ALTERNATIVE TO COHERENT SIGNAL SUBSPACE BASED METHOD FOR BURIED OBJECTS LOCALIZATION

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ABSTRACT

In this paper, a new method for simultaneous range and bearing estimation for buried objects in the presence of an unknown colored noise is proposed. We propose a method based on MUSIC (MUltiple SIgnal Classification) which evaluates the time delay of arrival (TDOA) of the reflection of an emitted signal on objects. This is achieved by exploiting the wideband of the signal and the assumption that the emitted signal spectrum is known. The bearing and the range objects are expressed as a function of those TDOA. The proposed algorithm is compared with a classical coherent signal subspace method on experimental data recorded during underwater acoustic experiments.

Index Terms— Source localization, wide band signals, time delay estimation, high resolution algorithm, correlation

1. INTRODUCTION

During the past years, many high resolution algorithms (HRA), such as Multiple Signal Classification (MUSIC) [1] or estimation of signal parameters via rotational invariance techniques (ESPRIT) [2], have been developed in direction of arrival (DOA) estimation of multiple sources localization. These algorithms use the spectral matrix with the assumption that the signals impinging on the array are not fully correlated or coherent. Under uncorrelated conditions, the source spectral matrix is full rank, which is the basis of the eigendecomposition. So they provide super resolution DOA estimation when sources are not or lowly correlated. In practice, signals are correlated due to multipaths, reflexions, ... The correlation significantly degrades the performance of the traditional high resolution methods, diminishing the source spectral matrix rank under the number of sources, making localization erroneous. To handle the problem of correlated signals, a variety of methods [3–11] have been proposed in the literature during the last decades. Although many different methods are available, their principle can be roughly categorized into two classes: 1. spatial smoothing techniques, for the case of a uniform linear antenna (ULA), based on a preprocessing scheme that divides the total array into overlapping sub-arrays and then averages the sub-array outputs spectral matrices to form the spatially smoothed spectral matrix [3] but inducing a high number of sensors. 2. frequential smoothing techniques, which are achieved by the number of frequencies contained in the frequency band. Many methods have been developed for the wide-band signals analysis [12–14]. They are grouped into two categories: Incoherent signal subspace methods and coherent signal subspace methods. Incoherent signal subspace methods suffer from a significant computational load and a lack of robustness. The coherent subspace methods transform the subspaces in a predefined subspace by the focusing matrices.
Other methods are proposed, in [15] only quasi-stationary signals are considered and in [16] the test of orthogonality of projected subspaces (TOPS) is extended for better decorrelation.
In this paper, the proposed method for solving the problem of buried objects localization, exploits advantage of the wide band of these signals. The performance of the proposed algorithm is evaluated using several numerical simulations and real-world signals recorded during an underwater acoustic experiment.

Throughout the paper, the superscript "T" is used for transpose operation, "H" is used for transjugate, $E[.]$ denotes the expectation operator, $||.||_F$ denotes Frobenius norm and "$*$" denotes complex conjugate.

2. PROBLEM FORMULATION

Consider a transmitter that generates a wideband signal with an angle θ_{inc}. The incident wave will propagate and be reflected by P objects. For example, located in the seabed of a tank filled with sand and water (see Fig. 1 P=1). An array composed of N sensors receives the P reflected and correlated signals ($P < N$). The received signals are grouped into a vector $\mathbf{r}(f)$, which is the Fourier transform of the array output vector at frequency bin f and is written as [5]:

$$\mathbf{r}(f) = \mathbf{A}(f)\mathbf{s}(f) + \mathbf{n}(f) \quad (1)$$

Thanks to DGA agency for funding.

