NOISE UNCERTAINTY ANALYSIS OF ENERGY DETECTOR: BOUNDED AND UNBOUNDED APPROXIMATION RELATIONSHIP

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ABSTRACT

Energy detection is a simple non-coherent approach used for spectrum sensing that offers linear computational complexity and low latency. Its main shortcoming is the well-known noise uncertainty problem, which results in failed detection in the very low signal-to-noise ratio. In this paper, we give a theoretical analysis of the noise uncertainty modeling with energy detection to drive a relationship between bounded and unbounded uncertainty approximation.

***Index Terms*—** spectrum sensing, energy detector, noise uncertainty, log-normal approximation, SNR$_{wall}$

1. INTRODUCTION

Spectrum sensing has been identified as the first step in cognition cycle, and the most important functionality for the establishment of cognitive radio [1, 2]. It is performed to find the spectrum holes which in turn tells that whether the primary user is present or not. Various spectrum sensing techniques have been proposed including matched filter method, feature detection based on Higher-order statistics like cyclostationary detection, energy detection method, and some emerging methods such as eigenvalue-based sensing and wavelet-based sensing [3, 4]. The Higher-order statistics based spectrum sensing detection [5] can effectively separates noise from received signals and operates at very low SNR regions. However it often requires a large number of data to obtain the accurate estimations of the relevant statistics. The energy detection approach has been widely studied for primary user signal detection [6]. It is a non-coherent detector and has low implementation complexity [7, 8]. In addition, the energy detector does not require any prior knowledge about the primary users signal. However, the detection efficiency of energy detection degrades heavily under low signal-to-noise ratio (SNR) and noise uncertainty conditions [9], which can restrict its efficiency for cognitive radio. There exists an SNR$_{wall}$ value of the signal-to-noise ratio beyond which detection is theoretically impossible.

According to Tandra and Sahai paper [10], the error in estimating the noise power spectral density N_0 is assumed to be bounded. In fact, from practical point of view, N_0 would need to be estimated by the detector. So this estimated value, $\tilde{N_0}$, belongs to $[(1+\rho)^{-1}N_0, (1+\rho)N_0]$ interval where $10log_{10}(1+\rho)$ is the noise uncertainty factor in dB, $\rho \geq 0$. There are several factors that increase noise uncertainty such as thermal noise change due to temperature and/or calibration errors. An example given by IEEE 802.22 in [11] shows that the noise uncertainty may be much larger than 1 dB, and we can not reduce it by increasing sensing time. In more recent study [12], Jouini proposes a new results on energy detector performances when the noise uncertainty is modeled by an unbounded log-normal distribution. Moreover, new expression of the SNR$_{wall}$ is presented as a function of the noise uncertainty parameter.

In this paper, we analyze the energy detector performance under bounded and unbounded noise uncertainty's model and try to obtain a simple relationship between them. The remainder of this paper is organized as follows. In Section II, the system model of the spectrum sensing is described, and the ideal energy detection is introduced. In Section III, the bounded and unbounded noise uncertainty models are presented and the performances are analyzed. Section IV seeks to find the relationship between the two uncertainty's models and some simulation results are discussed. Finally, we conclude this study in Section V.

2. SYSTEM MODEL

Spectrum sensing is actually used to identify the presence of primary users signal in a desired frequency band. In this paper, we consider non-cooperative spectrum sensing model. The information of the bandwidth is known and a bandpass filter is used to select the interested bandwidth. In this section, a general system model and ideal energy detection are introduced.

2.1. Basic hypothesis model

Spectrum sensing can be modeled as a hypothesis test problem. There are two possible hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$:

$$y(k) = \begin{cases} b(k) & \text{for} \quad \mathcal{H}_0 \\ x(k) + b(k) & \text{for} \quad \mathcal{H}_1 \end{cases} \tag{1}$$

where hypothesis $\mathcal{H}_0$ refers to the presence of vacant frequency bands and hypothesis $\mathcal{H}_1$ refers to the presence of a primary user, $y(k)$ is the received signal by the cognitive user, $b(k)$ is a white Gaussian noise with zero-mean and variance σ_b^2, $x(k)$ represents a transmitted signal in the given frequency band and assumed to be independent and identically distributed with zero mean and σ_x^2 variance. We define the average signal-to-noise ration as SNR= σ_x^2/σ_b^2.

