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GAME-THEORETIC LEARNING FOR ACTIVATION OF DIFFUSION LEAST MEAN SQUARES

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ABSTRACT

This paper presents a game-theoretic approach to node activation control in parameter estimation via diffusion least mean squares (LMS). The energy-aware activation control is a noncooperative repeated game where nodes autonomously decide when to activate based on a utility function that captures the trade-off between node's contribution and energy expenditure. The proposed two time-scale stochastic approximation algorithm ensures the parameter estimates weakly converge to the true parameter across the network, yet the global activation behavior along the way tracks the set of correlated equilibria of the underlying activation control game.

Index Terms— Adaptive networks distributed estimation, game theory, stochastic approximation.

1. INTRODUCTION

This paper deals with a novel game-theoretic mechanism whereby nodes learn how to activate diffusion least mean squares (LMS). The diffusion LMS [2] aims for a group of nodes to estimate/track an unknown common parameter by taking noisy measurements and relying solely on in-network processing. Due to limited communication capability, nodes can exchange information only with neighbors. At successive times, each node: (i) exchanges estimate with neighbors and fuses the collected data; (ii) uses the fused data and local measurements to refine its estimate via an LMS-type adaptive filter. Thus, nodes' estimates are functions of both temporal and spatial data, thereby, enabling the adaptive network to respond in real-time to temporal and spatial evolution of the data statistics [2]. Due to noise suppression through ensemble averaging, such diffusion LMS schemes improve the estimation performance, yet yield savings in computation, communication and energy expenditure.

The intertwined role of adaptation and learning, crucial to the self-configuration of networks, makes game-theoretic learning an appealing framework, wherein sophisticated global behavior can be achieved as the long-run outcome of nodes locally groping for optimality. In this paper, we devise an energy-aware diffusion LMS by equipping nodes with a game-theoretic algorithm that prescribes nodes when to activate. Due to interdependence of nodes' activation, it is formulated as a noncooperative repeated game. Associated with the activation decision, there corresponds a reward/penalty that captures the trade-off between the "potential value" of the node contribution and its energy costs. A novel two time-scale stochastic approximation algorithm is then proposed that combines the diffusion LMS (slow timescale) with a learning algorithm of regret-matching type [3] (fast timescale) for activation control. The proposed energy-aware diffusion LMS is scalable, robust to delays, and requires minimal message passing among nodes. Our results show that the proposed algorithm is asymptotically consistent, yet the global activation behavior along the way tracks the set of *correlated equilibria* [4] of the underlying game. There are only few works (e.g., [5, 6]) that propose game-theoretic methods for energy-efficient data acquisition. What differentiates this work is that direct interaction of the energy saving scheme with the parameter estimation algorithm is studied.

Correlated equilibrium is a generalization of Nash equilibrium and describes a condition of competitive optimality. It is, however, more preferable for online learning due to its structural and computational simplicity, and coordination capability. This coordination can potentially lead to higher utilities than if nodes make activation decisions independently (as required by the Nash equilibrium).

2. GAME-THEORETIC ACTIVATION CONTROL FOR DIFFUSION LMS

2.1. Parameter Estimation Problem

Consider a set of K nodes $\mathcal{K} = \{1, \ldots, K\}$ deployed to perform parameter estimation in the local regression model:

$$y_n^k = h_n^k \overline{\theta} + v_n^k, \quad k = 1, \ldots, K. \tag{1}$$

This paper is a short version of [1], where the interested reader can find details not discussed here.

Here, $\overline{\theta}$ is the $M \times 1$ unknown parameter, $\{h_n^k\}$ the sequence of $1 \times M$ random vector of regressors locally accessible to each node k, y_n^k the scalar local measurements and $\{v_n^k\}$ the sequence of zero-mean local measurement noise. The network of nodes then seeks $\overline{\theta}$ that solves

$$\min_\theta \ \mathbb{E}\left\|Y_n - H_n^c\theta\right\|^2. \tag{2}$$

Here, $H_n^c = \text{col}\{h_n^1, \ldots, h_n^K\}$, $Y_n = \left[y_n^1, \ldots, y_n^K\right]'$, where $(\cdot)'$ denotes the transpose operator. Further, $\mathbb{E}$ and $\|\cdot\|$ denote the expectation and Euclidean norm operators, respectively.