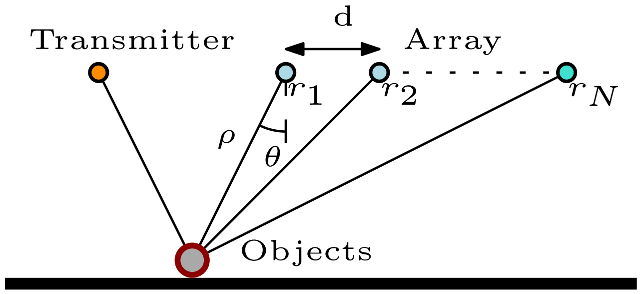

Fig. 1. Configuration of the sources

where $\mathbf{A}(f) = [\mathbf{a}(f,\theta_1,\rho_1), \mathbf{a}(f,\theta_2,\rho_2), ..., \mathbf{a}(f,\theta_P,\rho_P)]$, matrix of dimensions $(N \times P)$ is the transfer matrix of the source-sensor array systems with respect to some chosen reference point with (θ_p, ρ_p) the bearings and ranges of the objects, $\mathbf{s}(f) = [s_1(f), s_2(f), ..., s_P(f)]^T$ is the vector of signals, $\mathbf{n}(f) = [n_1(f), n_2(f), ..., n_N(f)]^T$ is the vector of noise. We define the spectral matrix by:

$$\mathbf{\Gamma}(f) = E[\mathbf{r}(f)\mathbf{r}^H(f)] \quad (2)$$

The noises $\mathbf{n}(f)$ and the source signals $\mathbf{s}(f)$ are assumed uncorrelated, thus the spectral matrix $\mathbf{\Gamma}(f)$ can be written:

$$\mathbf{\Gamma}(f) = \mathbf{A}(f)\mathbf{\Gamma}_s(f)\mathbf{A}^H(f) + \mathbf{\Gamma}_n(f), \quad (3)$$

where $\mathbf{\Gamma}_s(f) = E[\mathbf{s}(f)\mathbf{s}^H(f)]$ is the spectral matrix associated to the object signals, $\mathbf{\Gamma}_n(f) = E[\mathbf{n}(f)\mathbf{n}^H(f)]$ is the spectral matrix of noises.

2.1. Usual Algorithm for Bearing and Range Estimation of Buried Objects

Generally, the eigendecomposition of the spectral matrix Eq.(3) is performed. Then the eigenvector matrix $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_N]$ is separated into two matrices :
$\mathbf{V}_s = [\mathbf{v}_1, \ldots, \mathbf{v}_P]$ corresponding to the P eigenvectors associated with the P largest eigenvalues span the signal subspace and the remaining $N-P$ eigenvectors $\mathbf{V}_n = [\mathbf{v}_{P+1}, \ldots, \mathbf{v}_N]$ span the orthogonal space named the noise subspace.
Then, MUSIC algorithm exploits those $N-P$ eigenvectors to estimate the bearings and ranges of the waves associated with the narrow-band (at the chosen frequency f) sources. The ranges and DOAs are estimated by taking the local maximum points of the conventional spatial spectrum, $MUSIC(f,\theta,\rho) = \frac{1}{||\mathbf{a}^H(f,\theta,\rho)\mathbf{V}_n(f)||_F}$.
However, in the presence of P objects the $MUSIC(f,\theta,\rho)$ algorithm can not solve all the P angles because the signals are correlated. To address this issue, all the information contained in the wideband frequency (L frequency bins) of the signals is used simultaneously to estimate the coherent signal subspace at a chosen frequency $f_0 \in [f_1, \cdots, f_L]$, using $P_0 < P$ initial estimations of the bearings using the beamformer method, which leads to obtain the P ranges and DOAs, described by algorithm 1 [17].

