SUM RATE MAXIMIZATION IN MULTI-OPERATOR TWO-WAY RELAY NETWORKS WITH A MIMO AF RELAY VIA POTDC

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ABSTRACT

We address the beamforming problem for maximizing the sum rate of a multi-operator two-way relay (TWR) network subject to the constraint on the total relay transmit power. This scenario is also known as relay sharing for multi-way relaying or TWR for multiple operators. The relay is assumed to be equipped with multiple antennas, and it uses the amplify-and-forward relaying strategy. It is shown that the corresponding optimization problem can be represented as a difference of convex functions (DC) programming problem which is NP-hard in general. Nevertheless, we develop an efficient polynomial time algorithm to solve the problem approximately. The performance comparison of the proposed polynomial time DC (POTDC) inspired algorithm to the existing state-of-the-art algorithms demonstrate that the proposed algorithm outperforms the existing algorithms especially in the case of non-symmetric networks.

1. INTRODUCTION

Relaying, as a mean of reducing the deployment cost, enhancing the network capacity, and mitigating shadowing effects, has strong potentials for future wireless networks. Although the traditional one-way relaying suffers from the half-duplex constraint and cannot utilize the radio resources in an efficient manner, two-way relaying (TWR) enhances the spectral efficiency and, thus, provides an attractive alternative [1]. Two relaying strategies are well studied, namely, amplify-and-forward (AF) and decode-and-forward (DF). Compared to the DF strategy, the digital AF strategy[1] is of a higher practical interest since it yields a much smaller delay and has a lower complexity [1]. Thus, it is also adopted in this paper.

Research on beamforming and power allocation algorithms for AF TWR ranges from the case of multiple single-antenna AF relays [2] to the multi-antenna relaying case [3]-[5]. Several linear preprocessing techniques have been proposed for the single-pair as well as multi-pair AF TWR [3], [4], [6]. The beamforming design for a multi-operator relay sharing scenario has also been considered in [8], where a sub-optimal algebraic algorithm has been proposed to accomplish the resource sharing by exploiting the multiple antennas at the relay.

In this paper, we address the beamforming design problem to maximize the sum rate of the multi-operator TWR with a multiple-input multiple-output (MIMO) AF relay subject to the relay transmit power constraint [9]. We first show that the corresponding optimization problem is the difference of convex functions (DC) programming problem which is non-convex and NP-hard in general. Afterwards, we derive an efficient polynomial time convex optimization-based algorithm to solve the problem approximately. This algorithm can be viewed as an extension of the **PO**lynomial **T**ime **DC** (POTDC) method which we recently proposed in [10] to maximize the sum rate in AF TWR with multiple antennas at the relay and just a single pair of users. For the latter problem, the POTDC algorithm, one step of which is based on semidefinite programming (SDP) relaxation, is exact, while in the case of multiple operators (multiple pairs of users that share the same relay), the randomization procedure has to be used that makes it approximate. To further evaluate the proposed algorithm, we compare its performance to the performance of some state-of-the-art algorithms and show that the proposed algorithm outperforms the existing algorithms especially in the case of non-symmetric networks.

2. DATA MODEL

The scenario under investigation is the same as in [8]. Pairs of users belonging to L different operators communicate with each other. However, due to the poor quality of the direct channels between these pairs of users, they can only communicate with the help of the relay. Each user has a single antenna and the relay is equipped with M_{R} antennas. We assume that the channel is flat fading. The channel between the kth user of the ℓth operator and the relay is denoted by $\boldsymbol{h}_k^{(\ell)} \in \mathbb{C}^{M_{\mathrm{R}}}$. Here, the index $k \in \{1,2\}$ is used to enumerate users and the index $\ell \in \{1,\dots,L\}$ is used to enumerate operators.

The adopted AF-based transmission protocol consists of two transmission phases. In the first phase, all the users transmit their data simultaneously to the relay. Let $x_k^{(\ell)}$ be the transmitted symbol of the kth user of the ℓth operator, which is independently distributed with zero mean and variance $P_k^{(\ell)}$. The received signal vector at the relay is then given

This work was supported in parts by the Natural Science and Engineering Research Council (NSERC) of Canada and by the Graduate School on Mobile Communications (GSMobicom), Ilmenau University of Technology, which is partly funded by the Deutsche Forschungsgemeinschaft (DFG). This work was partly funded by the European Union FP7-ICT project EMPhAtiC (http://www.ict-emphatic.eu) under grant agreement no. 318362.

