EFFECTS OF NOISE CORRELATION ON LEAST SQUARES FILTERING IN MULTIPATH DETECTION FOR GNSS

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ABSTRACT

In GNSS (Global Navigation Satellite System) multipath (MP) results to be one of the main error sources affecting the GNSS solution.
In this paper a Linear Adaptive Filter (LAF) technique [1] is applied, based on Least Squares (LS), to estimate the MP coefficients and delay by using a post-correlation approach. An assumption using LAFs [1] is the noise to be a white process, but considering post-correlation data the hypothesis of uncorrelation among the samples is not valid. The LAF is a stat-of-the-art technique, but not in the GNSS-MP-detection and mitigation field. With the objective of using this method for this purpose, the effects of the noise correlation in LS filters are studied in this paper, when the technique is applied to GNSS channel estimate in post-correlation. In this paper, a preliminary analysis is done, by means of simulations. Comparisons are shown between data affected by correlated and uncorrelated noise, using realistic GNSS data.

***Index Terms*—** GNSS, multipath, adaptive filter, LS, correlated noise.

1. INTRODUCTION

Nowadays, due to the improvements on GNSS that allow to mitigate most of the error sources, multipath (MP) results to be one of the main error sources [2]. MP introduces a distortion in the correlation function, causing a shift of the zero crossing point of the discriminator S-curve and an error on the propagation path delay measurement. As a result, a bias is introduced on the pseudorange estimate. MP mitigation is a challenging topic and many techniques are proposed in literature, including novel tracking loops and antenna technologies.

In this paper we consider Linear Adaptive Filter (LAF) techniques [1] to estimate the MP in GNSS applications.

The coefficients of a FIR (Finite Impulse Response) filter are adapted to minimize an error function, represented by the difference between the FIR output and a target signal to be estimated. In the case of GNSS the target signal is related to the received signal. The expected ideal signal is the input of the FIR, which computes combinations of weighted sums of delayed replicas of the input. The FIR outputs an estimate of the direct path to which the MP is added, whose weights depend on the power with which MP reaches the receiver antenna. Different approaches are considered in literature, as in [3], where the LAFs are applied pre-correlation. The technique can be applied post-correlation, as proposed e.g. in [4]. LAFs include methods based on Least Squares (LS), Least Mean Squares (LMS) and Recursive Least Squares (RLS) [1].

We use a post-correlation approach, that include the possibility to apply the LS (Least Square) method, since the correlation matrixes required by the LS can be easily modeled.

In LAFs, one of the assumption [1] is the noise to be a white process. Considering post-correlation data, the zero-mean condition is satisfied, but the hypothesis of uncorrelation among the samples is not valid. The topic of data correlation in LAFs has been considered in literature, for instance in [5], or in [6] and [7]. The aim of this paper is to investigate the problem of the noise correlation in LAFs applied to post-correlation GNSS signals. In fact, the application of the technique in post-correlation highly simplify the implementation, but the noise correlation must be taken into account since one of the basic assumption of the LAF's theory is to have white noise.

2. SIGNAL MODEL

A GNSS signal is transmitted by using the Code Division Multiple Access (CDMA) format, [8]. Therefore, after down-conversion and sampling, it can be written as

$$y\left(nT_s\right) = \sum_{m=1}^{N_v} y_m(nT_s) \tag{1}$$

where

$$\begin{aligned} y_m(nT_s) \quad &= \quad \sqrt{2P_m} C_m(nT_s - \tau_m) d_m(nT_s - \tau_m) \\ & \quad\quad \cos(2\pi(f_{IF} + f_{d,m})nT_s + \varphi_m) \end{aligned} \tag{2}$$

and N_v is the number of satellites in view, P_m is the received power, $C_m(nT_s - \tau_m) = c_m(nT_s - \tau_m)s_b(nT_s - \tau_m)$ is the product of the satellite spreading pseudo random noise (PRN) code $c_m(nT_s - \tau_m)$ and the subcarrier $s_b(nT_s - \tau_m)$ used in the new GNSS systems, such as in Galileo (if no subcarrier is present, then $s_b(nT_s - \tau_m)$=1), τ_m is the code delay, $d_m(nT_s - \tau_m)$ is the navigation data, f_{IF} is the intermediate frequency, $f_{d,m}$ is the Doppler frequency shift, φ_m is the phase of the carrier, and T_s is the sampling interval (the inverse of the sampling frequency f_s). The PRN code is a periodic sequence of rectangular chips (neglecting the front-end filter effects), with a period T_p, while T_c is the chip duration.