The diffusion LMS [2] is a *distributed* algorithm that adopts a peer-to-peer diffusion protocol to implement cooperation among individual nodes and ensures the sequence of parameter estimates θ_n^k across the network converge to (or track) the parameter $\overline{\theta}$ ($\overline{\theta}_n$) that solves (2).

2.2. Diffusion LMS

We start by spelling out the conditions imposed on the measurement model (1):

(C1) There exists a symmetric and positive-definite matrix R_h^k such that $R_h^k \triangleq \mathbb{E}h_n^{k\prime}h_n^k$, and

$$\left\|\sum_{\tau=n}^{\infty}\mathbb{E}_n^k h_\tau^{k\prime}v_\tau^k\right\| \le C, \ \ \left\|\sum_{\tau=n}^{\infty}\mathbb{E}_n^k h_\tau^{k\prime}h_\tau^k - R_h^k\right\| \le C.$$

Here, $\mathbb{E}_n^k$ denotes the conditional expectation with respect to $\{h_\tau^k, y_\tau^k, \tau < n\}$. The above condition allows us to work with correlated signals whose remote past and distant future are asymptotically independent. With suitable regularity conditions (such as uniform integrability), a large class of processes $\{h_n, v_n\}$ can be considered, e.g., Markovian or moving average processes.

Each node k can only communicate with neighbors determined by the connectivity graph $\mathcal{G} = (\mathcal{E}, \mathcal{V})$. Here, $\mathcal{G}$ is an undirected graph, where $\mathcal{V} = \mathcal{K}$ is the set of nodes and

$$(k,l) \in \mathcal{E} \text{ if there exists a link between nodes } k \text{ and } l. \tag{4}$$

The *open* and *closed neighborhoods* for each node k are then defined by $\mathcal{N}_k = \{l \in \mathcal{V}; (k,l) \in \mathcal{E}\}$ and $\mathcal{N}_k^c = \mathcal{N}_k \cup \{k\}$, respectively. Let further $\mathcal{S}_k = \mathcal{K}\backslash\mathcal{N}_k^c$ represent the set of non-neighbors of node k.

Nodes exchange estimates over the graph at each period. Each node k then fuses the local estimates $\{\theta_n^l\}_{l\in\mathcal{N}_k^c}$ by means of a linear combiner:

$$\phi_n^k = \textstyle\sum_{l\in\mathcal{N}_k^c} w_{kl}\theta_n^l. \tag{5}$$

We pose the following condition on the weights w_{kl} that constitute a stochastic matrix $W = [w_{kl}]$.

$$\text{(C2)} \qquad \begin{array}{c} W = I_K + \mu Q, \ \ 0 < \mu \ll 1, \\ Q\mathbb{1}_K = \mathbf{0}, |q_{ij}| \le 1, \ \ \forall i,j, \ \ q_{ij} \ge 0, \ \ \text{for} \ \ i \ne j. \end{array}$$

Here, $\mathbb{1}_K = [1, \ldots, 1]_{K\times 1}$. Note that W is dominated by the diagonal elements representing the weights that nodes put on their own estimate. This is essentially because smaller step-sizes require smaller fusion weights to ensure fusion and assimilation of measurements are performed at the same timescale. The fused estimates are then fed back into the local LMS-type adaptive filter. More precisely, each node runs:

$$\begin{array}{c} \theta_n^k = \phi_{n-1}^k + \mu h_n^{k\prime}\left[y_n^k - h_n^k\phi_{n-1}^k\right], \\ \phi_{n-1}^k = \sum_{l\in\mathcal{N}_k^c} w_{kl}\theta_{n-1}^l. \end{array} \tag{6}$$

The estimate at each node is thus a function of both its temporal data as well as the spatial data across the neighbors. This cooperation among nodes enables the adaptive network to respond in real-time to the temporal and spatial variations in the statistical profile of the data and leads to savings in communication and energy resources [2].