[1]By "digital" we mean that the signal processing is performed in the base band.

as

$$\boldsymbol{r} = \sum_{\ell=1}^{L}\sum_{k=1}^{2} \boldsymbol{h}_k^{(\ell)} x_k^{(\ell)} + \boldsymbol{n}_{\mathrm{R}} \in \mathbb{C}^{M_{\mathrm{R}}} \tag{1}$$

where $\boldsymbol{n}_{\mathrm{R}} \in \mathbb{C}^{M_{\mathrm{R}}}$ denotes the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise vector and $\mathbb{E}\{\boldsymbol{n}_{\mathrm{R}}\boldsymbol{n}_{\mathrm{R}}^{\mathrm{H}}\} = \sigma_{\mathrm{R}}^2 \boldsymbol{I}_{M_{\mathrm{R}}}$ with $\boldsymbol{I}_M$ denoting an $M \times M$ identity matrix.

In the second phase, the relay amplifies the received superposition of signals and then forwards it to all the users simultaneously. Then the signal transmitted by the relay can be expressed as

$$\bar{\boldsymbol{r}} = \boldsymbol{G}\boldsymbol{r} \tag{2}$$

where $\boldsymbol{G} \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{R}}}$ is the relay amplification matrix. Since the total transmit power at the relay is limited, the transmit power constraint at the relay must be fulfilled so that

$$\mathbb{E}\{\|\bar{\boldsymbol{r}}\|^2\} = \sum_{\ell=1}^{L}\sum_{k=1}^{2} P_k^{(\ell)} \|\boldsymbol{G}\boldsymbol{h}_k^{(\ell)}\|^2 + \sigma_{\mathrm{R}}^2 \|\boldsymbol{G}\|_{\mathrm{F}}^2 \le P_{\mathrm{R}} \tag{3}$$

where P_{R} denotes the total power at the relay, $\mathbb{E}\{\cdot\}$ stands for the expectation operator, and the Euclidean norm of a vector and the Frobenius norm of a matrix are denoted by $\|\cdot\|$ and $\|\cdot\|_{\mathrm{F}}$, respectively.

For notational simplicity, we assume that the reciprocity between the first- and second-phase channels holds. This assumption is fulfilled in a time-division duplex (TDD) system if the RF chains are calibrated.[2] The received signal $y_k^{(\ell)}$ for the kth user of the ℓth operator can be written as

$$\begin{aligned} y_k^{(\ell)} &= \boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \bar{\boldsymbol{r}} + n_k^{(\ell)} \\ &= \underbrace{\boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\boldsymbol{h}_{3-k}^{(\ell)} x_{3-k}^{(\ell)}}_{\text{desired signal}} + \underbrace{\boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\boldsymbol{h}_k^{(\ell)} x_k^{(\ell)}}_{\text{self-interference}} \\ &+ \underbrace{\sum_{\substack{\bar{\ell}\neq\ell \\ \bar{k}=\{1,2\}}} \boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\boldsymbol{h}_{\bar{k}}^{(\bar{\ell})} x_{\bar{k}}^{(\bar{\ell})}}_{\text{inter-operator interference}} + \underbrace{\boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\boldsymbol{n}_{\mathrm{R}} + n_k^{(\ell)}}_{\text{effective noise}} \end{aligned} \tag{4}$$

where $n_k^{(\ell)}$ denotes the ZMCSCG noise symbol with variance $\sigma_k^{(\ell)^2}$ and $\{\cdot\}^{\mathrm{T}}$ stands for transpose.

Assuming that perfect channel knowledge can be acquired at each user, the self-interference term can be subtracted [3]. Denote $\eta_k^{(\ell)}$ as the signal-to-interference-plus-noise ratio (SINR) at the kth user of the ℓth operator, which is defined as

$$\eta_k^{(\ell)} = \frac{P_{3-k}^{(\ell)} |\boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\boldsymbol{h}_{3-k}^{(\ell)}|^2}{\sum_{\substack{\bar{\ell}\neq\ell \\ \bar{k}=\{1,2\}}} P_{\bar{k}}^{(\bar{\ell})} |\boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\boldsymbol{h}_{\bar{k}}^{(\bar{\ell})}|^2 + \sigma_{\mathrm{R}}^2 \|\boldsymbol{h}_k^{(\ell)^{\mathrm{T}}} \boldsymbol{G}\|^2 + \sigma_k^{(\ell)^2}}. \tag{5}$$