2.2. Energy detector

The discrete-time received signal can be represented in vector notation by $\mathbf{y} = [y(1), y(2), ..., y(M)]$. The energy detector utilizes square of the magnitude of the received data samples to acquire received signal energy and this estimated energy is compared with the threshold λ and the decision is made whether the signal is present or not as follows:

$$\begin{aligned} \mathcal{T}_{ED} &= \frac{1}{M}\sum_{k=1}^{M} |y(k)|^2 \qquad (2) \\ &= \begin{cases} > \lambda & \text{under } \mathcal{H}_1 \\ < \lambda & \text{under } \mathcal{H}_0 \end{cases} \end{aligned}$$

The threshold λ is obtained for a given probability of false alarm P_{fa}, which is defined as the probability that the receiver decides a target is present when it is not. In case of large M and according to Central Limit Theorem (CLT), the probability distribution of $\mathcal{T}_{ED}$ can be approximated by Gaussian distribution. The expressions for probability of detection and false alarm are calculated in [10, 12] as:

$$P_{fa} \approx \mathcal{Q}(\frac{\lambda - \sigma_b^2}{\sqrt{2/M}\sigma_b^2}) \tag{3}$$

$$P_d \approx \mathcal{Q}(\frac{\lambda - (\sigma_b^2 + \sigma_x^2)}{\sqrt{2/M}(\sigma_b^2 + \sigma_x^2)}) \tag{4}$$

where $\mathcal{Q}(z) = \frac{1}{\sqrt{(2\pi)}}\int_z^{\infty} e^{-\frac{t^2}{2}} dt$ denotes tail probability of the normal PDF. As we can see in equations (3) and (4), the threshold λ depends on parameters such as variance, SNR, number of the samples M, and required detection or false alarm probability.

3. NOISE UNCERTAINTY MODELS

As mentioned above, the threshold λ used in energy detector depends on the noise variance, hence any errors in noise variance, will lead to a significant performance degradations. The lack of accurate knowledge of the noise variance is well known as noise uncertainty problem.

3.1. Bounded uncertainty model

In their paper [13], Tandra and Sahai consider that the distribution function for noise can be summarized in a bounded interval $[(1+\rho)^{-1}\sigma_b^2, (1+\rho)\sigma_b^2]$, where σ_b^2 is the nominal noise power and ρ is a parameter that quanties the level of the uncertainty. We have already discussed and analyzed the case without noise uncertainty. Now, if we consider the case with uncertainty in the noise model, the new detection probability P_d and false alarm probability P_{fa} are defined as:

$$\begin{aligned} P_{fa} &\approx \underset{\tilde{\sigma}_b^2 \in [(1+\rho)^{-1}\sigma_b^2, (1+\rho)\sigma_b^2]}{\arg\max} \mathcal{Q}(\frac{\lambda - \tilde{\sigma}_b^2}{\sqrt{2/M}\tilde{\sigma}_b^2}) \\ &\approx \mathcal{Q}(\frac{\lambda - (1+\rho)\sigma_b^2}{\sqrt{2/M}(1+\rho)\sigma_b^2}) \qquad (5) \\ P_d &\approx \underset{\tilde{\sigma}_b^2 \in [(1+\rho)^{-1}\sigma_b^2, (1+\rho)\sigma_b^2]}{\arg\min} \mathcal{Q}(\frac{\lambda - (\tilde{\sigma}_b^2 + \sigma_x^2)}{\sqrt{2/M}(\tilde{\sigma}_b^2 + \sigma_x^2)}) \\ &\approx \mathcal{Q}(\frac{\lambda - ((1+\rho)^{-1}\sigma_b^2 + \sigma_x^2)}{\sqrt{2/M}((1+\rho)^{-1}\sigma_b^2 + \sigma_x^2)}) \qquad (6) \end{aligned}$$

According to these formulas, the M-SNR relationship for given P_{fa} and P_d is:

$$M = 2[\beta\mathcal{Q}^{-1}(P_{fa}) - (\beta^{-1} + SNR)\mathcal{Q}^{-1}(P_d)]^2(SNR - (\beta^2 - 1/\beta))^2 \tag{7}$$

where $\beta = 1 + \rho$. As the SNR approaches the SNR$_{wall}$, defined as $\frac{\beta^2-1}{\beta}$, the number M of received signal samples goes to infinity for a given (P_{fa}, P_d).