2.3. Activation Control Game

The objective is to combine the diffusion LMS with a game-theoretic mechanism which enables the nodes to decide when to activate. Nodes in the sleep mode do not take measurements, nor do they fuse data from neighbors and update estimates. Therefore, they save energy in both sensing and transmitting their estimates to neighbors. The key feature that characterizes our study as a game is the interdependence of individual node's behavior. The usefulness of a node's information, channel quality, required packet transmission energy all depend on the activity of other nodes in the network.

The problem of each node k is to pick actions a_n^k from the set $\mathcal{A} = \{0(\text{sleep}), 1(\text{activate})\}$ at successive times $n = 1, 2, \ldots$ to strategically optimize its utility. Denote by $\mathbf{a} = (a^1, \ldots, a^K)$ the joint action profile of all nodes, which can be rearranged as $\mathbf{a} = (a^k, \mathbf{a}^{-k})$. Here, $\mathbf{a}^{-k}$ denotes the joint activation decision of all nodes excluding node k. Let further $\boldsymbol{\theta} = [\theta^1, \ldots, \theta^K]'$ denote the K-tuple of nodes' estimates each obtained locally via the diffusion LMS. The utility function captures the trade-off between energy expenditure and the "value" of each node k's contribution, and is comprised of a *local* and a *global* term:

$$U^k\big(a^k, \mathbf{a}^{-k}; \boldsymbol{\theta}\big) = U_{\text{loc}}^k\big(a^k, \mathbf{a}^{\mathcal{N}_k}; \boldsymbol{\theta}\big) + U_{\text{glob}}^k\big(a^k, \mathbf{a}^{\mathcal{S}_k}\big). \tag{7}$$

In (7), $\mathbf{a}^{-k}$ is rearranged as $\mathbf{a}^{-k} = \big(\mathbf{a}^{\mathcal{N}_k}, \mathbf{a}^{-\mathcal{N}_k^c}\big)$, where $\mathbf{a}^{\mathcal{N}_k}$ and $\mathbf{a}^{\mathcal{S}_k}$ denote the joint action profile of neighbors $\mathcal{N}_k$ and non-neighbors $\mathcal{S}_k$, respectively. The utility for each node k further depends on the vector of parameter estimates $\boldsymbol{\theta}$.

The *local* utility function $U_{\text{loc}}^k(\cdot)$ captures the trade-off between the value of the measurements collected by node k and the energy costs associated to it. If too many of node k's neighbors activate simultaneously, excessive energy is consumed due to the spatial-temporal correlation of measurements. Additionally, the probability of successful transmission reduces due to channel congestion. On the other hand, if too few of node k's neighbors activate, their fused estimates lack "innovation." The *local* utility of node k is

then given by: [see (8) at the bottom of the page], where $\eta^k(a^k, \mathbf{a}^{\mathcal{N}_k}) = a^k + \sum_{l\in\mathcal{N}_k} a^l$. In (8), $K_{l,1}$ and γ_l are the pricing parameters related to the 'reward' associated with data collected by node k; $K_{l,2}$ is the pricing parameter related to the energy costs associated to activation E_{Act} and broadcasting measurements $E_{\text{Tx}}(\cdot)$; see [6, p. 6099] for the above details of the Chipcon CC2420 transceiver chipset that implements the CSMA/CA scheme and is used in the numerical example in Sec. 5. Higher activity η^k of nodes neighboring node k lowers its local utility due to: (i) reducing success of transmission attempts, and (ii) inefficient usage of energy resources (due to spatial-temporal correlation).