Applying the vec$\{\cdot\}$ operator that stacks the columns of a matrix into a vector, the actual relay transmit power (3) and the SINR (5) can be expressed as

$$\mathbb{E}\{\|\bar{\boldsymbol{r}}\|^2\} = \boldsymbol{g}^{\mathrm{H}} \boldsymbol{C} \boldsymbol{g} \tag{6}$$

and

$$\eta_k^{(\ell)} = \frac{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{D}_k^{(\ell)} \boldsymbol{g}}{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{E}_k^{(\ell)} \boldsymbol{g} + \sigma_k^{(\ell)^2}} \tag{7}$$

respectively, where $\boldsymbol{g} = \mathrm{vec}\{\boldsymbol{G}\}$, $\{\cdot\}^{\mathrm{H}}$ stands for the Hermitian transpose, and as shown in [8]

$$\begin{aligned} \boldsymbol{C} &= \sum_{k,\ell} P_k^{(\ell)} ((\boldsymbol{h}_k^{(\ell)} \boldsymbol{h}_k^{(\ell)^{\mathrm{H}}})^{\mathrm{T}} \otimes \boldsymbol{I}_{M_{\mathrm{R}}}) + \sigma_{\mathrm{R}}^2 \boldsymbol{I}_{M_{\mathrm{R}}} \\ \boldsymbol{D}_k^{(\ell)} &= P_k^{(\ell)} (\boldsymbol{h}_{3-k}^{(\ell)^{\mathrm{T}}} \otimes \boldsymbol{h}_k^{(\ell)^{\mathrm{T}}})^{\mathrm{H}} (\boldsymbol{h}_{3-k}^{(\ell)^{\mathrm{T}}} \otimes \boldsymbol{h}_k^{(\ell)^{\mathrm{T}}}) \\ \boldsymbol{E}_k^{(\ell)} &= \sum_{\bar{k},\bar{\ell}\neq\ell} P_{\bar{k}}^{(\bar{\ell})} (\boldsymbol{h}_{\bar{k}}^{(\bar{\ell})^{\mathrm{T}}} \otimes \boldsymbol{h}_k^{(\ell)^{\mathrm{T}}})^{\mathrm{H}} (\boldsymbol{h}_{\bar{k}}^{(\bar{\ell})^{\mathrm{T}}} \otimes \boldsymbol{h}_k^{(\ell)^{\mathrm{T}}}) \\ &+ \sigma_{\mathrm{R}}^2 (\boldsymbol{I}_{M_{\mathrm{R}}} \otimes (\boldsymbol{h}_k^{(\ell)} \boldsymbol{h}_k^{(\ell)^{\mathrm{H}}})^{\mathrm{T}}). \end{aligned} \tag{8}$$

Note that the $\boldsymbol{D}_k^{(\ell)} \in \mathbb{C}^{M_{\mathrm{R}}^2 \times M_{\mathrm{R}}^2}$ are positive semidefinite matrices and the $\{\boldsymbol{C}, \boldsymbol{E}_k^{(\ell)}\} \in \mathbb{C}^{M_{\mathrm{R}}^2 \times M_{\mathrm{R}}^2}$ are positive definite matrices, $\forall k, \ell$.

The overall sum rate of the system described above can be expressed as

$$R_{\mathrm{sum}} = \frac{1}{2} \sum_{\ell=1}^{L}\sum_{k=1}^{2} \log_2(1 + \eta_k^{(\ell)}), \tag{9}$$

where the factor 1/2 is due to the two transmission phases (half duplex).

Our goal is to find the relay amplification matrix $\boldsymbol{G}$ which maximizes the system sum rate subject to the relay transmit power constraint.

3. SOLUTION VIA POTDC

Mathematically, the constrained sum rate maximization problem for our system can be formulated as

$$\begin{aligned} \max_{\boldsymbol{g}} \quad & \tfrac{1}{2} \sum_{\ell=1}^{L}\sum_{k=1}^{2} \log_2 \left(1 + \frac{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{D}_k^{(\ell)} \boldsymbol{g}}{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{E}_k^{(\ell)} \boldsymbol{g} + \sigma_k^{(\ell)^2}} \right) \\ \text{subject to} \quad & \boldsymbol{g}^{\mathrm{H}} \boldsymbol{C} \boldsymbol{g} \le P_{\mathrm{R}}. \end{aligned} \tag{10}$$

It can be easily seen that the inequality constraint in (10) has to be satisfied with equality at optimality. Otherwise, the optimal $\boldsymbol{g}$ can be scaled up to satisfy the constraint with equality while increasing the objective function. Using this observation, the relay transmit power constraint in (10) can be rewritten as equality constraint.