Algorithm 1 Usual Algorithm for Bearing and Range Estimation of Buried Objects

1. use the beamformer method to find an initial estimate $\widehat{\theta_{0k}}$ of θ_p, where $k = 1, \cdots, P_0$, with $P_0 \leq P$.
2. compute the initial values of ρ_k using geometrical assumptions
3. for each frequency $f_l \in [f_1, \ldots, f_L]$
   (a) estimate the transfer matrix $\hat{\mathbf{A}}(f_l)$
   (b) estimate the spectral matrix $\hat{\mathbf{\Gamma}}(f_l) = \frac{1}{Nr}\sum_{nr=1}^{Nr}\mathbf{r}_{nr}(f_l)\mathbf{r}_{nr}^H(f_l)$, where Nr represents the number of realizations.
   (c) estimate noise variance $\hat{\sigma}^2(f_l) = \frac{1}{N-P_0}\sum_{i=P_0+1}^{N}\lambda_i(f_l)$, where $\lambda_i(f_l)$ is the i^{th} eigenvalue of $\Gamma(f_l)$.
   (d) calculate the spectral matrix of the signals reflected on the objects by $\hat{\mathbf{\Gamma}}_s(f_l) = (\hat{\mathbf{A}}^H(f_l)\hat{\mathbf{A}}(f_l))^{-1}\hat{\mathbf{A}}^H(f_l) \times [\hat{\mathbf{\Gamma}}(f_l) - \hat{\sigma}^2(f_l)\mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix and use SVD to obtain $\hat{\mathbf{V}}_s(f_l)$,
4. compute the average of the spectral matrices $\bar{\mathbf{\Gamma}}_s(f_0) = \frac{1}{L}\sum_{l=1}^{L}\hat{\mathbf{\Gamma}}_s(f_l)$, where L represents the number of frequencies and f_0 is the center frequency of the spectrum of wideband signals,
5. calculate $\hat{\mathbf{\Gamma}}(f_0) = \hat{\mathbf{A}}_0(f_0)\bar{\mathbf{\Gamma}}_s(f_0)\hat{\mathbf{A}}_0^H(f_0)$, using SVD to obtain $\hat{\mathbf{V}}_s(f_0)$,
6. for each f_l, calculate $\mathbf{T}_s(f_0, f_l) = \hat{\mathbf{V}}_s(f_0)\hat{\mathbf{V}}_s^H(f_l)$
7. form the average matrix $\bar{\mathbf{\Gamma}}(f_0) = \frac{1}{L}\sum_{l=1}^{L}\mathbf{T}_s(f_0, f_l)[\hat{\mathbf{\Gamma}}(f_l) - \hat{\sigma}^2(f_l)\mathbf{I}_N]\mathbf{T}_s{}^H(f_0, f_n)$ and calculate $\bar{\mathbf{V}}_n(f_0)$ by SVD,
8. calculate the spatial spectrum of the $MUSIC$ method for bearing and range object estimation: $MUSIC(f_0,\theta,\rho) = \frac{1}{||\mathbf{a}^H(f_0,\theta,\rho)\bar{\mathbf{V}}_n(f_0)||_F}$,
9. The estimated bearings and ranges $(\hat{\theta}_p, \hat{\rho}_p)$ correspond to the arguments of $MUSIC(f_0,\theta,\rho)$ maxima.

2.2. Proposed Algorithm for Bearing and Range Estimation of Buried Objects

The hereinbefore method is based on a spatial diversity model and coherently focuses the wideband information ($\mathbf{\Gamma}(f_l)$, $f_l \in [f_1, \cdots, f_L]$) into a single frequency $\bar{\mathbf{\Gamma}}(f_0)$.

In this paper we want to address the issue of correlation in a different way, by using more efficiently all information. We propose a sensor-by-sensor model based on frequential diversity, that estimate TDOA. The ranges and DOAs of the objects are expressed as a function of those TDOA, using the spatial diversity.
For a given sensor j, the Fourier transform of the observed signal can be written as:

$$\tilde{\mathbf{r}}(j) = \mathbf{\Lambda}\mathbf{A}(j)\mathbf{c}(j) + \tilde{\mathbf{n}}(j),\ j = 1, \ldots, N \tag{4}$$