The *global* utility function $U^k_{\text{glob}}(\cdot)$ concerns the connectivity of the network and diffusion of estimates in a geographic region. Let $\mathcal{R}^k_r$ denote the set of nodes within radius r from node k excluding the neighborhood $\mathcal{N}^c_k$. Define the number of active nodes in $\mathcal{R}^k_r$ by $\zeta^k\big(a^k, \mathbf{a}^{\mathcal{S}_k}\big) = a^k + \sum_{l\in\mathcal{R}^k_r} a^l$. The *global* utility of node k is defined as:

$$U^k_{\text{glob}}\big(a^k, \mathbf{a}^{\mathcal{S}_k}\big) = \begin{cases} 0, & a^k = 0 \text{ (sleep)}, \\ K_g\big(e^{-\gamma_g \zeta^k}\big), & a^k = 1 \text{ (activate)}. \end{cases} \tag{9}$$

In (9), K_g and γ_g are the pricing parameters. Higher ζ^k lowers the global utility due to: (i) smaller role of node k in diffusion of estimates and keeping connectivity in $\mathcal{R}^k_r$, and (ii) reducing the success of transmission for other nodes in the neighborhood which affects global diffusion of estimates; see [1] for more details.

Each node k realizes $\eta^k_n = \eta^k(a^k, \mathbf{a}^{\mathcal{N}_k}_n)$ as a consequence of receiving neighbors' estimates $\{\theta^l_n\}_{l\in\mathcal{N}_k}$, therefore, is able to evaluate its local utility. Nodes, however, do not observe non-neighbors' actions, therefore, do not realize $\zeta^k_n = \zeta^k(a^k_n, \mathbf{a}^{\mathcal{S}_k}_n)$ and are unable to evaluate global utilities. To this end, some sort of centralization is required by implementing cluster heads that monitor the activity of nodes in their locale and deliver global utilities to the nodes. Hereafter, $U^k_{\text{glob},n}(a^k_n) = U^k_{\text{glob}}\big(a^k_n, \mathbf{a}^{\mathcal{S}_k}_n\big)$ denotes the realized global utility for node k at period n by choosing action a^k_n.

The activation control problem is then formulated as a noncooperative repeated game with neighborhood structure:

$$G = \Big(\mathcal{K}, \big(\mathcal{A}^k\big)_{k\in\mathcal{K}}, \mathcal{G}, \big(U^k\big)_{k\in\mathcal{K}}, \boldsymbol{\theta}\Big), \tag{10}$$

where the set of actions $\mathcal{A}^k = \mathcal{A} = \{0 \text{ (sleep)}, 1 \text{ (activate)}\}$ for all nodes $k \in \mathcal{K}$. The set of *correlated ϵ-equilibrium* $\mathcal{C}_\epsilon(\boldsymbol{\theta})$ for the game G is defined as follows:

Definition 2.1. *Let $\boldsymbol{\pi}$ denote a joint distribution on the joint action space $\mathcal{A}^{\mathcal{K}} = \times_{k\in\mathcal{K}}\mathcal{A}^k$, i.e., $\pi(\mathbf{a}) \geq 0$, $\forall \mathbf{a} \in \mathcal{A}^{\mathcal{K}}$ and $\sum_{\mathbf{a}\in\mathcal{A}^{\mathcal{K}}} \pi(\mathbf{a}) = 1$. The set of* correlated equilibria $\mathcal{C}_\epsilon(\boldsymbol{\theta})$ *is the polytope: [see (11), shown at the bottom the page], where $\pi^k(i, \mathbf{a}^{-k})$ denotes the probability that node k picks action i and the rest $\mathbf{a}^{-k}$.*

Note that $\mathcal{C}_\epsilon(\boldsymbol{\theta})$ is a function of $\boldsymbol{\theta}$—it slowly evolves as the local parameter estimates $\boldsymbol{\theta}_n$ change over time. Correlated equilibrium allows nodes to coordinate their activation while retaining their autonomy. This coordination leads to potentially higher utilities than if they take actions independently (as required by Nash equilibrium) [4]. It is further natural due to the correlation among nodes' activation decisions.

3. ENERGY-AWARE DIFFUSION LMS

In light of the above game-theoretic model, we proceed here to present the energy-aware diffusion LMS algorithm. The node activation mechanism can be described as follows: Each node k generates and updates two regret matrices: (i) $\boldsymbol{\alpha}^k_{2\times 2}$, that records average local-regrets, and (ii) $\boldsymbol{\beta}^k_{2\times 2}$, which is an unbiased estimate of the average global-regrets. (The interested reader is referred to [1] for related details.) Positive overall-regrets $\alpha^k_n(i,j) + \beta^k_n(i,j)$ imply the opportunity to achieve higher utilities by switching from action i to j in future. Define $|x|^+ = \max\{0, x\}$. Nodes activate according to a randomized policy $\boldsymbol{\psi}^k_n$, determined as follows: At each period, with probability $1-\delta$, node k chooses its action with probabilities proportional to $\big|\boldsymbol{\alpha}^k_n + \boldsymbol{\beta}^k_n\big|^+$. With the remaining probability δ, it uniformly randomizes among its action set $\mathcal{A}$. This can be interpreted as "exploration".