3.2. Unbounded uncertainty model

The Unbounded uncertainty model is firstly proposed by Sonnenschein and Fishman in [9]. This model consider the case of a log-normal approximated noise uncertainty and, after some mathematical development, they reduce the analysis of the noise uncertainty to a bounded distribution. However, Jouin in [12] revisit this log-normal approximation model and suggest to redene the uncertainty based on the estimated noise distribution's variance. The estimated noise level $\tilde{\sigma}_b^2$ is assumed to follow log-normal distribution with expectation $\mathbb{E}[\tilde{\sigma}_b^2] = \sigma_b^2$ and variance $\mathbb{V}[\tilde{\sigma}_b^2] = u\sigma_b^4$, where u the defined uncertainty parameter. For large number of received samples M, the statistic $\mathcal{T}_{ED}$ can be accurately approximated by a Log-Normal distribution as following [12]:

$$\mathcal{T}_{ED} \sim logN(\mu, \nu) \begin{cases} \nu = log(1 + \frac{2}{M}) \\ \mu = 2log(\sigma_{\mathcal{T}_{ED}}) - \mu/2 \end{cases}$$

where $\sigma_{\mathcal{T}_{ED}}$ is the value of the power level of the collected samples under hypothesis $\mathcal{H}_0$ or $\mathcal{H}_1$. Let's now consider the statistics $\mathcal{W}_{ED} = \mathcal{T}_{ED}/\tilde{\sigma}_b^2$ which follow a normal distribution. Thus, we can define the false alarm probability P_{fa} and the detection probability P_d as:

$$\mathcal{W}_{ED} \quad \lessgtr_{\mathcal{H}_1}^{\mathcal{H}_0} \quad log(\lambda) \tag{8}$$

$$P_{fa} \quad \approx \quad \mathcal{Q}\left(\frac{log(\lambda\sqrt{\frac{1+2/M}{1+u}})}{\sqrt{log(1+2/M)(1+u)}}\right) \tag{9}$$

$$P_d \quad \approx \quad \mathcal{Q}\left(\frac{log(\frac{\lambda}{1+SNR}\sqrt{\frac{1+2/M}{1+u}})}{\sqrt{log(1+2/M)(1+u)}}\right) \tag{10}$$

So for a given probability of false alarm, we can determine the energy detector threshold $log(\lambda)$.

4. BOUNDED AND UNBOUNDED NOISE UNCERTAINTY RELATIONSHIP

The principal idea is to find a match between the noise uncertainty interval, $[log((1+\rho)^{-1}\sigma_b^2), log((1+\rho)\sigma_b^2)]$, defined by Tandra and Sahai and the confidence interval of the normal distribution of the estimated noise level $log(\tilde{\sigma}_b^2)$ defined as $I_c = [log(\frac{\sigma_b^2}{\sqrt{1+u}}) - \alpha_1\frac{\sqrt{log(1+u)}}{\sqrt{M}}; log(\frac{\sigma_b^2}{\sqrt{1+u}}) + \alpha_2\frac{\sqrt{log(1+u)}}{\sqrt{M}}]$. Thus, we evaluate the correspondence between above two intervals and we get:

$$\begin{cases} \sqrt{log(1+u)} = \frac{\alpha_2-\alpha_1}{\sqrt{M}} & \alpha_2 > \alpha_1 \\ log(1+\rho) = \frac{\alpha_2+\alpha_1}{2\sqrt{M}}\sqrt{log(1+u)} & \rho > 0 \end{cases}$$

Therefore, the noise uncertainty parameter can be calculated by fixing α_1 and α_2.

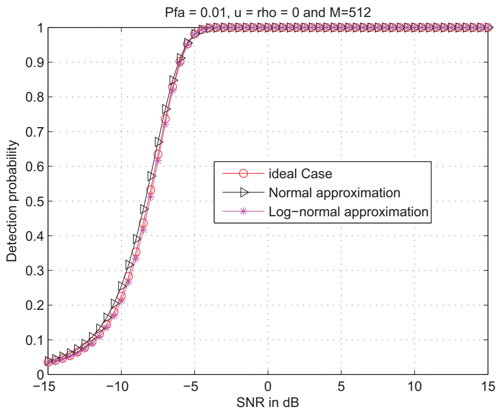

Fig. 1. Detection probability as function of the SNR for a fixed false alarm (Pfa=0.01), ideal case of Energy Detector ($\rho = u = 0$).

