BOUNDED SELECTIVE SPANNING WITH EXTENDED FAST ENUMERATION FOR MIMO-OFDM SYSTEMS DETECTION

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ABSTRACT

Selective Spanning Fast Enumeration (SSFE) is a promising detection approach for Multiple-Input Multiple-Output (MIMO) systems which allows detection performance to be traded off against computational complexity depending on the communications environment. However, SSFE suffers from two drawbacks: its symbol enumeration strategy cannot properly span modulation schemes of 16 QAM or greater, leading to increased BER, and is computationally inefficient as a result of enumerating redundant symbols. This paper presents Bounded Selective Spanning with Extended Fast Enumeration (BSS-EFE), an adapted SSFE which resolves these issues, avoiding redundant symbol enumeration and enabling increased detection performance and lower complexity for SSFE-based detection systems employing 16 QAM or greater modulation schemes. Experiments show that BSS-EFE this enables to 1 dB BER gain whilst reducing computational complexity up to 16.3% as compared to SSFE.

Index Terms— Sphere Decoder, Selective Spanning with Fast Enumeration (SSFE), Multiple-Input Multiple-Output (MIMO) detection

1. INTRODUCTION

The high spectral efficiency of MIMO makes it a key enabling technology for next-generation high data rate wireless communications [1]. However, the complexity of a number of MIMO reception tasks, in particular symbol detection has prompted the invention of low-complexity sub-optimal algorithms for deployment on embedded detectors [2].

A number of Sphere Decoding (SD) [3] variants have proven to be very promising sub-optimal detection algorithms. Amongst these, SSFE is particularly notable not only for its high detection performance and ability to balance computational complexity and detection performance depending on the detector's communications and embedded environments, but also due to its deterministic behaviour, which makes it highly suited to efficient embedded realisation [4]. However, SSFE suffers from two restrictions: when employed for 16 QAM (Quadrature Amplitude Modulation) or greater, it is unable to enumerate large parts of the QAM constellation potentially restricting detection performance and it may enumerate redundant symbols, reducing computational efficiency.

This paper proposes BSS-EFE, an adapted SSFE scheme which overcomes these issues by demonstrating:

1) a novel selective spanning approach which avoids enumerating redundant points, improving BER by an average of 1 dB (25.9%) SNR as compared to SSFE
2) a symbol enumeration approach which can span all members of a QAM constellation enhancing performance by a further 1 dB SNR whilst reducing computational complexity by 16.3% as compared to SSFE.

Section 2 introduces SSFE and motivates BSS-EFE, which is described in Section 3 . Section 4 demonstrates the quoted performance and complexity gains.

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2. BACKGROUND

For a $N_t \times N_r$ ($N_t \leq N_r$) MIMO system with N_t transmit and N_r receive antennas communicating over a multipath fading channel $\mathbf{H} \in \mathbb{C}^{\mathbf{N_r} \times \mathbf{N_t}}$ with Additive White Gaussian Noise (AWGN) $\mathbf{v} \in \mathbb{C}^{N_r \times 1}$ at the receiver, the distorted received symbols $\mathbf{y} \in \mathbb{C}^{\mathbf{N_r} \times \mathbf{1}}$ resulting from transmitted symbol vector $\mathsf{s} \in \mathbb{C}^{\mathbf{N_t} \times \mathbf{1}}$ can be expressed as

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{s} + \mathbf{v}. \tag{1}$$

Sphere Decoding (SD) is a popular optimal technique for estimating s given $\mathbf{y}$ [2] [5]. However, SD is highly computationally complex and infeasible for practical MIMO systems [6] [7]. As a result a series of sub-optimal SD approaches with reduced computational complexity have emerged. SSFE is one such technique which allows the complexity and performance to be traded-off against one another by specifying the number of candidate symbols enumerated at each stage of the detection process.

Each SSFE configuration is uniquely identified by a vector $\mathbf{m} = \{m_1, ..., m_{N_t}\}$, which signifies the number of symbols enumerated at each stage nt of the SD search tree. Fig. 1 shows an example SSFE topology for $\mathbf{m} = \{1, 2, 2, 4\}$.