Algorithm 1:

Step 0) **Initialization:** Set the exploration factor $0 < \delta < 1$ and $\xi^k > 2|U^k_{\max} - U^k_{\min}|$, where $U^k_{\max}$ and $U^k_{\min}$ denote the upper and lower bounds on node k's utility function. Choose the adaptation rates $0 < \mu, \rho \ll 1$ such that $\mu = O(\rho^2)$. Initialize $\theta^k_0 = \phi^k_0 = \mathbf{0}$, $\boldsymbol{\psi}^k_0 = [\frac{1}{2}, \frac{1}{2}]^T$, $\boldsymbol{\alpha}^k_0 = \boldsymbol{\beta}^k_0 = \mathbf{0}_{2\times 2}$.

For $n = 1, 2, \ldots$, repeat:

Step 1) **Node Activation:** Select action a^k_n according to the distribution $\boldsymbol{\psi}^k_n$: [see (12) at the bottom of the page], where a^k_{n-1} denotes the action picked in the last period.

Step 2) **Diffusion LMS:**

$$\theta^k_n = \begin{cases} \theta^k_{n-1}, & a^k_n = 0, \\ \phi^k_{n-1} + \mu h^{k\prime}_n \big[y^k_n - h^k_n \phi^k_{n-1}\big], & a^k_n = 1. \end{cases} \tag{13}$$

$$U^k_{\text{loc}}\big(a^k, \mathbf{a}^{\mathcal{N}_k}; \boldsymbol{\theta}\big) = \begin{cases} 0, & a^k = 0 \text{ (sleep)} \\ K_{l,1}\big(1 - e^{-\gamma_l \|\theta^k - \phi^k\|^2 s(\eta^k)}\big) - K_{l,2}\big(E_{\text{Tx}}\big(\eta^k\big) + E_{\text{Act}}\big), & a^k = 1 \text{ (activate)} \end{cases} \tag{8}$$

$$\mathcal{C}_\epsilon\big(\boldsymbol{\theta}\big) = \left\{ \boldsymbol{\pi}^c_{\boldsymbol{\theta}} : \sum_{\mathbf{a}^{-k}} \pi^{c,k}_{\boldsymbol{\theta}}\big(i, \mathbf{a}^{-k}\big) \big[U^k\big(j, \mathbf{a}^{-k}; \boldsymbol{\theta}\big) - U^k\big(i, \mathbf{a}^{-k}; \boldsymbol{\theta}\big)\big] \leq \epsilon,\ \forall i, j \in \mathcal{A}, \forall k \in \mathcal{K} \right\} \tag{11}$$

$$\psi^k_n(i) = \begin{cases} (1-\delta)\min\left\{\frac{1}{\xi^k}\left|\alpha^k_{n-1}\left(a^k_{n-1}, i\right) + \beta^k_{n-1}\left(a^k_{n-1}, i\right)\right|^+, \frac{1}{2}\right\} + \frac{\delta}{2}, & i \neq a^k_{n-1} \\ 1 - \sum_{j\in\mathcal{A}\setminus\{i\}} \psi^k_n(j), & i = a^k_{n-1} \end{cases} \tag{12}$$

***Step 3)* Estimate Exchange:** If $a_n^k = 1$: (i) Transmit θ_n^k to neighbors $\mathcal{N}_k$ and collect $\{\theta_n^l\}_{l\in\mathcal{N}_k}$; (ii) Form neighbors' action profile $\boldsymbol{a}_n^{\mathcal{N}_k} = (a_n^l)_{l\in\mathcal{N}_k}$ and $\{\widehat{\theta}_n^l\}_{l\in\mathcal{N}_k}$ such that

$$\widehat{\theta}_n^l = \begin{cases} \text{received estimate}, & \text{if } a_n^k = 1 \text{ (sleep)}, \\ \theta_{n-1}^l, & \text{if } a_n^k = 0 \text{ (active)}. \end{cases} \tag{14}$$