Thanks to the PRN code orthogonality the receiver operations can be analyzed taking into account a single space vehicle (SV) signal. Therefore only the signal in (2) will be considered in the remainder of this paper.

The power of the received signal is very weak and then the IF signal in (2) is buried in the noise, then the signal to be processed can be modeled as

$$r[n] = r(nT_s) = y_m(nT_s) + \eta(nT_s) \qquad (3)$$

where $\eta(nT_s)$ is a discrete time noise obtained by sampling the IF noise, which is in turn obtained by filtering a white Gaussian noise (WGN), with power spectral density $S(f) = N_0/2$, through the IF front-end filters. Therefore $\eta(nT_s)$ is a Gaussian discrete-time random process with zero mean and variance $\sigma^2 = N_0 B_{IF}$, where B_{IF} is the front-end bandwidth. Equation (3) models the so called *Additive White Gaussian Noise* (AWGN) propagation channel. To characterize the amount of noise the carrier-to-noise ratio defined as $C/N_0 = P_m/N_0$ is generally used.

2.1. Multipath channel

In GNSS applications an important source of error to be mitigated is multipath (MP), due to the reflection of the transmitted signal from surrounding buildings or other obstructions. As a consequence the AWGN model in (3) is not sufficient to describe the propagation channel, and a more complex model has to be introduced. Because of the motion of both SV and user, the channel response to any signal transmitted through changes with time, and then the impulse response of the channel is not time-invariant. To model it many factors have to be taken into account, as described, for example, in [9], where the concepts of fading, frequency selective channel, and frequency nonselective channel are highlighted. Here we consider the case of an MP channel, which can be modeled as an FIR transversal filter with M taps and coefficients $\beta_k(t)$, $k = 0, 1, \cdots, M-1$, which may change with time.

In GNSS applications only the contribution of the line of sight (LOS) is of interest, since the information we want to extract from the received signal is the LOS propagation time, [8]. Therefore we want to identify the reflected paths, in order to eliminate, or at least mitigate, their effect on the computation of the user position. The receiver subsystem devoted to the fine estimation of the propagation delay is the Delay Lock Loop (DLL). The goal of a DLL is to estimate only a residual part of the delay, with a duration in the order of a chip T_c. The output of the DLL is at some fixed epochs $t_l = lT_{DLL}$, where T_{DLL} is generally a multiple of the code period T_p. Typical values are $T_{DLL} = 20$ms for GPS CA code, and $T_{DLL} = 4$ms for Galileo E1 signal.

In this paper we analyse the effects of multipaths, which are approximately constant inside the time interval T_{DLL}, therefore it is widely used to model the MP channel with a linear time-invariant FIR filter, with an impulse response of the type $h_c(t) = \sum_{k=0}^{M-1} \beta_k \delta(t - \tau_k)$. Since a DLL works with the discrete-time signal $y_m(nT_s)$, given in (2), we consider a channel model in the discrete-time domain of the type

$$h_c[n] = h_c(nT_s) = \sum_{k=0}^{M-1} \beta_k \delta[n-k] \qquad (4)$$

where only MP delays of the type $\tau_k = kT_s$ can be represented. In section 5 we will discuss the effects of this assumption.

3. DLL MODEL WITH AN MP CHANNEL

As introduced in Section 1, in GNSS a widely used system is the DLL, a closed loop scheme [8] able to estimate the fractional delay from the peak of the correlation between the incoming PRN code $C_m(nT_s - \tau_m)$, and a code $c_{loc}[n] = c_{loc}(mT_s)$, locally generated. The incoming PRN code is extracted from the incoming signal $y_m(nT_s)$, after carrier frequency wipe-off, that, even if it is not perfect, implies in general a negligible residual error.