Using such an equality constraint, changing to the natural logarithm, and also omitting the constant $\frac{1}{2}$ in the objective function, the constrained optimization problem (10) can be turned into the following *unconstrained* optimization problem

$$\max_{\boldsymbol{g}} \sum_{\ell=1}^{L}\sum_{k=1}^{2} \log \left(1 + \frac{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{D}_k^{(\ell)} \boldsymbol{g}}{\boldsymbol{g}^{\mathrm{H}} \boldsymbol{E}_k^{(\ell)} \boldsymbol{g} + \boldsymbol{g}^{\mathrm{H}} \frac{\sigma_k^{(\ell)^2}}{P_{\mathrm{R}}} \boldsymbol{C} \boldsymbol{g}} \right). \tag{11}$$

[2] Note that our method is not limited to the reciprocity assumption which is considered only for notation simplicity.

[3] In practice the channel knowledge can be estimated, for example, using the method discussed in [8].

Moreover, after some straightforward algebra, the problem (11) can be shown to be equivalent to the following optimization problem

$$\max_{\boldsymbol{g}} \log\left(\prod_{\ell=1}^{L}\prod_{k=1}^{2}\frac{\boldsymbol{g}^{\mathrm{H}}\boldsymbol{A}_k^{(\ell)}\boldsymbol{g}}{\boldsymbol{g}^{\mathrm{H}}\boldsymbol{B}_k^{(\ell)}\boldsymbol{g}}\right) \tag{12}$$

where $\{\boldsymbol{A}_k^{(\ell)}, \boldsymbol{B}_k^{(\ell)}\} \in \mathbb{C}^{M_{\mathrm{R}}^2\times M_{\mathrm{R}}^2}$ are positive semidefinite matrices defined as

$$\begin{aligned} \boldsymbol{A}_k^{(\ell)} &= \boldsymbol{E}_k^{(\ell)} + \frac{\sigma_k^{(\ell)^2}}{P_{\mathrm{R}}}\boldsymbol{C} + \boldsymbol{D}_k^{(\ell)} \\ \boldsymbol{B}_k^{(\ell)} &= \boldsymbol{E}_k^{(\ell)} + \frac{\sigma_k^{(\ell)^2}}{P_{\mathrm{R}}}\boldsymbol{C}. \end{aligned} \tag{13}$$

The problem (12) is a homogeneous quadratically constrained quadratic programming (QCQP) problem which is NP-hard in general.

Introducing the new notation $\boldsymbol{X} = \boldsymbol{g}\boldsymbol{g}^{\mathrm{H}}$ and taking the logarithm of the objective function, the problem (12) can be equivalently written as

$$\begin{aligned} \max_{\boldsymbol{X}} \quad & \sum_{\ell=1}^{L}\sum_{k=1}^{2}\left(\log(\mathrm{Tr}\{\boldsymbol{A}_k^{(\ell)}\boldsymbol{X}\}) - \log(\mathrm{Tr}\{\boldsymbol{B}_k^{(\ell)}\boldsymbol{X}\})\right) \\ \text{subject to} \quad & \mathrm{rank}(\boldsymbol{X}) = 1 \\ & \boldsymbol{X} \succeq \boldsymbol{0} \end{aligned} \tag{14}$$

where $\mathrm{Tr}\{\cdot\}$ and $\mathrm{rank}\{\cdot\}$ denote the trace and rank of a matrix, respectively.

Moreover, using SDP relaxation, i.e., removing the non-convex rank-1 constraint in (14), the relaxed problem can be shown to be a DC programming problem, which is still non-convex. Hereafter, for notational simplicity, we define an index m to substitute the indices ${}_k^{(\ell)}$ such that $m = 2(\ell-1)+k, \forall k,\ell$ (i.e., $m \in \{1,2,\cdots,2L\}$). Then the relaxed problem (14) with new simplified indices can be rewritten as

$$\begin{aligned} \max_{\boldsymbol{X},\{\alpha_m,\beta_m\}} \quad & \log(\mathrm{Tr}\{\boldsymbol{A}_1\boldsymbol{X}\}) - \log(\mathrm{Tr}\{\boldsymbol{B}_1\boldsymbol{X}\}) + \sum_{m=2}^{2L}\log(\alpha_m) \\ & - \sum_{m=2}^{2L}\log(\beta_m) \\ \text{subject to} \quad & \mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}\} = \alpha_m,\ m = 2,3,\cdots,2L \\ & \mathrm{Tr}\{\boldsymbol{B}_m\boldsymbol{X}\} = \beta_m,\ m = 2,3,\cdots,2L \\ & \boldsymbol{X} \succeq \boldsymbol{0}. \end{aligned} \tag{15}$$

