of this vertex have been coded (left pair of drawings in Figure 2), the parts of faces that become visible can be rendered readily. However, if a neighbor is missing at the display end (right pair in Figure 2), parts of faces which become visible after decoding appear as a gap unless this neighbor is additionally coded or extrapolated from decoded vertices. Gap filling with these approaches was not pursued in this paper.

Visible geometry is partitioned by METIS [10] into regions balanced in number of vertices ($\approx$ 500) as in [1] so that regions with varied complexities are allocated different rates to benefit coding performance. Without loss of generality to other coding systems, for each region, the spectral transform [1] is applied and the coefficients are coded by CSPECK (Color Set Partitioning Embedded Block Coder) [11] to generate a bitstream as in [3]. The concatenated bitstreams form the bitstream of the mesh. The transforms and their inverses are derived from the separately coded connectivity data.

Since the CSPECK coding decisions bound the coefficient value errors and thereby the vertex coordinate errors, but not the view image distortion, the reconstructed view quality is optimized by distributing rate to regions.

3. RATE ALLOCATION TO VISIBLE REGIONS

For N regions (sources) containing P vertices (samples), let b_n be the number of bits allocated to n'th region. The bit allocation vector $\vec{b} = (b_1, b_2 ..., b_N)$ is optimized as

$$\vec{b}^* = \text{argmin}_{\vec{b}\in B} D(\vec{b}) \tag{1}$$

over the convex set $B = \{\vec{b} : R(\vec{b}) = \frac{1}{P}\sum_{n=1}^{N} b_n \leq R_c\}$ where $D(.)$ is the distortion measure and R_c is the rate constraint. An equivalent alternative is the nonparametric Lagrangian rate allocation method [12] that is optimal even if a region has a non-convex operational rate-distortion function.

The rate-distortion values for each region are collected at all operating points in an initial CSPECK run. Since the sources are coded independently, given λ, $b_n^*(\lambda) = argmin_b(J_n(b))$ is solved for n'th region where $J_n(b) = D_n(b) + \lambda R_n(b)$. The allocation vector $\vec{b}^*(\lambda) \in B$ associated with the smallest λ becomes the solution. Finally, n'th region is coded with $b_n^*(\lambda)$ bits in a second CSPECK run.

4. DISTORTION MEASURES FOR VIEW-DEPENDENT RATE ALLOCATION

This section presents the distortion measures that can be used in rate allocation in view dependent coding. View distance, angle between view direction and surface normals, lighting conditions and surface materials play roles in transforming the 3-D geometry distortion to a distortion of the view image.

In the following, let N be the number of visible regions, P_n be the index set of visible vertices in nth region, $\vec{v}_p$ and $\vec{v}'_p$ be the original and reconstructed positions, respectively, and $\vec{n}_p$ be the unit surface normal vector of the pth vertex in P_n.

1. Distance Based Distortion Measure: Motivated by the observations that the magnitude of the projection the error vector of a vertex on the screen is inversely proportional to its view distance and faces closer to the viewpoint occupy a larger screen space, [7] defines the distance based measure as

$$D_{dist} = \sum_{n=1}^{N} \frac{D_{3DMSE,n}}{\bar{d}_n^2} = \frac{1}{P}\sum_{n=1}^{N} \frac{\sum_{p\in P_n} \|\vec{v}_p - \vec{v}'_p\|^2}{\bar{d}_n^2} \tag{2}$$

where $\bar{d}_n$ is the average distance of these vertices to the viewpoint. For a fair comparison with (2), the next two measures also incorporate weights that are proportional to $(\bar{d}_n^2)^{-1}$.

2. Visibility Based Distortion Measure: The use of this measure allocates more bits to more visible vertices or regions. The visibility priority score [6] is defined as

$$D_{vis} = \sum_{n=1}^{N} \frac{\sum_{p\in P_n} (max((\vec{n}_p \cdot \vec{V}), 0)^2 \cdot (\|\vec{v}_p - \vec{v}'_p\|)^2)}{\bar{d}_n^2}$$

where the view direction vector is denoted as $\vec{V}$.