The effects that can seriously degrade the DLL performance, i.e. the delay estimate, are the noise and in particular the multipath. Therefore, an adequate model of the correlation evaluated by a DLL is

$$R_{DLL}(mT_s) = \sum_{k=0}^{M-1} \beta_k R(mT_s + kT_s) + w_R(mT_s) \qquad (5)$$

where $w_R(mT_s)$ is a noise component due to the correlation of the input noise with the local code $c_{loc}[n]$, and $R(mT_s)$ is the ideal correlation function in the absence of noise and multipath. Thanks to the code orthogonality the correlation $R(mT_s)$ is ideally zero outside a time interval $(-T_c, T_c)$ around the peak. For a GPS CA code $R(mT_s)$ is a triangle (neglecting the effects due to the fact that the codes are not perfectly orthogonal), while for the other GNSS signals the shape of the main lobe of the correlation function depends also on the subcarrier.

Only the multipaths affecting the main lobe degrade the system, therefore we can limit the number of taps of our channel model so as to cover a delay spread equal to $2T_c$. The the DLL is generally initialized with a delay coarsely estimated by the acquisition system, therefore the initial LOS can be in any point inside the delay spread $2T_c$. This means that we can include the acquisition error in the MP channel and we say that in the absence of noise and multipath $R_{DLL}(mT_s) = \beta_{k_0} R(mT_s + k_0 T_s)$.

In general, the acquisition is done by computing the correlation between the incoming signal code and carrier, and local replicas with different delays and Doppler frequencies. The criterion to detect the signal delay and Doppler is to find the correlation peak, looking for the maximum in the so called search-space and fixing a threshold on the first and the second maximum correlation peak. If the ratio between the two highest peaks is above the threshold, a detection is done, otherwise it is said that the signal is not present.

In this paper the analysis is done considering the signal correlation, fixing the Doppler frequency, so that the estimate is only on the code-delay.

4. LS METHOD FOR GNSS MULTIPATH ESTIMATE

In Sections 2.1 and 3, the effects of the multipath on a GNSS channel and, as a consequence, on the receiver's DLL are described. In this section the signal model of LAFs is recalled, showing how it can be suitable for the GNSS multipath channel described in Section 2.1. Then, the LS method is briefly described, which provides a way to estimate the MP parameters. In the LAF theory, as in [1], the signal model, consistent with (5) is:

$$d[i] = \sum_{k=0}^{M-1} w_{0k}^* u[i-k] + n_0[i] \tag{6}$$

where $d[i]$ is here the measured correlation at the time i and is a combination of delayed replicas of the signal $u[i]$, weighted by the parameters w_{0k}^*, which represents the coefficients of a FIR filter and are unknown. In this application, $u[i]$ is the correlation of the direct signal, while the replicas, with amplitude weighted by w_{0k}^*, represents the contributions of the multipath. $n_0[i]$ is the measurement error, unobservable, that in the LAF theory is assumed to be a zero-mean white random process. M is the length of the coefficient vector. The bigger M, the better the model is and the better the error can be minimized (which means better the estimate), but on the other side bigger M means also a higher computational effort. Equation (6) can be used to describe an MP channel, a sit uses a sum of weighted delayed replicas of the transmitted signal, as well as (5) which represents the DLL estimate as a sum of weighted delayed replicas plus a component of white noise. Therefore, since the MP channel model is consistent with the LAF's signal model described in (6), the LAF's technique can be suitable for the estimation of GNSS signals affected by multipath.

The goal of a LAF is to find an estimate w_k^* of the unknown coefficients w_{0k}^*, which minimize the residual error $e[i] = d[i] - y[i]$, where $y[i]$ is:

$$y[i] = \sum_{k=0}^{M-1} w_k^* u[i-k] \tag{7}$$

and $u[i]$ is the ideal correlation at the input of the FIR.

As described in [1], different linear adaptive filtering techniques exist. In this paper, the Least Square (LS) approach is considered. Differently from the LMS and RLS, the LS computes the solution analytically as:

$$\hat{\mathbf{w}} = \mathbf{\Phi}^{-1} \boldsymbol{\theta} \tag{8}$$

where $\mathbf{w}^* = [w_0^*, ..., w_{M-1}^*]$ is the vector of the estimated coefficients, $\mathbf{\Phi}$ is a square matrix of dimension M which contains the correlation terms of $u[i]$. $\boldsymbol{\theta}$ is a vector which contains cross-correlation terms between $u[i]$ and $d[i]$ [1]. It is $\boldsymbol{\theta} = [\theta(0)\theta(-1)...\theta(-M)\theta(-M+1)]^T$ where each element is $\theta(-k) = \sum_{i=1}^{M} u[i-k] d^*[i]$.