Due to the Rayleigh-quotient structure of (12), the problem does not change by setting $\boldsymbol{g}^{\mathrm{H}}\boldsymbol{B}_1\boldsymbol{g} = \mathrm{Tr}\{\boldsymbol{B}_1\boldsymbol{X}\} = 1$. Furthermore, the objective function in (15) turns into a convex function by replacing the concave elements, i.e., the elements with the minus sign by scalar variables. Then the reformulated problem, which is equivalent to (15), is written as

$$\begin{aligned} \max_{\boldsymbol{X},\{\alpha_m,\beta_m,t_m\}} \quad & \log(\mathrm{Tr}\{\boldsymbol{A}_1\boldsymbol{X}\}) + \sum_{m=2}^{2L}\log(\alpha_m) - \sum_{m=2}^{2L}t_m \\ \text{subject to} \quad & \mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}\} = \alpha_m,\ m = 2,3,\cdots,2L \\ & \mathrm{Tr}\{\boldsymbol{B}_m\boldsymbol{X}\} = \beta_m,\ m = 2,3,\cdots,2L \\ & \log(\beta_m) \le t_m,\ m = 2,3,\cdots,2L \\ & \mathrm{Tr}\{\boldsymbol{B}_1\boldsymbol{X}\} = 1,\ \boldsymbol{X} \succeq \boldsymbol{0}. \end{aligned} \tag{16}$$

As compared to the problem (15) with non-convex DC-type objective function, the non-convexity in the equivalent problem (16) is localized in the inequality constraints $\log(\beta_m) \le t_m,\ m = 2,3,\cdots,2L$. To deal with these non-convex constraints, we propose to use a linear approximation of the log function, e.g., the second order Taylor series of the log function, which uses the same philosophy as the original POTDC algorithm in [10]. The second order Taylor polynomial approximation of $\log(\beta)$ at β_0 is defined as

$$\log(\beta) \approx \log(\beta_0) + \frac{\beta - \beta_0}{\beta_0}. \tag{17}$$

Using (17), the optimization problem (16) can be reformulated as

$$\begin{aligned} \max_{\boldsymbol{X},\{\alpha_m,\beta_m,t_m\}} \quad & \log(\mathrm{Tr}\{\boldsymbol{A}_1\boldsymbol{X}\}) + \sum_{m=2}^{2L}\log(\alpha_m) - \sum_{m=2}^{2L}t_m \\ \text{subject to} \quad & \mathrm{Tr}\{\boldsymbol{A}_m\boldsymbol{X}\} = \alpha_m,\ m = 2,3,\cdots,2L \\ & \mathrm{Tr}\{\boldsymbol{B}_m\boldsymbol{X}\} = \beta_m,\ m = 2,3,\cdots,2L \\ & \log(\beta_{0,m}) + \tfrac{\beta_m - \beta_{0,m}}{\beta_{0,m}} \le t_m,\ m = 2,3,\cdots,2L \\ & \mathrm{Tr}\{\boldsymbol{B}_1\boldsymbol{X}\} = 1,\ \boldsymbol{X} \succeq \boldsymbol{0}. \end{aligned} \tag{18}$$

It can be seen that for a given set of initial values $\{\beta_{0,2},\beta_{0,3},\cdots,\beta_{0,m}\}$, the problem (18) is an SDP problem that can be solved efficiently using the interior-point algorithms if it is feasible [11]. Since the best set of initial values is unknown, it is natural to use an iterative method and update the initial values in each iteration. Here, the initial values $\{\beta_{0,2}^{(p)},\beta_{0,3}^{(p)},\cdots,\beta_{0,m}^{(p)}\}$ at the pth step are the optimal values of β_m which are obtained by solving the problem (18) at the $(p-1)$th step. It is worth stressing that, if the problem (18) is feasible at the pth step, then the optimal solution for the problem (18) denoted as $f^{\star(p)}$ should be larger or equal to the optimal solution for the same problem at the previous $(p-1)$th step, i.e., $f^{\star(p-1)}$. Otherwise, if $f^{\star(p)} < f^{\star(p-1)}$, it is contradictory to the objective function. Summarizing, the proposed iterative algorithm for solving the optimization problem (16) can be described as in Table 1.