where $\tilde{\mathbf{r}}(j) = [\tilde{r}(f_1)(j), \ldots, \tilde{r}(f_L)(j)]^T$ is the vector of L frequencies of the Fourier transform of the signal recorded on sensor j, $\mathbf{\Lambda} = diag\,(\tilde{s}(f_1)(j), \ldots, \tilde{s}(f_L)(j))$, is the diagonal matrix of the emitted signal Fourier transform, $\mathbf{c}(j) = [c_1(j), \ldots, c_P(j)]^T$ where $c_p(j)$ is a complexe amplitude, $\tilde{\mathbf{n}}(j) = [\tilde{n}(f_1)(j), \tilde{n}(f_2)(j), \ldots, \tilde{n}(f_L)(j)]^T$ is the vector of the noise Fourier transform, $\mathbf{A}(j)$ is the $(M \times P)$ transfer matrix of the source-j^{th} sensor system for the observed frequencies, whose column are the steering vectors, $\mathbf{A}(j) = [\mathbf{a}(\tau_1(j)), \ldots, \mathbf{a}(\tau_P(j))]$, $\mathbf{a}(.) = \left[e^{-2i\pi f_1(.)}, \ldots, e^{-2i\pi f_L(.)}\right]^T$ where $\tau_p(j)$ is the TDOA of object p on sensor j.
MUSIC algorithm can be extended to estimate the TDOAs. However, the correlation of the signals is still a problem. Modified Spatial Smoothing Processing [18] estimate an unbiased covariance matrix of the observation and reduces the signal correlation. The spectrum is divided into K sub-bands of M frequencies and averaged. The bearing and range of the objects are functions of the TDOA. Algorithm 2 summarizes the proposed method.

This method offers two main assets. Firstly it does not require an initialization. Secondly the number of sensor N is substituted by M. It means that the assumption $P < N$ is not mandatory any more and the proposed method could be able to resolve indeterminate systems. Both methods are compared in Fig. 2.

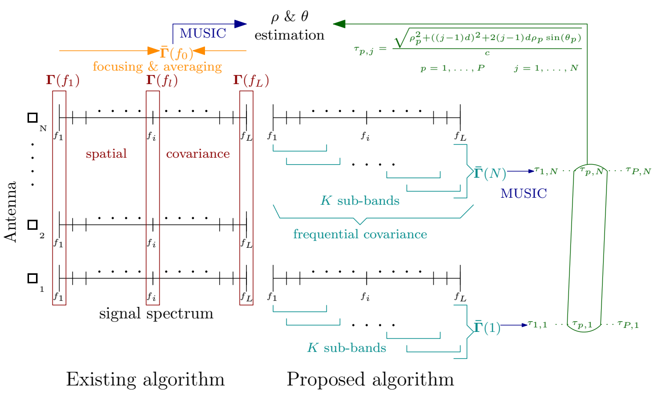

Fig. 2. Methods comparison

Algorithm 2 Proposed Algorithm for Bearing and Range Estimation of Buried Objects

1. for each sensor j
   (a) divide the frequential observations into K overlapping sub-bands of M frequencies
   (b) whiten the observation vector in each sub-band, $\mathbf{y}_k(j) = \mathbf{\Lambda}_k^{-1}\tilde{\mathbf{r}}_k(j)$, $k \in [1, \cdots, K]$ where $\mathbf{\Lambda}_k$ and $\tilde{\mathbf{r}}_k(j)$ are respectively made of the corresponding rows of $\mathbf{\Lambda}$ and $\tilde{\mathbf{r}}(j)$
   (c) compute $\hat{\mathbf{\Gamma}}_k(j) = \frac{1}{Nr}\sum_{l=1}^{Nr}\mathbf{y}_{knr}(j)\mathbf{y}_{knr}^H(j)$
   (d) calculate $\bar{\mathbf{\Gamma}}(j) = \frac{1}{2K}\sum_{k=1}^{K}(\mathbf{\Gamma}_k(j) + \mathbf{J}\mathbf{\Gamma}_k^*(j)\mathbf{J})$ where $\mathbf{J}$ stands for the anti-diagonal matrix of permutation that helps generating the observation vector in the backward direction. Use SVD to obtain $\bar{\mathbf{V}}_n(j)$.
   (e) calculate the spatial spectrum of the $MUSIC$ method for TDOA estimation:$MUSIC(\tau) = \frac{1}{||\mathbf{a}(\tau)^H\bar{\mathbf{V}}_n(j)||_F}$, where $\bar{\mathbf{V}}_n(j)$ is the $M-P$ eigenvector matrix of $\bar{\mathbf{\Gamma}}(j)$.
   (f) The estimated TDOA $(\hat{\tau}_k(j))$ correspond to the arguments of $MUSIC(\tau)$ maxima.
2. estimate the ranges and DOAs using the so estimated TDOA assuming that $\tau_{p,j} = \frac{\sqrt{\rho_p^2+((j-1)d)^2+2(j-1)d\rho_p\sin(\theta_p)}}{c}$ where c is the wave velocity.