3. Screen Space Based Distortion Measure: The screen space distortions were used in [8] for view dependent wavelet based progressive coding of semiregular meshes and [13] for view dependent refinement. Both emphasize the vertex error vector component orthogonal to the view vector. In [13], if larger, the component of the error vector along the surface normal direction replaces it. Better objective decoded view image quality is obtained (see Section 6) with the screen space distortion measure adapted from the test in [13] as

$$\begin{aligned} D_{scrsp,n} \quad = \quad & \sum_{n=1}^{N} \frac{1}{\bar{d}_n^2} \sum_{p \in P_n} max(\|\vec{v}_p - \vec{v}_p'\|^2, \\ & \|proj(\vec{v}_p - \vec{v}_p')_{\vec{n}_p} \times \frac{\vec{v}_p - \vec{e}}{\|\vec{v}_p - \vec{e}\|}\|^2) \qquad (3) \end{aligned}$$

than the one in [8] where $\vec{e}$ is the viewpoint.

4. Illumination Based Distortion Measure: The Blinn-Phong reflection model suggests the squared illumination error along the segment normal $\vec{N}$ projected onto $\vec{V}$,

$$\begin{aligned} D_{ill,p} \quad = \quad & \max(0, (\vec{N} \cdot \vec{V}))Sa(p) \\ & (\sum_{c \in lights} k_{dif}(max(0, \vec{n}_p \cdot \vec{l}_c) - max(0, \vec{n}_p' \cdot \vec{l}_c)) \\ & + k_{spec}(max(0, \vec{n}_p \cdot \vec{h}_c)^\alpha - max(0, \vec{n}_p' \cdot \vec{h}_c)^\alpha))^2 \end{aligned}$$

to be the distortion contribution of $\vec{v}_p$, where k_{dif} and k_{spec} are the diffuse and specular reflection constants, $\vec{l}_c$ is the direction vector to c'th light, $\vec{h}_c = \frac{\vec{l}_c + \vec{V}}{\|\vec{l}_c + \vec{V}\|}$, $Sa(p)$ is the area of the faces around $\vec{v}_p$ [8] and α is the shininess constant.

5. Image Rendering (MSE) Based Distortion Measure: Let $I(x, y)$, $I'(x, y)$ be the luminance values at the (x, y) coordinates of the original and reconstructed view images of the mesh(es), respectively. The image rendering (MSE) based distortion measure is defined as

$$D_{2DMSE} = \frac{1}{X \cdot Y} \sum_{x \in [1,X], y \in [1,Y]} (I(x, y) - I'(x, y))^2$$

The rendering parameters (the viewpoint, lighting conditions and surface material) used at the decode/display end have to match the ones used in rate allocation. Otherwise, the view image of the reconstructed mesh may not be the best in the MSE sense for the bit budget expended.

When this measure is used in the optimal rate allocation framework, the exhaustive rendering of the view images (of a reasonable resolution) of the reconstructed meshes for all operating points is computationally predominant compared to other parts of the rate allocation process. However, the reconstructed view image PSNR vs. rate performance is superior to the performances achieved by using other measures.

5. LOG SEARCH RATE ALLOCATION METHOD

A logarithmic search bit allocation method for the image rendering based measure is proposed here that balances performance and computational complexity. Since a rate increase is rarely accompanied by a (transient) increase of D_{2DMSE}, the admissible b_n are on a hyperplane $\mathcal{H} = \{b_n : \sum_{n=1}^{N} b_n = PR_c\}$ in the first quadrant of $\mathcal{R}^N$ with normal $(1, 1, 1, ...)$.

For initialization, the method of Section 3 is first applied on three operating points equally spaced in rate over $[0, PR_c]$ for each region. The resulting coarse bit allocation vector on $\mathcal{H}$ is then improved iteratively by walking on $\mathcal{H}$.