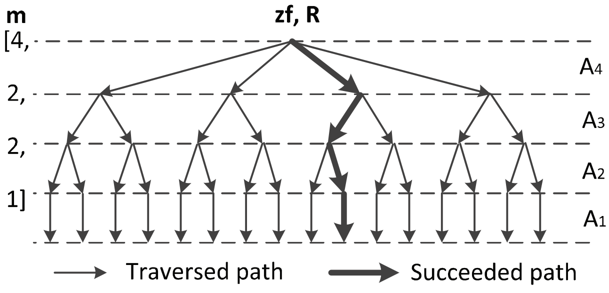


Fig. 1: The SSFE Algorithm Example of $\mathbf{m} = [1, 2, 2, 4]$

As this shows, SSFE begins with a zero-forcing equalized received signal, $\mathbf{zf} = (\mathbf{H}^H \cdot \mathbf{H})^{-1} \cdot \mathbf{H^H} \cdot \mathbf{y}$ [1] and $\mathbf{R}$, an upper triangular matrix obtained by QR decomposition of $\mathbf{H}$.

At each SSFE enumeration stage, the Partial Euclidean Distance (PED) is accumulated as given by (2), with $\widehat{zf}_{nt,i}$ given by (3).

$$APED_{nt,i} = APED_{nt+1,i} + \sum_{l=nt}^{N_t} r_{l,l}^2 \cdot \left\| \widehat{zf}_{nt,i} - \widehat{s}_{nt,i} \right\|^2, \quad APED_{N_t,i} = 0, \tag{2}$$

$$\widehat{zf}_{nt,i} = zf_{nt,i} - \sum_{l=nt+1}^{N_t} \frac{r_{nt,l}}{r_{l,l}} \cdot (zf_{l,i} - \widehat{s}_{l,i}), \quad \widehat{zf}_{N_t,i} = zf_{N_t,i}. \tag{3}$$

In SSFE the candidate symbols for enumeration $\widehat{s}_{nt,i}$ are identified from the valid QAM constellation symbols via a 'fast' derivation heuristic. This heuristic selects a set of candidates based on the sliced value of the ZF equalized symbol $\widehat{s}_{nt,0} = \mathcal{Q}\left(\widehat{zf}_{nt,i}\right)$ from (3). The candidates are selected by identifying the set of symbols closest to $\widehat{s}_0$ [2] via the heuristic described in (4), where $d = \widehat{zf}_{nt,i} - \mathcal{Q}\left(\widehat{zf}_{nt,i}\right)$, $\phi = \Re(d) > \Im(d)$, $R_{sgn} = \text{sgn}(\Re(d))$ and $I_{sgn} = \text{sgn}(\Im(d))$

$$\begin{cases} \widehat{s}_1 = \widehat{s}_0 + 2 \cdot (R_{sgn} \cdot \phi + j \cdot I_{sgn} \cdot (!\phi)), \\ \widehat{s}_2 = \widehat{s}_0 + 2 \cdot (R_{sgn} \cdot (!\phi) + j \cdot I_{sgn} \cdot \phi), \\ \widehat{s}_3 = \widehat{s}_0 + 2 \cdot (R_{sgn} + j \cdot I_{sgn}), \\ \widehat{s}_4 = \widehat{s}_0 - 2 \cdot j \cdot I_{sgn}, \\ \widehat{s}_5 = \widehat{s}_3 - 4 \cdot j \cdot I_{sgn}, \\ \widehat{s}_6 = \widehat{s}_0 - 2 \cdot R_{sgn}, \\ \widehat{s}_7 = \widehat{s}_3 - 4 \cdot R_{sgn}, \end{cases} \tag{4}$$

Consider the behaviour of this enumeration process in the case of an example received symbol with arbitrary ZF value of $(-3.5, -2.1)$, as illustrated in Fig. 2. As this shows, ZF values results in $\widehat{s}_0$ taking a value at the edge of the valid constellation, with the connected points denoting the candidate symbols for the detection process.