It should also be stressed that the initial set of $\{\beta_{0,2}^{(0)},\beta_{0,3}^{(0)},\cdots,\beta_{0,m}^{(0)}\}$ has to be feasible. Taking into account the generalized Rayleigh quotient structure and recalling that $\boldsymbol{g}^{\mathrm{H}}\boldsymbol{B}_1\boldsymbol{g} = 1$, β_m can be any value between the maximum and minimum generalized eigenvalues of the matrix pair $\boldsymbol{B}_m$ and $\boldsymbol{B}_1$, i.e, $\beta_m \in \{\lambda_{\min}\{\boldsymbol{B}_1^{-1}\boldsymbol{B}_m\}, \lambda_{\max}\{\boldsymbol{B}_1^{-1}\boldsymbol{B}_m\}\}$. For example, $\beta_{0,m}^{(0)}$ can be chosen in a random way such that

$$\beta_{0,m}^{(0)} = \frac{\boldsymbol{a}^{\mathrm{H}}\boldsymbol{B}_m\boldsymbol{a}}{\boldsymbol{a}^{\mathrm{H}}\boldsymbol{B}_1\boldsymbol{a}}\ m = 2,3,\cdots,2L, \tag{19}$$

where $\boldsymbol{a} \in \mathbb{C}^{M_{\mathrm{R}}^2} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{M_{\mathrm{R}}^2})$.

The algorithm in Table 1 provides only an approximate solution to the relaxed problem (10) in terms of the matrix variable $\boldsymbol{X}$. This solution is the same as the solution of the original problem (10) only if $\boldsymbol{X}$ is a rank-1 matrix. In other

Table 1: Algorithm I: Iterative algorithm for solving the optimization problem (16)

Initialization step: input: $\boldsymbol{A}_1$, $\boldsymbol{B}_1$, $\boldsymbol{A}_m$, $\boldsymbol{B}_m$, set $\{\beta_{0,2}^{(0)}, \beta_{0,3}^{(0)}, \cdots, \beta_{0,m}^{(0)}\}$, $f^{\star(0)}$ maximum iteration number $N_{\max}$ and the threshold value ε.
Main steps:
for $p = 1$ to $N_{\max}$ **do**
Solve the problem (18) in order to find the optimal value $f^{\star(p)}$ and $\beta_m^{(p)}$.
$\beta_{0,m}^{(0)} = \beta_m^{(p)}$, $m = 2, 3, \cdots, 2L$
if $\left\lvert f^{\star(p)} - f^{\star(p-1)} \right\rvert \le \varepsilon$ **then**
break
end if
end for

Table 2: Algorithm II: Iterative algorithm for approximately solving the problem (10)

Initialization step: input: $\boldsymbol{A}_1$, $\boldsymbol{B}_1$, $\boldsymbol{A}_m$, $\boldsymbol{B}_m$, set $\{\beta_{0,2}^{(0)}, \beta_{0,3}^{(0)}, \cdots, \beta_{0,m}^{(0)}\}$, $f^{\star(0)}$, $R_{\mathrm{sum},0}$, maximum iteration number $N_{\max}$, N_{iter} and the threshold value ε.
Main steps:
Solve problem (16) finding $\boldsymbol{X}$ with arbitrary rank
Calculate the eigen-decomposition of $\boldsymbol{X}$ as $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^{\mathrm{H}}$;
for $j = 1$ to N_{iter} **do**
Generate $\hat{\boldsymbol{g}}_j = \boldsymbol{U}\boldsymbol{\Sigma}^{1/2}\boldsymbol{z}_j$ where $\boldsymbol{z}_j \in \mathbb{C}^{M_{\mathrm{R}}^2} \sim \mathscr{CN}(\boldsymbol{0}, \boldsymbol{I}_{M_{\mathrm{R}}^2})$.
$\tilde{\boldsymbol{g}}_j = \frac{\hat{\boldsymbol{g}}_j \sqrt{P_{\mathrm{R}}}}{\sqrt{\hat{\boldsymbol{g}}_j^{\mathrm{H}} \boldsymbol{C} \hat{\boldsymbol{g}}_j}}$.
Insert $\tilde{\boldsymbol{g}}_j$ into (9) to calculate $R_{\mathrm{sum},j}$.
if $R_{\mathrm{sum},j} > R_{\mathrm{sum},(j-1)}$ **then**
$\boldsymbol{g}_{\mathrm{opt}} = \tilde{\boldsymbol{g}}_j$.
end if
end for