The iterations are organized in multiple levels. The allocations after level $l - 1$ satisfy $\sum_{n=1}^{N} b_{n,l-1} = PR_c$. At the m'th iteration of l'th level, ϵ_l bits are transferred from $n_l^-(m)$'th region to $n_l^+(m)$'th region such that the net distortion decrease among such region pairs is maximized. The new allocations for the two regions are $b_{n_l^+(m),l} = b_{n_l^+(m),l-1} + \epsilon_l$ and $b_{n_l^-(m),l} = b_{n_l^-(m),l-1} - \epsilon_l$. This may be viewed as a step of size ϵ_l along the line $b_{n_l^-(m)} + b_{n_l^+(m)} = PR_c - \sum_{\{n:n \neq n_l^-(m), n_l^+(m)\}} b_{n,l-1}$. When the largest possible distortion decrease in an iteration is negative, the iterations of level l are stopped, and level $l + 1$ is started with $\epsilon_{l+1} = \epsilon_l / 2$. This regime spans a large rate interval and zooms in on a rate of desired precision at the end. Since two small steps in the same direction are equivalent to one larger step, at most one step for each region is allowed at each level. The set of candidate allocations for n'th region at level l is $B_{n,l} = \{b_{n,l-1} - \epsilon_l, b_{n,l-1}, b_{n,l-1} + \epsilon_l\}$. The bit allocation grid at level l is $\mathcal{B}_l = \{\prod_{n=1}^{N} B_{n,l} \cap \mathcal{H}\}$. The theorem below guides the selection of the two regions at each iteration:

Theorem: Let $\delta D_{n,l}$ be the distortion contribution change due to a rate contribution change of $\delta R_{n,l}$ of n'th region in level l. If the regions are selected in pairs iteratively according to

$$n_l^+(m) = \arg \min_{n \in \mathcal{N}_l(m)} \frac{\delta D_{n,l}}{\delta R_{n,l}} \text{ for } \delta b_{n,l} = P\delta R_{n,l} = \epsilon_l \quad (4)$$

$$n_l^-(m) = \arg \max_{n \in \mathcal{N}_l(m)} \frac{\delta D_{n,l}}{\delta R_{n,l}} \text{ for } \delta b_{n,l} = P\delta R_{n,l} = -\epsilon_l$$

for $m = 1, \ldots$ where $\mathcal{N}_l(m) = \{1, \ldots, N\} \setminus \{n_l^+(1), n_l^-(1), \ldots, n_l^+(m-1), n_l^-(m-1)\}$, pairwise bit transfers yield the optimal allocation on $\mathcal{B}_l$.

Proof: Since $\frac{\delta J_{n,l}}{\delta R_{n,l}} = \frac{\delta D_{n,l}}{\delta R_{n,l}} + \lambda$, (4) is equivalent to
$n_l^+(m) = \arg\min_{n \in \mathcal{N}_l(m)} \frac{\delta J_{n,l}}{\delta R_{n,l}}$ for $\delta b_{n,l} = \epsilon_l$ and
$n_l^-(m) = \arg\max_{n \in \mathcal{N}_l(m)} \frac{\delta J_{n,l}}{\delta R_{n,l}}$ for $\delta b_{n,l} = -\epsilon_l$

Let $q_{n,l}^\alpha = \frac{\delta J_{n,l}}{\delta R_{n,l}}$ when $\delta b_{n,l} = \alpha\epsilon_l$ for $n = 1, \ldots, N$. and $\alpha \in \{+, -\}$. The cost changes $q_{n,l}^+$ are sorted in increasing order to yield the index list $n_l^+(m)$ $m = 1, \ldots, N$ and $q_{n,l}^-$ are sorted in decreasing order to yield the index list $n_l^-(m)$ $m = 1, \ldots, N$. Let $M_l = max\{m : q_{n_l^+(m),l}^+ < q_{n_l^-(m),l}^-\}$ The optimum allocation allocates ϵ_l more bits to regions with indices $n_l^+(m)$ $m = 1, \ldots, M_l$ and ϵ_l less bits to regions with indices $n_l^-(m)$ $m = 1, \ldots, M_l$ so that the maximum cost reduction

$$\delta J_l = \sum_{m=1}^{M_l} \frac{\epsilon_l}{P}(q_{n_l^+(m),l}^+ - q_{n_l^-(m),l}^-) \qquad (5)$$

is obtained. This may be achieved by forming M_l region pairs with indices $(n_l^+(m), n_l^-(m))$ $m = 1, \ldots, M_l$ and transferring ϵ_l bits from one in each pair to the other. Each pair contributes a negative amount (term in the sum of (5)) to the cost change. Any more pairs increase the cost change and should be avoided. A systematic way to ensure that the first M_l indices of each list are used to form the pairs is to determine $n_l^+(m)$ and $n_l^-(m)$ by (4). To avoid $n_l^+(m) = n_l^-(m)$, second to maximum or minimum candidate regions are selected.