If $a_n^k = 0$, go to Step 3.
***Step 4)* Data Fusion:**

$$\phi_n^k = \textstyle\sum_{l\in\mathcal{N}_k^c} w_{kl}\widehat{\theta}_n^l. \tag{15}$$

***Step 5)* Regret Update:** For all $i, j \in \mathcal{A}$:
5.1: Local-Regret Update [see (16) at the bottom of the page], where $I_{\{\cdot\}}$ denote the indicator function.
5.2: Global-Regret Update [see (17)at the bottom of the page].
***Step 6)* Recursion:** Set $n \leftarrow n+1$ and go to Step 1.

The above algorithm is simply a non-linear adaptive filter (16)–(17) run in the fast timescale (step-size ρ), which governs node activation, coupled with the diffusion LMS (13) run in the slow timescale (step-size $\mu = \mathcal{O}(\rho^2)$), which carries out the parameter estimation task. Step 1) simply updates the activation strategy of nodes based on the regret updates (Step 5) in the last period. The exploration factor δ in (12) forces all nodes to activate with a minimum frequency and is essential as nodes continuously learn their global utility functions. As will be discussed later, larger δ will lead to convergence of the nodes' global activation behavior to a larger ϵ-distance of the correlated equilibria set. Taking the minimum with $1/2$ and ξ^k also ensures that $\boldsymbol{\psi}_n^k$ forms a valid probability distribution. Step 2) shows that nodes run the diffusion LMS and update their estimates only when they choose to activate. Step 3) describes the exchange of information with neighbors.

Remark 3.1. The computational burden of the activation mechanism is $O(1)$ and does not grow with the number of nodes K, nor with the dimension of the parameter estimation problem M. It is negligible as compared to the diffusion LMS, which requires $O(M)$ calculations per iteration.

4. GLOBAL NETWORK PERFORMANCE

4.1. Global Behavior

The emergent global behavior of the network can be captured by two inter-connected discrete-time processes:

Define

$$\begin{array}{l} B = W \otimes I_M, \quad \widehat{B} = Q \otimes I_M, \quad S \triangleq \mu I_{KM}, \\ V_n = [v_n^1, \ldots, v_n^K]', \quad Y_n = \left[y_n^1, \ldots, y_n^K\right]', \\ M_n = \left[\text{diag}\{a_n^1 I_M, \ldots, a_n^K I_M\}\right], \\ H_n = \left[\text{diag}\{h_n^1, \ldots, h_n^K\}\right], \end{array} \tag{18}$$

where $\otimes$ denotes the Kronecker product. The global diffusion LMS update associated with Algorithm 1 can be expressed in the compact state-space form as

$$\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1} + SM_n\left[\widehat{B}\boldsymbol{\theta}_{n-1} + H_n'\left(Y_n - H_n B\boldsymbol{\theta}_{n-1}\right)\right]. \tag{19}$$

The collective activation behavior of nodes, $\boldsymbol{z}_n$, is defined as the discounted *empirical frequency* of joint activation decisions of all nodes up to period n. Formally,

$$\boldsymbol{z}_n = \rho \textstyle\sum_{\tau\le n} (1-\rho)^{n-\tau}\, \boldsymbol{e}_{\mathbf{a}_\tau}, \tag{20}$$

where $\boldsymbol{e}_{\mathbf{a}_\tau}$ denotes the unit vector in the space of Cartesian product $\times_{k\in\mathcal{K}}\mathcal{A}$ with the element corresponding to $\mathbf{a}_\tau$ being equal to one. In (20), ρ serves as a forgetting factor to foster adaptivity to the evolution of the local parameter estimates.