words, $\hat{\boldsymbol{g}}$ is optimal for (10) only if there exists $\boldsymbol{X}^\star = \hat{\boldsymbol{g}}\hat{\boldsymbol{g}}^{\mathrm{H}}$, where $\boldsymbol{X}^\star$ is the solution obtained based on the algorithm in Table 1. However, according to [12] (*Theorem 3.2 and Corollary 3.4*), there is no guarantee that the matrix $\boldsymbol{X}$ found using the algorithm in Table 1 has rank-1. Indeed, the latter would be guaranteed only if the number of constraints in the SDP relaxed optimization problem would be less or equal to 3. In our problem, the number of constraints is clearly larger than 3 when ($L \ge 2$), i.e., when the number of operators is larger than one. For such a situation, a good rank-1 approximation can be obtained by using the randomization techniques [13]. Thus, using also randomization for obtaining rank-1 approximate solution to the problem (10), the overall algorithm for finding an approximate solution to the sum-rate maximization problem in multi-operator TWR networks with AF relay equipped with multiple antennas can be summarized as in Table 2.

4. SIMULATIONS

In this section, the performance of the proposed algorithms is evaluated via Monte-Carlo simulations. They are also compared to other methods, namely, the projection based separation of multiple operators (ProBaSeMO) scheme (specifically the two variations block diagonalization (BD) together with algebraic norm-maximizing (ANOMAX) scheme (BA) and regularized BD (RBD) together with rank-restored ANOMAX scheme (RR)) in [8] and the MMSE method in [7]. The simulated MIMO flat fading channels $\boldsymbol{h}_k^{(\ell)}$ are spatially uncorrelated Rayleigh fading channels. They are fixed during two time slots. The transmit power at each user and at the relay are identical and $P_k^{(\ell)} = P_{\mathrm{R}} = 1$, $\forall k, \ell$. The noise variance at each user and at the relay are also identical, i.e., $\sigma_{\mathrm{R}}^2 = {\sigma_k^{(\ell)}}^2 = \sigma_{\mathrm{n}}^2$, $\forall k, \ell$. For the ProBaSeMO algorithms, the weighting factor β is set to 0.5 in all simulations [8]. All the simulation results are obtained by averaging over 1000 channel realizations.

The ProBaSeMO scheme is selected for the comparison because it outperforms the other sub-optimal designs in a symmetric scenario as shown in [8], e.g., the MMSE method in [7]. Moreover, it has much lower computational complexity especially when compared to the iterative solution inspired by the power method [8]. The complexity of the ProBaSeMO schemes can be roughly estimated as follows. For L pairs ProBaSeMO requires L singular value decompositions (SVDs) of complex matrices of size $M_{\mathrm{R}} \times 2(L-1)$ and L SVDs of complex matrices of size $M_{\mathrm{R}}^2 \times 2$. Assuming that the SVD of a $M \times N$ real matrix has the complexity of $O(MN^2)$ and taking into account that a $M \times N$ complex matrix can be written equivalently as a $2M \times 2N$ real matrix, then the complexity of ProBaSeMO can be estimated as $O(L(32M_{\mathrm{R}}^3 + 32M_{\mathrm{R}}(L-1)^2))$. The complexity of the proposed POTDC-type algorithm is a product of a number of required iterations to the complexity of solving the SDP problem (18), which is higher than the complexity of the SVD.

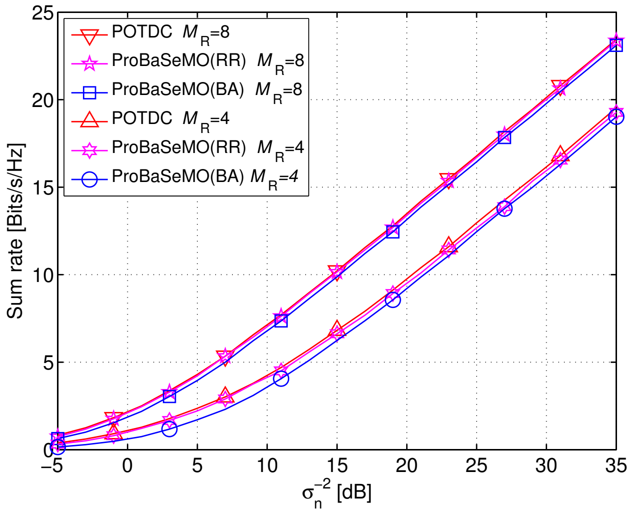

Figure 1: Sum rate comparison of ProBaSeMO ({BA, RR}) and POTDC approaches for $L = 2$ in a symmetric scenario, where each user in the network has equal distance to the relay.