The search for the number of bits allocated to the regions is a coupled search down tertiary trees. For n'th region, the intermediate node at level $l-1$ associated with allocation $b_{n,l-1}$ has three offsprings at level l associated with $B_{n,l}$. The selected offspring corresponds to the allocation $b_{n,l}$. The tree is traversed from the root node (initial allocation) down to level L with the final step size $\epsilon_L = \epsilon_1/2^{L-1}$ giving the desired allocation precision. The final step size for the log search method is set equal to the number of bits spacing between the operating points of the globally optimal method to compare these methods in a fair way. The number of bits spacing (final step size) and rate spacing are related by $\delta\epsilon_L = P\delta R_{n,L}$.

In the proposed method, the renderings at the initial sparse set of three operating points of each region are performed at a quarter of the resolution, while the ones at the lower search tree nodes are performed at the full resolution of the original view image. This conserves some complexity while degrades PSNR by < 0.05dB. Let the full and quarter resolution rendering complexities be C_f and C_q, respectively. For each region, the total rendering complexities of the log-search method with final step size of ϵ_L ($L \geq 1$) and the globally optimal rate allocation method with a number of bits spacing of ϵ_L are $4C_q + 2(L+1)C_f$ and $2^{L+2}C_f$, respectively.

6. IMPLEMENTATION DETAILS AND EXPERIMENTS

Five view images (of 512x512 full resolution) of five scenes have been used in the experiments that were conducted on a 2GHz Core2 T7200 CPU system with GeForce Go7400 GPU. Scenes with multiple objects offer a wider range of region distances. For the distortion computation of a region, the reconstructed view image is rendered by using the decoded vertices of the region along with the original vertices of other regions. The framebuffer object feature of OPENGL is used to achieve low rendering complexity by rendering to offscreen buffers. For all view images of a specific resolution rendered during rate allocation, a single pair of color render buffer and depth buffer is created and used.

From Figure 3, the view image PSNR vs. Rate performance of the optimal rate allocation with the illumination based distortion measure [8] is seen to be inferior to the performance of the optimal rate allocation with the screen space based distortion measures proposed in [8] and adapted from [13] to (3), since the illumination based measure only depends on the changes in the normal vectors, but not the effects of the changes in the contour of the object such as occlusions or unocclusions due to lossy geometry coding. Therefore, the screen space measure is considered by itself here and not linearly combined with the illumination based measure as in [8]. Secondly, since the screen space definition of (3) is consistently the winner over the definition in [8], (3) is used in the rest of the experiments. Finally, even with a larger rate spacing (lower precision), the performance of the optimal rate allocation with the image rendering based measure is superior to its performances with the other distortion measures.

Table 1 reports the average run times for the coding of 'Horse' with the two rate allocation methods and various distortion measures considered. The times for face culling, surface partitioning, spectral transformation and final coding run are excluded. In the first and fifth rows, the complexities for the optimal rate allocation method and the log-search method are seen to be proportional to the inverse of ϵ_L and the logarithm of the inverse of ϵ_L, respectively. A small ϵ_L does not benefit the view image quality when distortion measures other than the image rendering based measure are used.

Since optimal rate allocation ($\epsilon_L = 3.2P$) and log-search rate allocation ($\epsilon_L = 1.6P$) with the image rendering based measure have similar complexities (Table 1), the comparison of their view image coding performances in Figure 4 is fair. At low rates ($\leq$3bpv), log-search rate allocation has as much as a few dB's PSNR advantage over optimal rate allocation.

For $\epsilon_L = 0.1P$, the reconstructions in Figure 5 show that the use of the image rendering based measure in rate allocation yields a discernable average quality gain over the use of the three distortion measures considered.

7. CONCLUSIONS

In this work, it has been shown experimentally that the image rendering based distortion measure gives the best coding performance in the optimal rate allocation framework. However, if optimal rate allocation needs to be performed quickly or updated frequently due to changes in viewpoint or lighting conditions, the use of this measure with a small rate spacing may not be feasible. To address this issue, a low complexity log-search rate allocation method is proposed for use with the image rendering based measure which enables a coding performance close to that of the optimal rate allocation method and well over the performances of the other measures.