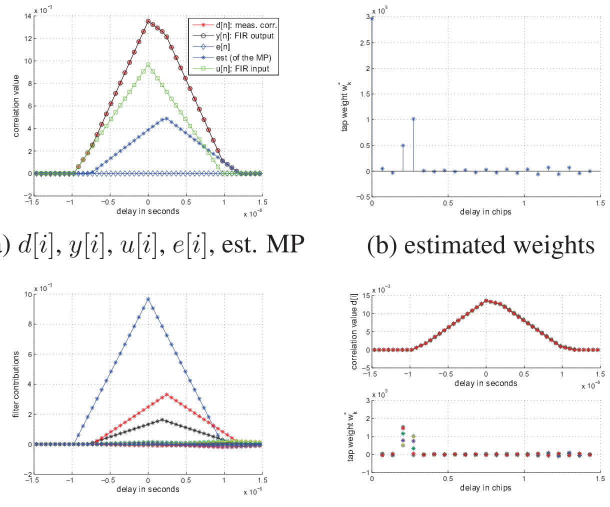
Thanks to the LS method, an estimate can be done of the multipath components, which introduce an additive delay on the DLL estimate. A correction can be applied on the resulting pseudorange measurement, taking into account the multipath contribution, anyhow the focus of this paper is not on this issue that will be analyzed in depth in the future. Notice that the LOS contribution is represented by the replica with the smallest delay, given the hypothesis of MP with bigger delay than the LOS. Different cases will be treated in future.

5. SIMULATION RESULTS

Some simulation results are shown, obtained with MatLab using realistic data generated with N-Fuels [10], a GNSS signal simulator. Simplifying assumptions are done, than anyway are widely used since generally valid. Only the case of delayed MP is considered, and not the particular and rare case in which the MP rays are received by the antenna before the direct signal. Then, MP is assumed to have the same Doppler frequency shift of the direct path. Last, the MP power is lower than the direct signal power. Results are shown in the case of 1 MP, with a delay of $0.2T_c$ and correlation peak amplitude $0.5A$, where A is the LOS correlation peak.

In (5), the noise component is assumed to be white. This is not the case if all the samples of the GNSS correlation $d[i]$ are computed by correlating a fixed signal segment with shifted-versions of the local code, so generating a correlated noise component $n_0[i]$. A way to have uncorrelated noise is to compute each sample of $d[i]$ using different segments of the incoming signal, but in order to have the same integration time and then the same performance in terms of Signal to Noise Ratio (SNR), many subsequent signal segments have to be considered.

Another important aspect is the resolution. The spacing between the correlator samples determines the resolution with which the MP can be characterized. In this paper, we consider $N = 49$ correlation samples and $M = 22$ FIR taps. Notice that N is the number of samples in the correlation function, i.e. the number of correlators needed. Future activities will focus on how this number can be optimized.

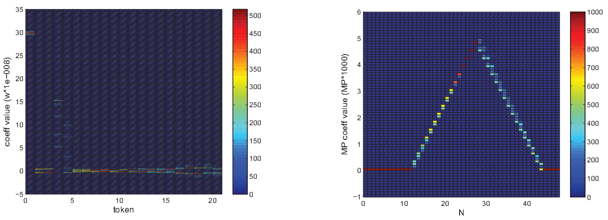


(a) $d[i]$, $y[i]$, $u[i]$, $e[i]$, est. MP (b) estimated weights

(c) FIR contributions (d) 10 MC simulations

Fig. 1. Ideal case without noise, 1 MP.

The ideal case of the absence of noise is first considered, with an integration time $T_{DLL} = 1$ ms. Figure 1 (a) shows the measured correlation $d[i]$, the filter estimate $y[i]$, the error $e[i] = d[i] - y[i]$, the ideal correlation $u[i]$ and the estimate of the MP contributions, obtained as $y[i] - w_0^* \cdot u[i]$, that is by subtracting the zero-delay contribution (assumed as the LOS) from the filter estimate $y[i]$.