3. EXPERIMENTAL SETUP: RESULTS AND DISCUSSIONS

The studied signals are recorded during an underwater acoustic experiments in order to estimate the developed method performance. The experiment is carried out in an acoustic tank under the conditions similar to those in a marine environment. The bottom of the tank is filled with sand. The experimental device is presented in Fig. 3. The tank is topped by two mobile carriages. The first carriage supports a transducer emitter and the second supports a transducer receiver pilot by the computer. This tank is filled of water with $W_h = 0.5$ m (Fig. 3) and its bottom is filled with homogeneous fine sand, where three cylinder couples ((O_3, O_4),(O_5, O_6),(O_7, O_8)) and one sphere couple (O_1, O_2), see Fig. 4) are buried between 0 and 0.05 m under the sand. The considered objects have the following characteristics, where δ represents the distance between the two objects of the same couple and $\emptyset_a$ the outer radius (the inner radius $\emptyset_b = \emptyset_a - 0.001$ m):

1. the 1^{st} couple (O_1, O_2): spherical shells, $\emptyset_a = 0.3$ m, $\delta = 0.33$ m, full of air,

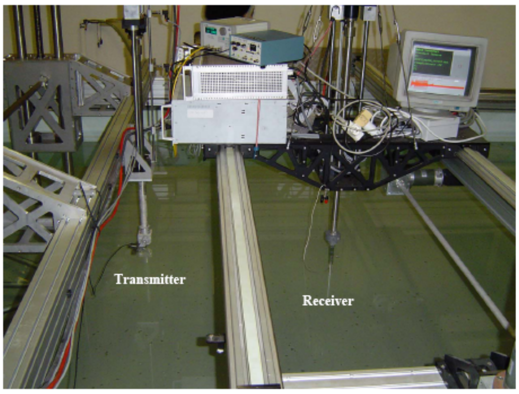

Fig. 3. Experimental tank

2. the 2^{nd} couple (O_3, O_4): cylindrical shells, $\emptyset_a = 0.01$ m, $\delta = 0.13$ m, full of air,

3. the 3^{rd} couple (O_5, O_6): cylindrical shells, $\emptyset_a = 0.018$ m, $\delta = 0.16$ m, full of water,

4. the 4^{th} couple (O_7, O_8): cylindrical shells, $\emptyset_a = 0.02$ m, $\delta = 0.06$ m, full of air,

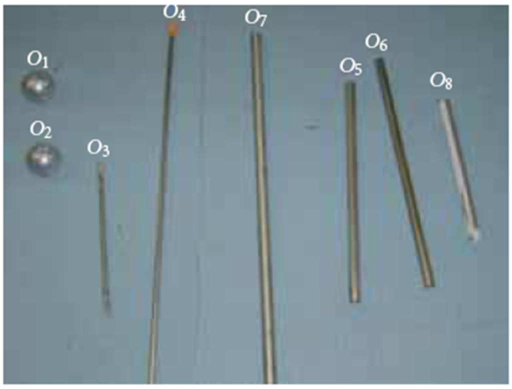

Fig. 4. Experimental Objects.