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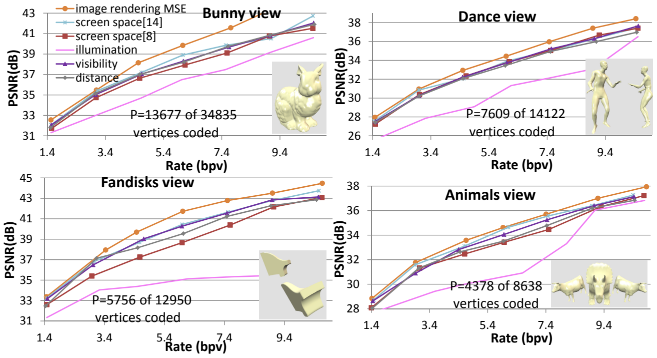

Fig. 3. Coding performances with optimal rate allocation employing various distortion measures ($\epsilon_L = 0.1$ for all except image rendering MSE distortion measure for which $\epsilon_L = 0.8$.

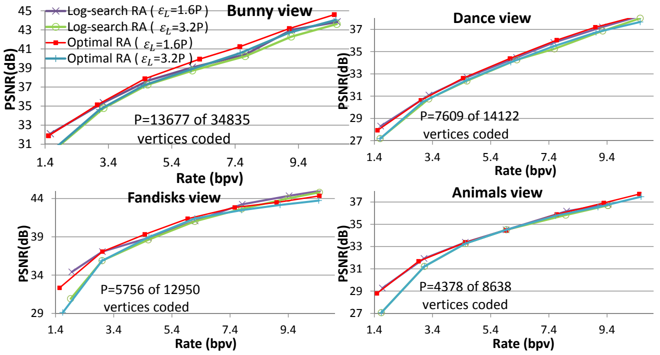

Fig. 4. Coding performances of the rate allocation methods employing image rendering MSE distortion measure

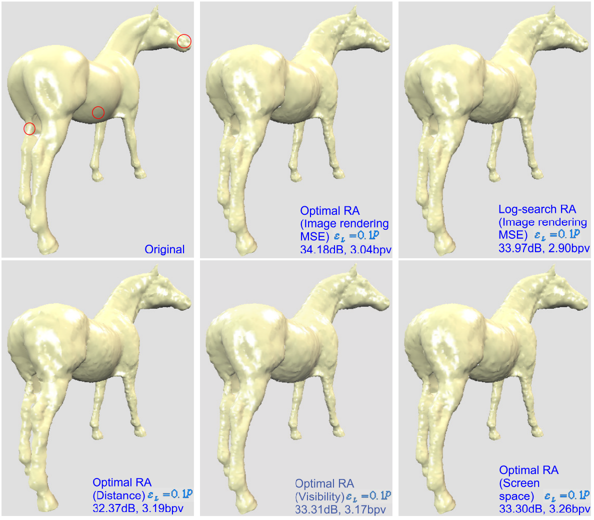

Fig. 5. Original and reconstructed view images of horse. Red circles show areas of interest for comparison.

	$\epsilon_L = 0.1P$	$\epsilon_L = 0.4P$	$\epsilon_L = 0.8P$	$\epsilon_L = 1.6P$	$\epsilon_L = 3.2P$
Optimal	47.6	12.5	6.9	4.2	2.6
imageMSE	*34.18,3.04*	*34.11,3.17*	*33.90,3.02*	*33.77,3.12*	*32.99,3.14*
Optimal	21.7	5.5	2.9	1.6	0.8
distance	*32.37,3.19*	*32.37,3.10*	*32.63,3.10*	*32.70,3.08*	*30.56,3.16*
Optimal	38.5	9.8	5.2	2.9	1.5
scr. space	*33.30,3.26*	*33.35,3.13*	*33.32,3.15*	*33.29,3.09*	*32.10,3.15*
Optimal	35.7	9.1	4.8	2.6	1.4
visibility	*33.31,3.18*	*33.28,3.12*	*33.26,3.10*	*33.25,3.11*	*31.42,3.15*
Logsearch	4.9	3.7	3.2	2.6	2.1
imageMSE	*33.97,2.90*	*33.87,2.90*	*33.78,2.90*	*33.61,2.92*	*32.55,2.96*

Table 1. Run times(sec.), *(PSNR (dB), Rate (bpv))* for horse

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