4.2. Main Result: From Individual to Global Behavior

We now characterize the global behavior emerging by each node individually following Algorithm 1. Let $\widetilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \overline{\boldsymbol{\theta}}$. Define the piecewise constant continuous-time interpolation processes $\widetilde{\boldsymbol{\theta}}^\mu(\cdot)$ and $\boldsymbol{z}^\rho(\cdot)$ associated to Algorithm 1:

$$\begin{array}{ll} \widetilde{\boldsymbol{\theta}}^\mu(\cdot) = \widetilde{\boldsymbol{\theta}}_n, & \text{for } t \in [n\mu, (n+1)\mu), \\ \boldsymbol{z}^\rho(\cdot) = \boldsymbol{z}_n, & \text{for } t \in [n\rho, (n+1)\rho). \end{array} \tag{21}$$

Before proceeding with the theorem, let us recall that weak convergence is a generalization of convergence in distribution to a function space.

Theorem 4.1. *Suppose every node follows Algorithm 1. Then, for each $\epsilon > 0$, there exists $\widehat{\delta}(\epsilon)$ such that if $\delta < \widehat{\delta}(\epsilon)$ in Algorithm 1 (Step 1) and $\mu = \mathcal{O}(\rho^2)$,*
1) As $\rho \to 0$, $\boldsymbol{z}^\rho(\cdot)$ converges weakly to $\mathcal{C}_\epsilon(\boldsymbol{\theta}(\cdot))$.
2) Under (C1)–(C2), $\widetilde{\boldsymbol{\theta}}^\mu(\cdot)$ converges weakly, as $\mu \to 0$, to $\widetilde{\boldsymbol{\theta}}(\cdot)$ that is a solution to

$$\tfrac{d\widetilde{\boldsymbol{\theta}}(t)}{dt} \in \left\{ M_{\boldsymbol{\pi}_{\boldsymbol{\theta}}(t)}(\widehat{B} - R_h^d)\widetilde{\boldsymbol{\theta}}(t); \boldsymbol{\pi}_{\boldsymbol{\theta}(t)} \in \mathcal{C}_\epsilon(\boldsymbol{\theta}(t)) \right\}, \tag{22}$$

where $M_{\boldsymbol{\pi}_{\boldsymbol{\theta}}} = diag\{m_{\boldsymbol{\pi}_\theta}^1 I_M, \ldots, m_{\boldsymbol{\pi}_\theta}^K I_M\}$, $I_{KM} > M_{\boldsymbol{\pi}_{\boldsymbol{\theta}}} > \kappa \cdot I_{KM}$ for some $\kappa > 0$, $\widehat{B}$ is defined in (18), and $R_h^d = diag\{R_h^1, \ldots, R_h^K\}$.
3) System (22) is globally asymptotically stable and

$$\lim_{t\to\infty} \left\|\widetilde{\boldsymbol{\theta}}(t)\right\| = \mathbf{0}. \tag{23}$$

Proof. See [1] for detailed proof. □

$$\alpha_n^k(i,j) = \alpha_{n-1}^k(i,j) + \rho\left[\left(U_{\text{loc}}^k\left(j, \mathbf{a}_n^{\mathcal{N}_k}; \boldsymbol{\theta}_n\right) - U_{\text{loc}}^k\left(a_n^k, \mathbf{a}_n^{\mathcal{N}_k}; \boldsymbol{\theta}_n\right)\right) I_{\{a_n^k = i\}} - \alpha_{n-1}^k(i,j)\right] \tag{16}$$

$$\beta_n^k(i,j) = \beta_{n-1}^k(i,j) + \rho\left[\left[\frac{\psi_n^k(i)}{\psi_n^k(j)} U_{\text{glob},n}^k\left(a_n^k\right) I_{\{a_n^k = j\}} - U_{\text{glob},n}^k\left(a_n^k\right) I_{\{a_n^k = i\}}\right] - \beta_{n-1}^k(i,j)\right] \tag{17}$$