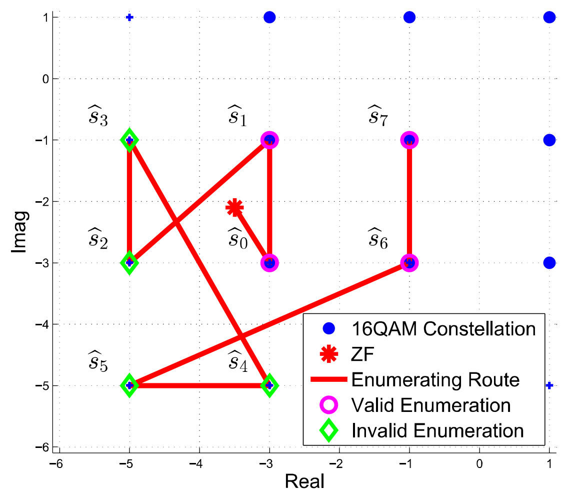


Fig. 2: SSFE Spanning Procedure [4]

There are two issues which arise in this scenario. Firstly, only 8 candidates are enumerated. This is an insufficient number to span the entirety of the QAM constellation for systems employing 16 QAM or greater. Whilst enumerating all points is infeasible, restricting the selection to just over 50% of the possible transmitted symbols potentially restricts detection performance. Furthermore, in the illustrated case 50% of the enumerated symbols, $\widehat{s}_2$ - $\widehat{s}_5$, are known to be impossible since they lie outside the valid constellation. However the SSFE enumeration process is unable to detect the invalidity of these points and accordingly evaluates them nevertheless. These represent redundant APED computations, leading to computational inefficiency in the SSFE algorithm.

Combined, these issues restrict the potential performance of the SSFE detection approach and reduce its computational efficiency. To overcome these issues, novel selective spanning and complete enumeration approaches for SSFE are required. Two such approaches are described in Section 3.

3. BOUNDED AND EXTENDED SSFE

3.1. Bounded Selective Spanning (BSS)

In Fig. 3, the enumerated symbols $\widehat{s}_2$ - $\widehat{s}_5$ are invalid yet will be evaluated despite the fact that their occurrence is known not to be possible. This leads to computational redundancy. This section describes a Bounded Selective Spanning (BSS) process which maps these points to valid 16 QAM points.

To avoid redundant enumeration invalid points outside the constellation must be mapped to valid points inside. Letting

[1] $(\cdot)^H$ denotes complex conjugate transpose

[2] Note that a since a similar enumeration process is repeated for each transmit antenna the index nt present in (3) is omitted hereafter for clarity

M_c represents the cardinality of the modulation scheme (for example, 16 for 16 QAM) an enumeration bound may be defined as

$$TH = \sqrt{M_c} - 1. \tag{5}$$

This bound acts as a threshold which, when exceeded, will indicate that a specific symbol lies outside of the valid constellation. This is employed to calculate an $offset$ which maps the invalid point to a valid alternative within the constellation, as defined by (6).

$$\text{offset} = \begin{array}{l} -2 \cdot scale \cdot \left(\left(\Re\left(\widehat{s}_{ind}\right) > TH \right) \cdot \text{sgn}\left(\Re\left(\widehat{s}_1\right)\right) \right. \\ \left. +j \cdot \left(\Im\left(\widehat{s}_{ind}\right) > TH \right) \cdot \text{sgn}\left(\Im\left(\widehat{s}_1\right)\right) \right), \end{array} \tag{6}$$

where $scale = \left\lfloor \sqrt{m_{nt} - 0.1} \right\rfloor + 1$.

Since $\widehat{s}_0$ is always sliced to the valid constellation the additive offset given by (6) is only applied to the remaining symbols. The effect of this bounded spanning approach is illustrated in Fig. 3, which shows how the invalid points from Fig. 2, $\widehat{s}_2 - \widehat{s}_5$ are mapped to valid alternatives. Note that the mapped symbols are as close as possible to the initial point whilst increasing distance from it, in keeping with the original SSFE strategy.