Figure 1 demonstrates the sum rate comparison of the ProBaSeMO schemes and the proposed POTDC approach in a symmetric scenario. That is, each user has equal distance to the relay. The proposed POTDC only slightly outperforms the ProBaSeMO schemes. When the noise variance is small and the number of antennas at the relay is large, the performance difference almost vanishes.

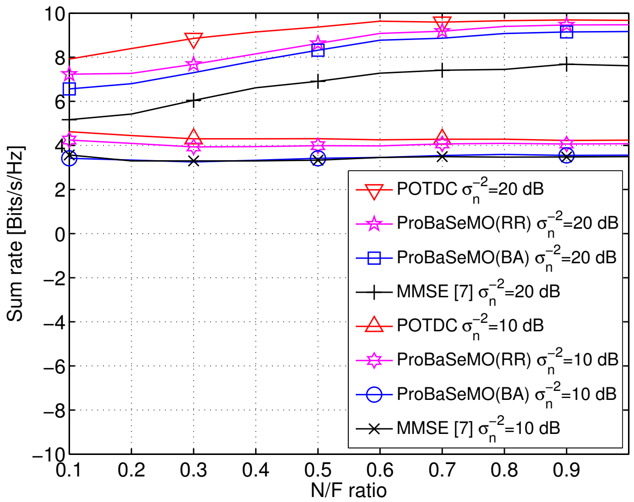

Figure 2: Sum rate comparison of ProBaSeMO ({BA, RR}), MMSE [7], and POTDC approaches for $M_R = 4$ and $L = 2$ in an asymmetric scenario.

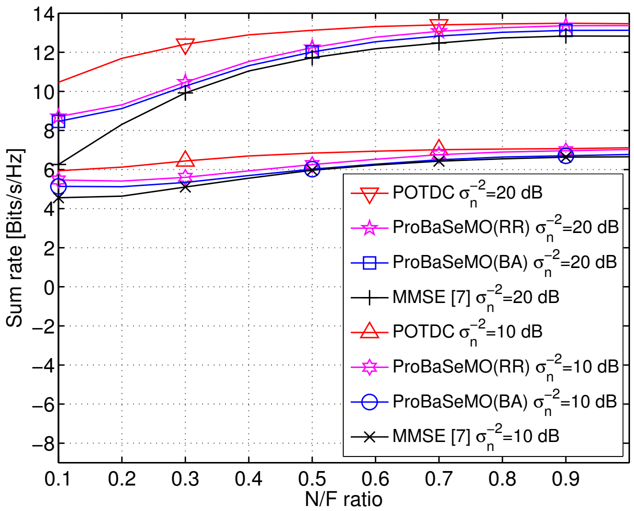

Figure 3: Sum rate comparison of ProBaSeMO ({BA, RR}), MMSE [7], and POTDC approaches for $M_R = 8$ and $L = 2$ in an asymmetric scenario.

However, the superiority of the proposed POTDC approach is revealed in a non-symmetric scenario, i.e., when the users have different distances to the relay. To show this, we define a path loss model $P_L = 20\log_{10}(d_k^{(\ell)})$ where $d_k^{(\ell)}$ is the normalized distance between the relay and the kth user of the ℓth operator. For simplicity, we further assume an inter-operator symmetry, i.e., $d_1^{(\ell)} = d_1$ and $d_2^{(\ell)} = d_2\ \forall \ell$. The near-far (N/F) ratio is defined as d_2/d_1. As shown in Figures 2 and 3, compared to the POTDC approach, the ProBaSeMO scheme and the MMSE method in [7] suffer more from the asymmetry of the system especially when the near-far ratio is far away from 1. When the number of antennas at the relay increases, the performance difference between the ProBaSeMO approach and the POTDC approach is even enlarged.

5. CONCLUSION

This paper addresses the beamformer design to maximize the sum rate of a multi-operator AF TWR network with multiple antennas at the relay subject to the constraint on the total relay transmit power has been addressed. The corresponding optimization task has been represented as a DC programming problem which is NP-hard in general. To solve such a problem approximately, the efficient polynomial time algorithm POTDC has been extended to the multi-operator case. It has been demonstrated in terms of simulations that the proposed algorithm performs better than the existing state-of-the-art algorithms especially in the case of non-symmetric networks.

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