The objective is to estimate the bearing and range of the sources during the experiment. The signals are received on an uniform linear array. The observed signals come from various reflections on the objects being in the tank. Generally the aims of acousticians is the detection, localization and identification of these objects. In this experiment we have recorded the reflected signals by a single receiver. This receiver is moved along a straight line between position $X_{min} = 50mm$ and position $X_{max} = 150mm$ with a step of $\alpha = 1mm$ in order to create a uniform linear array. We have measured eight times $E_i(O_{ii}, O_{ii+1})$ with $i = 1, \ldots, 8$ and $ii = 1, 3, 5, 7$. At first, the receiver horizontal axis XX' is fixed at 0.2 m, we performed the experiments $E_1(O_1, O_2), \ldots, E_4 O_7, O_8)$ associated to the 1^{st}, 2^{nd}, 3^{rd} and the 4^{th} couple, respectively. Then we performed the other four experiments $E_5(O_1, O_2), \ldots, E_8(O_7, O_8)$ with XX' fixed at 0.4 m. RR' is the vertical axis which goes through the center of the fist object of each couple. For each experiment, the transmitted signal had the following properties: pulse duration is 15 μs, the frequency band is, the frequency of the band is $[f_{min} = 150, f_{max} = 250]$ kHz and the center frequency f_0 is $f_0 = 200$ kHz. The sampling rate is 2 MHz. The duration of the received signal was 700 μs. The obtained values of θ and ρ are given in Tab. 1. These values confirm the efficiency of the proposed method.

Table 1. The expected (exp) and estimated (est) values of range and bearing objects using usual method (UM) and proposed method (PM)

	$E_{1(O_1,O_2)}$	$E_{2(O_3,O_4)}$	$E_{3(O_5,O_6)}$	$E_{4(O_7,O_8)}$
$\theta_{1exp}(^\circ)$	20	23	33.2	32.4
ρ_{1exp}(m)	0.3	0.24	0.26	0.26
$\theta_{2exp}(^\circ)$	22	9.2	20	5.8
ρ_{2exp}(m)	0.32	0.22	0.24	0.22
$\theta_{1estUM}(^\circ)$	20	23	33	32
ρ_{1estUM}(m)	0.31	0.25	0.29	0.28
$\theta_{2estUM}(^\circ)$	22.5	9	20	6
ρ_{2estUM}(m)	0.33	0.25	0.25	0.23
$\theta_{1estPM}(^\circ)$	20	23	33	32
ρ_{1estPM}(m)	0.31	0.245	0.25	0.26
$\theta_{2estPM}(^\circ)$	22.1	8.95	20	5.9
ρ_{2estPM}(m)	0.32	0.23	0.23	0.22

	$E_{5(O_1,O_2)}$	$E_{6(O_3,O_4)}$	$E_{7(O_5,O_6)}$	$E_{8(O_7,O_8)}$
$\theta_{1exp}(^\circ)$	-50	-52.1	-70	-51.6
ρ_{1exp}(m)	0.65	0.65	1.24	0.65
$\theta_{2exp}(^\circ)$	-22	-41	-65.3	-49
ρ_{2exp}(m)	0.45	0.56	1.17	0.64
$\theta_{1estUM}(^\circ)$	-49	-52	-70	-52
ρ_{1estUM}(m)	0.65	0.63	1.21	0.63
$\theta_{2estUM}(^\circ)$	-22	-40	-65	-50
ρ_{2estUM}(m)	0.44	0.53	1.2	0.63
$\theta_{1estPM}(^\circ)$	-50	-52	-70	-51
ρ_{1estPM}(m)	0.65	0.64	1.25	0.64
$\theta_{2estPM}(^\circ)$	-22	-41	-65	-49
ρ_{2estPM}(m)	0.45	0.55	1.2	0.64

4. CONCLUSION

One of the main targets of array processing is the estimation of the parameters: DOAs of the buried objects and the objects-sensors ranges. In this paper, we have proposed a new localization algorithm to estimate both the ranges and the bearings of objects. We developed a new organization and processing of the recorded data, aiming to use the frequential and spatial information more efficiently. By treating the recorded signals sensor by sensor, the different TDOA of the received signals are estimated. Then, by considering the spatial evolution of the TDOA the object ranges and bearings are obtained. This method could also work in a situation where there is less sensors than sources. The proposed method performance is investigated through scaled tank tests associated with some cylindrical and spherical shells buried in an homogeneous fine sand. The obtained results are promising, showing a better accuracy than for usual methods.

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