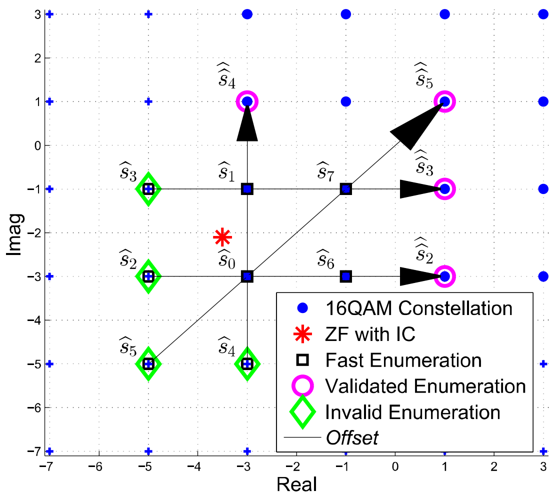


Fig. 3: Bounded Spanning

3.2. Extended Fast Enumeration (EFE)

As outlined in Section 2, $\mathbf{m}$ represents the unique configuration of each SSFE scheme with m_{nt} representing the number of symbols enumerated at stage nt in the detection tree. However, the enumeration strategy outlined in Section 2 was also limited to 8 cases, meaning that 16 QAM modulation schemes or greater may not be completely enumerated potentially restricting BER performance. In this section an Extended Fast Enumeration (EFE) scheme is proposed to overcome this restriction.

It is proposed that the enumeration conditions in (4) are reformulated as (7).

$$\left\{ \begin{array}{l} \widehat{s}_0 = \mathcal{Q}\left(\widehat{zf}_{nt,i}\right), \\ \widehat{s}_{k+1} = \widehat{s}_k + v_1 \cdot (-1)^{w+1}, \\ \widehat{s}_{l+1} = \widehat{s}_l + v_2 \cdot (-1)^{w+1}, \end{array} \right. \tag{7}$$

This approach ensures a consistently increasing distance from the initial symbol estimate but is not limited to 8 points but rather is directly controlled by $\mathbf{m}$ via the terms w, k and l in (8). Furthermore, whilst these control the scale of enumeration, they do not influence the direction, which may proceed in either a clockwise or anticlockwise direction around $\widehat{s}_0$. This direction is controlled by v_1 and v_2 in (7) as defined in (9).

$$\left\{ \begin{array}{l} w = \left[1, \left\lceil \sqrt{m_{nt} + 0.25} - 0.5 \right\rceil \right], \\ k = \left[(w-1) \cdot w + 1, w \cdot w\right], \\ l = \left[w \cdot w + 1, (w+1) \cdot w\right]. \end{array} \right. \tag{8}$$

$$\left\{ \begin{array}{l} v_1 = sgn\left(\Re\left(d\right)\right) \cdot \phi + j \cdot sgn\left(\Im\left(d\right)\right) \cdot (!\phi), \\ v_2 = sgn\left(\Re\left(d\right)\right) \cdot (!\phi) + j \cdot sgn\left(\Im\left(d\right)\right) \cdot \phi, \end{array} \right. \tag{9}$$

Fig. 4 illustrates the enumeration route for 15 QAM symbols enumerated in either a clockwise or anticlockwise direction from the same initial estimate as that in Fig. 2. As this illustrates, 15 points are enumerated with the invalid points $\widehat{s}_2 - \widehat{s}_6, \widehat{s}_{11} - \widehat{s}_{15}$ which would have occured without BSS mapped to equivalent positions within the constellation.