Figure 1 and figure 2 show the performance of energy detection algorithm for a given probability of false alarm, signal-to-noise ratio on X-axis and probability of detection on Y-axis. Noise uncertainty makes the energy detector very unreliable where good estimation of the noise power level is not available. Thus, the probability of detection vanishes very quickly when the SNR is becoming very small.

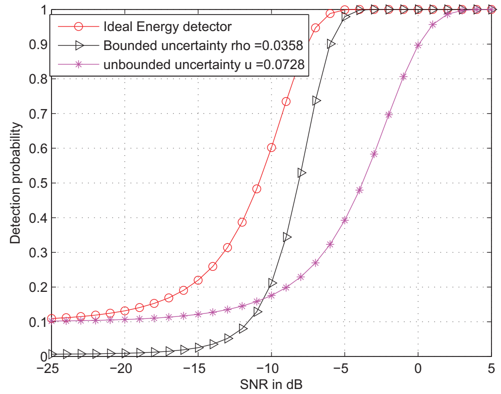

Fig. 2. Detection probability as function of the SNR for a fixed false alarm (Pfa=0.1) and M=512, Energy Detector under bounded and unbounded noise uncertainty.

Now, starting from the confidence interval I_c, as defined above, we can express the bounded noise uncertainty as $\tilde{\sigma}_b^2 \in [\frac{\sigma_b^2}{\sqrt{1+u}}e^{-\alpha_1\frac{\alpha_2-\alpha_1}{M}}; \frac{\sigma_b^2}{\sqrt{1+u}}e^{\alpha_2\frac{\alpha_2-\alpha_1}{M}}]$. Then P_d and P_{fa} can be evaluated as:

$$P_{fa} \quad \approx \quad \mathcal{Q}\left(\frac{\lambda - \frac{\sigma_b^2}{\sqrt{1+u}}e^{\alpha_2\frac{\alpha_2-\alpha_1}{M}}}{\sqrt{2/M}\frac{\sigma_b^2}{\sqrt{1+u}}e^{\alpha_2\frac{\alpha_2-\alpha_1}{M}}}\right) \tag{11}$$

$$P_d \quad \approx \quad \mathcal{Q}\left(\frac{\lambda - (1+SNR)\frac{\sigma_b^2}{\sqrt{1+u}}e^{-\alpha_1\frac{\alpha_2-\alpha_1}{M}}}{\sqrt{2/M}(1+SNR)\frac{\sigma_b^2}{\sqrt{1+u}}e^{-\alpha_1\frac{\alpha_2-\alpha_1}{M}}}\right) \tag{12}$$

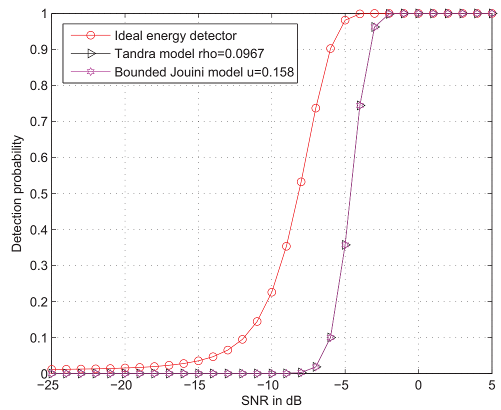

Fig. 3. Detection probability as function of the SNR for a fixed false alarm (Pfa=0.1) and M=512.

Figure 3 proves that the Tandra model is a particular case of the unbounded noise uncertainty log-normal distribution approximation.

5. CONCLUSION

In this paper, we have presented a study of the noise uncertainty models used in energy detection based spectrum sensing. By interpreting the two noise uncertainty bounded and unbounded models proposed respectively by Tandra and Jouini, we have proved that the bounded model is a specific case of general unbounded model suggested by Sonnenschein in [9] and Jouini in [12].

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