(a) w_k^* (b) MP est.

Fig. 2. LS estimate applied in post-correlation in the absence of noise. 1 MP. 1000 MC.

It is important to notice that due to the resolution, the simulated MP peak is in between two samples. In order to remedy this, the LAF estimates the MP contribution by combining two contributions with peaks on the samples immediately before and after, respectively, with respect to the true MP. This effect is clearly visible in Figure 1 (b) and in Figure 1 (c), which depict, respectively, the filter coefficients w_k^* and the correspondent filter contribution to the estimate $y[i]$. In Figure 1 (b) the coefficients correspondent to high delays should be ideally zero, but they are not, since the measured correlation function $d[i]$ is obtained by correlating the incoming signal in (1) after performing the carrier wipe-off, and this operation is not perfect. In fact carrier residuals remain due to the imperfect double frequency removal.

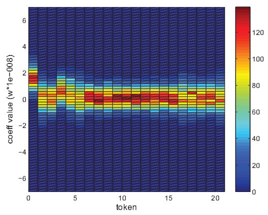
Another effect, due to the incommensurability between the sampling frequency and the chip rate, is shown in Figure 1 (d), where the coefficients w_k^* are shown for 10 Monte Carlo (MC) simulations. The effect is that the LAF estimate is not always the same in successive integration intervals, even in the absence of noise.

In the following, some results are shown, referred to different Monte Carlo simulations with number of runs $N_{MC} = 1000$. The results are presented in 2D, using a colormap, where the axes represent, respectively, the parameters we are interested in (estimated LAF coefficients and MP estimate) and the values that they could assume, while the color represents the occurrences.

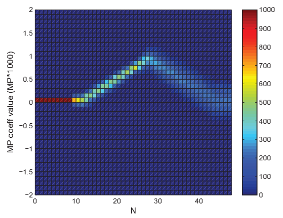
The parameters we are interested in are the coefficients w_k^* and the MP estimate $y[i] - w_0^* \cdot u[i]$. Results are shown in the absence and presence of noise. In the case of noise presence, a standard good value of carrier-to-noise ratio is used ($C/N_0 = 50$ dB-Hz). Comparisons are done between two cases, in which the correlation function $d[i]$ is computed in two different ways. In the first case, the integration time is $T_{DLL} = 1$ ms, and each sample of $d[i]$ is computed using the same signal interval, causing correlated noise in $d[i]$. In the second case, different signal intervals are considered to compute different samples of $d[i]$, so that the noise affecting $d[i]$ results to be uncorrelated. This comparison could be considered not fair, since in the first case the signal is extracted in a time window $T_w = T_{DLL} = 1$ ms, while in the second case a larger time window $T_w = ((N+1)/2) \cdot T_{DLL}$ ms is needed, where N is the number of correlation samples. Note that $(N+1)/2$ signal intervals are sufficient because in the two sides of the triangle the noise can be considered uncorrelated, even if computed using the same portion of signal, thanks to the PRN code orthogonality. However, this comparison can be considered fair because in both cases each sample of $d[i]$ is computed by using the same integration time T_{DLL}, and a larger T_w is the price to be paid to avoid noise correlation.

The first simulation is without noise (ideal case). Figure 2 (a) shows the estimated coefficients w_k^*. The highest peak corresponding to the LOS is clearly visible, and MP is detected. The distribution around the true delay is due to the sampling effect, combined with the incommensurability, as explained in the comments to Figure 1. As expected, as in Figure 1 (b), a distribution of values around zero is present for higher delays. Figure 2 (b) shows the MP estimate $y[i] - w_0^* \cdot u[i]$ ($N = 49$ correlation samples). Close to the true MP delay ($0.2T_c$), the histogram presents the highest occurrences. The

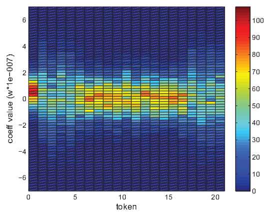
other samples are spreader, because of the possible different combinations of contributions that give similar estimates of the same MP, due to sampling and incommensurability.
More simulation results are shown in the case of noisy GNSS signal, with $C/N_0 = 50$ dB-Hz.