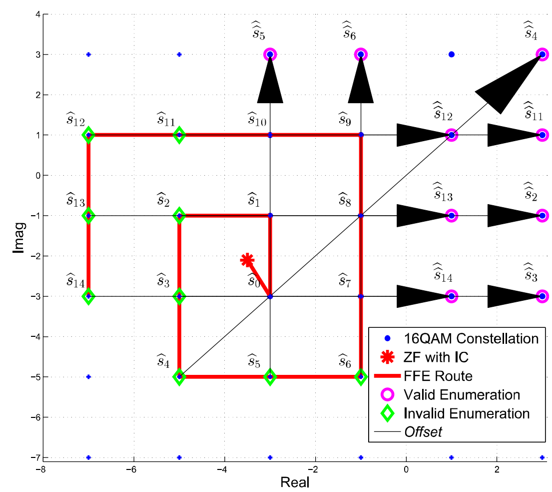


Fig. 4: Extended Enumeration with Bounded Spanning

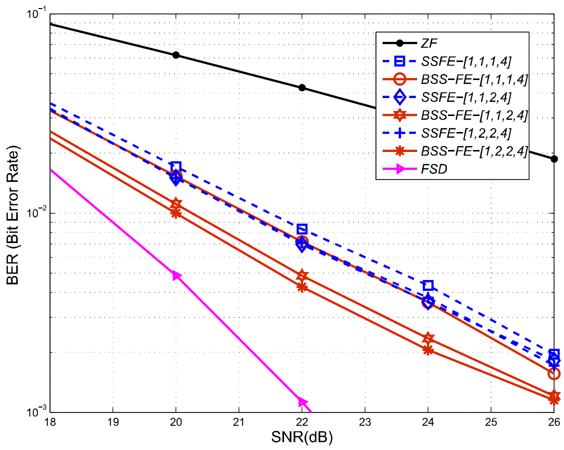
4. EXPERIMENTS

4.1. BSS-FE: BER and Complexity

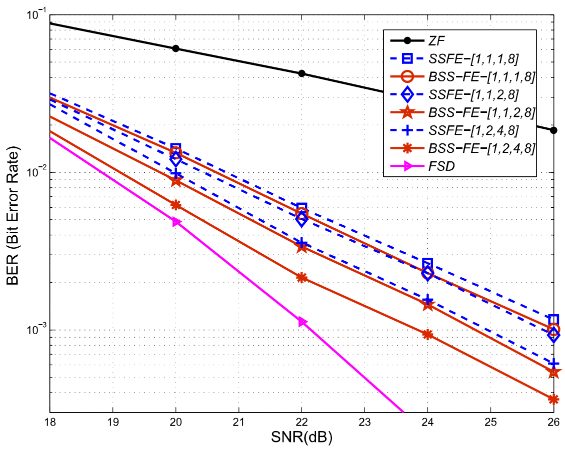
In this section, we analyse the BER performance and complexity of the augmented SSFE as compared to the original. The detection performance of both the BSS and basic

SSFE [4] schemes are analysed when applied to an ideal MIMO OFDM fading channel with AWGN when perfect complex Gaussian cyclic-prefix (CP) is employed. The uncoded hard decision for 4×4 MIMO employing 16 QAM is measured for 1×10^5 48-symbol frames, according to the $802.11n$ standard [8].

Fig. 5 shows the BER performance of BSS-FE and the original SSFE schemes for a series of values of $\mathbf{m}$ ranging from $\mathbf{m} = [1, 1, 1, 4]$ to $\mathbf{m} = [1, 2, 4, 8]$. For calibration purposes, Fig. 5 also illustrates the performance of the Fixed Complexity Sphere Decoder (FSD), a low complexity quasi-ML SD variant [7].



(a) $\mathbf{m} = [x, x, x, 4]$



(b) $\mathbf{m} = [x, x, x, 8]$

Fig. 5: BER Performance of BSS-FE

Fig. 5 shows that whilst quasi-ML performance is not obtained, BSS-FE offers superior BER performance to SSFE in all cases; for SSFE schemes enumerating more than one symbol on multiple levels, such as the $[1, 1, 2, 4]$, BSS-FE enables approximately $1.0 \sim 1.5$ dB SNR gain beyond SSFE. For SSFE schemes enumerating more than one symbol in a single layer, such as $[1, 1, 1, 4]$ performance gain varies between $0.4 \sim 0.6$ dB, with an average improvement of 0.5 dB.

Fig. 6 illustrates the computational cost increase of this enhanced performance. The complexity of BSS-FE is increased as compared to SSFE with approximate increases reducing from 10.2% to 4% (average 7.1%) as the enumeration varies between $\mathbf{m} = [1, 1, 1, 4]$ to $\mathbf{m} = [1, 2, 4, 8]$. This increased complexity is to be expected given the extra complexity of implementing the offset calculation to map invalid constellation points to valid alternatives. This increase is of course undesirable but is relatively mild given the substantial performance gains associated. As compared with FSD the complexity of the highest performance BSS-FE ($\mathbf{m} = [1, 2, 4, 8]$) is relatively high given the quasi-ML performance of FSD, however FSD cannot achieve the same lower complexity cost points which BSS-FE can in cases where sub-ML performance is acceptable.