Part 1) in Theorem 4.1 concerns the game-theoretic activation mechanism. It asserts that if all nodes individually follow the proposed mechanism, the global activation behavior tracks the time-varying polytope $\mathcal{C}_\epsilon(\boldsymbol{\theta}_n)$ of the underlying activation control game as the vector of local estimates $\boldsymbol{\theta}_n$ is refined over time. Hence, coordination is reached among nodes for activation. Part 2) gives the limit system representing the behavior of the energy-aware diffusion LMS as $\mu \to 0$. (A differential inclusion is of the form $dx/dt \in \mathcal{F}(x)$, where $\mathcal{F}(x)$ specifies a family of trajectories rather than a single trajectory as in the ODE $dx/dt = F(x)$.) It simply states that dynamics of the diffusion LMS along the way is based on the global activation behavior drawn from the set of correlated equilibrium. Part 3) finally concludes asymptotic consistency of the energy-aware diffusion LMS.

5. NUMERICAL EXAMPLE

We consider the network topology depicted in Fig. 1 and define $Q = [q_{kl}]$ in (6) as

$$q_{kl} = \begin{cases} \frac{1}{|\mathcal{N}_k^c|}, & l \in \mathcal{N}_k^c, \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

Fig. 1 illustrates the statistics of node's regressors, generated by a Gauss-Markov source with local correlation function $r(\tau) = \sigma_{h,k}^2 \nu_{h,k}^{|\tau|}$, where $\nu_{h,k}$ is the correlation index. We assume nodes are equipped with Chipcon CC2420 transceiver chipset which implements CSMA/CA protocol for exchanging estimates with neighbors. The reader is referred to [6, Appendix] for detailed model description and expressions for $E_{\text{Tx}}(\cdot)$ ($k_c = k_m = 1$) and E_{Act} in (8). We further set $\rho =$ 5e-2, $\mu =$ 5e-3, $K_{l,1} = K_g = 1$, and assume v_n^k is i.i.d., Gaussian with zero mean and variance $\sigma_{v,k}^2 = 10^{-3}$.

Define the network excess mean square error $\text{EMSE}^{\text{net}} \triangleq \frac{1}{K}\sum_{k=1}^{K} \text{EMSE}_k$, where $\text{EMSE}_k = \mathbb{E}|h_n^k(\overline{\theta} - \phi_{n-1}^k)|^2$. Fig. 2 demonstrates the trade-off between energy expenditure in the network and the rate of convergence of the diffusion LMS algorithm in terms of EMSE^{net} after 1000 iterations. Nodes become more conservative by increasing the energy consumption parameter $K_{l,2}$ and, accordingly, activate less frequently due to receiving lower utilities; see (8). This reduces the average proportion of active nodes and increases EMSE^{net} due to recording fewer measurements and less frequent fusion of neighboring estimates. Increasing the pricing parameters γ_l, in (8), and γ_g, in (9), has the same effect as can be observed in Fig. 2.

6. CONCLUSION

We considered energy-aware parameter estimation via diffusion LMS by equipping each node with a game-theoretic learning algorithm that updates its activation strategy and adapts it to the changes in the network. The convergence analysis revealed that the proposed energy-aware diffusion LMS is asymptotically consistent, yet the global activation behavior tracks the set of approximate correlated equilibria of the underlying activation control game.

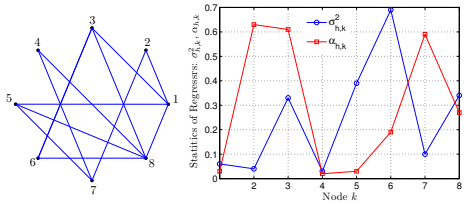

Fig. 1. Network topology and nodes' regressors statistics.

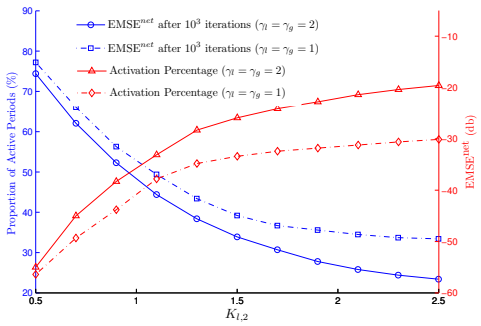

Fig. 2. Trade-off between energy expenditure and estimation accuracy after 1000 iterations.

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