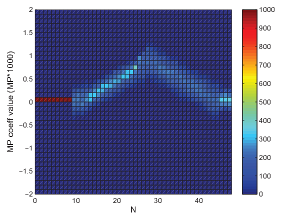
(a) w_k^*, standard corr.



(b) MP est., standard corr.



(c) w_k^*, decorrel.



(d) MP est., decorrel.

Fig. 3. 1 MP. 1000 MC, $C/N_0 = 50$ dB/Hz.

Figure 3 (a) and (b) refer to the case with correlated noise affecting $d[i]$, while Figure 3 (c) and (d) refer to the case where the noise is uncorrelated. Due to the noise correlation, in Figure 3 (a), the estimated coefficient values are less spread than in Figure 3 (c), anyhow the maximum value of the filter weights is in correspondence of the LOS (zero-delay). The other occurrences of the coefficients w_k^* are distributed around zero, apart around the MP, at $0.2T_c$ where higher values are present, which detects the presence of MP. With uncorrelated noise, also the MP estimate is spreader (Figure 3) (d), but the MP is still clearly identifiable. Notice that, due to the noise, the FIR coefficients can assume also negative values, as shown in Figure 3 (a) and in Figure 3 (c). This is how the filter responds when the target is a noisy signal, estimating $d[i]$ using a combination of positive and negative contributions. However, we are interested in the combined effect of all the coefficients, which gives an estimate of the MP correlation shape (triangle), as shown in Figure 3 (b) and Figure 3 (d).

6. CONCLUSIONS AND FUTURE DEVELOPMENTS

From this first qualitative analysis, it can be said that the noise correlation in the LAF applied to GNSS MP detection does not represent an impediment. On the contrary, it may have favorable impact, even if may generate a bias on the correlation value. Particular attention must be paid to the sampling rate used to measure the correlation function. In fact, at the decreasing of the sampling frequency, the performance of the channel effect estimate can highly degrade.

Future works include an analytical analysis of the effects of the noise correlation in this application, and joint analysis of the sampling frequency and of the noise effects and the determination of proper thresholds for the MP detection. Also, tests with real data and comparisons with established techniques for multipath estimation will allow to proof the method performance.

7. REFERENCES

[1] S. Haykin, *Adaptive filter theory*, Prentice-Hall, 1986.

[2] E. Kaplan, *Understanding GPS - Principles and Applications*, Boston: Artech House, 1996.

[3] Axelrad Penina Etter Delores M. Nelson, Lisa M., “Adaptive detection of code delay and multipath in a simplified gps signal model,” in *Proceedings of the ION GPS Conference, Kansas City, MO, September 1997*. ION, 1997, pp. 569–581.

[4] DingFan Y. Yanxin, Y. Dongkai, “A multipath-resistant precise tracking method for boc signal,” in *Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications, 2009 3rd IEEE International Symposium*. IEEE, 2009.

[5] E. Eweda, “Transient performance degradation of the lms, rls, sign, signed regressor, and sign-sign algorithms with data correlation,” *IEEE Transactions on circuits and systems*, vol. 46, August 1999.

[6] K.; Itoh Y.; Fukui Y. Kawamura, A.; Fujii, “A new noise reduction method using linear prediction error filter and adaptive digital filter,” in *IEEE International Symposium on Circuits and Systems, 2002. ISCAS 2002*. IEEE, 2002, vol. 3, pp. 488–491.

[7] J.R. Jr.; McCool J.M.; Kaunitz J.; Williams C.S.; Hearn R.H.; Zeidler J.R.; Eugene Dong Jr.; Goodlin R.C. Widrow, B.; Glover, “Adaptive noise cancelling: Principles and applications,” in *Proceedings of the IEEE*. IEEE, 1975, vol. 63, pp. 1692–1716.

[8] P. Misra and P. Enge, *Global Positioning System: Signals, Measure-ments, and Performance*, Lincoln, MA: Ganga-Jamuna Press, 2006.

[9] J. G. Proakis and M. Salehi, *COMMUNICATION SYSTEMS ENGI-NEERING*, Prentice-Hall, Inc., New Jersey, 2002.

[10] E. Falletti, D. Margaria, and B. Motella, “Educational library of gnss signals for navigation,” *Coordinates*, vol. 5, pp. 30–34, August 2009.