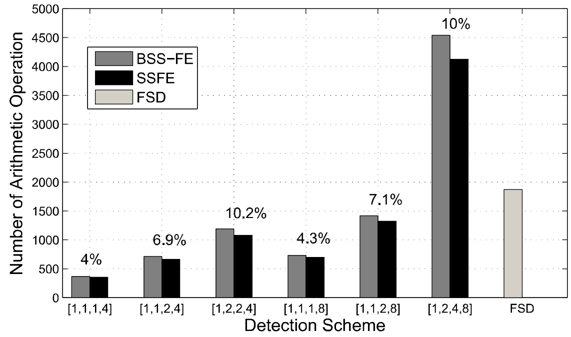


Fig. 6: BSS-FE v. SSFE Complexity Comparison

4.2. BSS-EFE: BER and Complexity

In this section we gauge the performance and complexity of the augmented SSFE with both the proposed BSS-EFE refinements. The same experimental conditions as employed in Section 4.1 are adopted once again to compare configurations of SSFE and BSS-EFE. Figs. 7 and 8 illustrate the benefits of BSS-EFE - both $\mathbf{m} = [1, 2, 2, 12]$ and $\mathbf{m} = [1, 1, 4, 12]$ offer performance gain of more than 1 dB BER as compared to $\mathbf{m} = [1, 2, 4, 8]$ SSFE, despite enabling reductions in computational complexity of more then 16% and 2% respectively.

5. CONCLUSION

This paper has proposed an augmented SSFE scheme, called BSS-EFE, which overcomes the limitation of existing SSFE approach by eliminating redundant computation caused by enumerating invalid constellation points and formalizing a

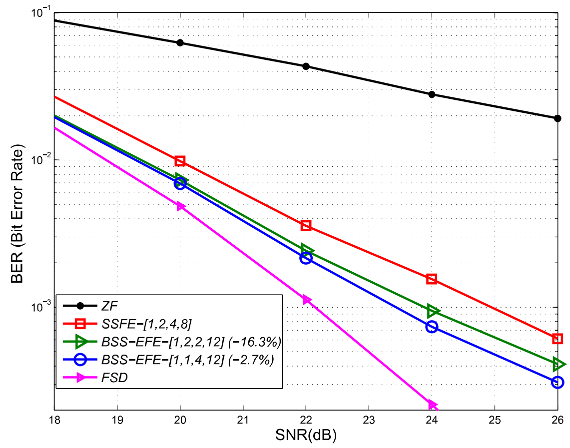


Fig. 7: BSS-EFE BER Performance

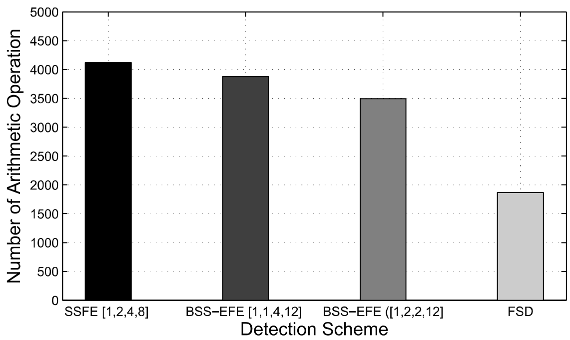


Fig. 8: SSFE/BSS-EFE Complexity Comparison

mechanism to enumerate more than eight points. By exploiting a modulation-related bound, an alternative enumeration approach has been presented which maintains fast enumeration and increases detection performance by 1 dB SNR at the cost of moderate complexity gain of around 7% on average. However, when combined with a complete enumeration approach which enables enumeration of more than 8 data points, a similar performance gain is enabled whilst reducing overall complexity by more than 16%. This scheme should become increasingly more relevant as the scale of MIMO systems and the order of modulation increase in future to include more antennas, demanding larger numbers of enumerated symbols to maintain diversity and detection performance.

6. REFERENCES

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