EVALUATION OF MIMO CHANNEL NON-STATIONARITY

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ABSTRACT

Several MIMO processing algorithms have been proposed that exploit long-term channel statistics, relaying on the critical assumption that this long-term information is valid long enough. In this paper, we consider the Correlation Matrix Distance (CMD) method previously proposed for the evaluation of MIMO channel non-stationarity. We highlight a couple of problems with the CMD measure and propose two new metrics that are more appropriate for non-stationarity evaluation. The performance of the CMD method and new correlation matrix distance metrics is investigated using measured 4x4 MIMO channels. Both Line-of-Sight (LOS) and Non-LOS (NLOS) environments are considered.

Index Terms— MIMO, Channel Non-Stationarity, Correlation Matrix Distance.

1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) technology is considered a key feature in the current wireless communication systems; e.g., long-term evolution (LTE) interface as well as the upcoming enhancement, LTE-Advanced [1][2]. The optimum transmission performance of MIMO communication systems can be achieved if the instantaneous channel gains are known at both the transmitter and receiver. For mobile channels, where the channel gains are varying rapidly with time, the instantaneous channel knowledge cannot be obtained at the transmitter. Therefore, channel statistics are often used instead of the instantaneous values at the expense of some performance degradation [2].

Advanced MIMO transmission schemes such as eigen-beamforming [3] and long-term adaptive pre-coding [4] are based on knowledge of the channel statistics. If the channel statistics are constant during some relatively large stationarity region (quasi-stationary), then these schemes can be successfully applied; otherwise, the receiver will not be able to feedback the correct channel statistical information to the transmitter. Therefore, it is very important to find a suitable method to estimate the stationarity regions of MIMO channels to see when these types of transmission schemes can be effectively employed.

Unlike Single-Input Single-Output (SISO) channels, MIMO channels have higher dependency on the spatial conditions of the link such as the antenna positions and multipath reflections [5]. In mobile MIMO communications, the channel non-stationarity is mainly due to the changes of these properties when either transmit or receive antennas move from one place to another. In Non-Line-of-Sight (NLOS) environments where the multipath components are rich, it is important to examine the non-stationarity of different MIMO spatial structures to design a reliable MIMO system.

The non-stationarity of SISO channels has been investigated in [6], where a Channel Correlation Function (CCF) was introduced to estimate the stationarity region of SISO channels. For the SIMO channels, where multiple antennas are employed at the receiver only, a stationarity measure has been introduced in [7].This measure, which is called F-eigen ratio, determines the similarity between the out-dated and new channel covariance matrices based on the largest F eigenvalues that can be selected depending on the transmission algorithm.

The non-stationarity of MIMO channels has been investigated in [5], where the Correlation Matrix Distance (CMD) metric has been introduced to measure the dissimilarity between two MIMO channel covariance matrices that correspond to two different time instants based on their inner product. In [8][9][10], the CMD has been used to measure the non-stationarity of realistic radio channels. In [10], the CMD was also used to determine local quasi-stationarity regions. In [11], the CMD was used to estimate the feedback interval for closed-loop MIMO systems which is useful to determine when the receiver needs to update channel information. Recently, the CMD has been proposed in [12] to evaluate the separation between multi-link MIMO channels.

Due to its previously mentioned importance and wide spread use in the measurement of non-stationarity of MIMO channels, the CMD is considered in this paper. Specifically, we have the following three main contributions.

- The CMD measure is first revisited, where we show that it is problematic to interpret the CMD for two important transmission scenarios. The first scenario arises when MIMO spatial multiplexing is made under rich scattering environments, where the maximum number of eigenmodes (parallel data streams) is achieved. In this case, we show that the CMD will always underestimate the non-stationarity. The second scenario arises under poor scattering environments, where few eigenmodes are used in the transmission. Since the CMD evaluates the non-stationarity with respect to all eigenmodes, it does not provide a relevant measure of the non-stationarity leading to improper non-stationarity estimation.

- New correlation metrics are then proposed to remedy the pitfalls of the CMD method. For the rich scattering environment scenario, we introduce a correction factor to the CMD to overcome the non-stationarity underestimation. This new correlation distance metric will be called the Normalized CMD (NCMD). For the second scenario, we introduce a new method that evaluates the non-stationarity based on the Distance between Equidimensional Subspaces (DES) algorithm presented in [13]. In the sequel, this new metric is termed the Correlation between Largest Eigenmodes (CLE) method. To the best of our knowledge, no attempts have been yet reported in the literature to consider the DES algorithm for MIMO channel non-stationarity estimation.
- The CMD, NCMD, and CLE methods are finally tested using measured MIMO channels in the context of estimating the non-stationarity and quasi-stationarity regions.

2. SYSTEM MODEL

Consider a MIMO system with n_T transmit antennas and n_R receive antennas. The linear time varying (LTV) MIMO channel is given by an $n_R \times n_T$ matrix, $\mathbf{H}(t, \tau)$. For Orthogonal Frequency Division Multiplexing (OFDM) signals, the channel for each subcarrier will only be time selective. If $\mathbf{s}(t)$ is a vector representing the transmitted signal with n_T antennas, then the $n_R \times 1$ received signal vector $\mathbf{r}(t)$ can be given as:

$$\mathbf{r}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{n}(t)$ is an $n_R \times 1$ additive noise vector. Unfortunately, due to channel fading, feeding back instantaneous channel gains to the transmitter under a time selective channel may require huge bandwidth to be practically useful. Alternatively, the channel in Eq. (1) can be modeled as a stationary stochastic process. Under the stationarity assumption, the statistics (i.e. the mean and covariance) of the channel matrix are constant and can be estimated at the receiver instead of the instantaneous channel values. However, if the MIMO channel is not stationary, then the channel statistics may also change very rapidly with respect to time and, hence, further investigation of the channel stationarity is needed to examine the feasibility of utilizing the channel statistics.

3. STATIONARITY OF MIMO CHANNELS

The stationary MIMO spatial correlation matrix $\mathbf{R}_H$ is given by [5]:

$$\mathbf{R}_H = E[\mathrm{vec}\{\mathbf{H}(t)\}\mathrm{vec}\{\mathbf{H}(t)\}^H] \tag{2}$$

where $E[.]$ is the expectation operation and $\mathrm{vec}\{\mathbf{H}(t)\}$is the vectorization operation of a matrix. Since $\mathrm{vec}\{\mathbf{H}(t)\}$ has n_R n_T elements, $\mathbf{R}_H$ will be of size $n_R\, n_T \times n_R\, n_T$. The size of $\mathbf{R}_H$ might become very large as it increases rapidly with respect to the number of antennas at the transmit or receive side. The so-called Kronecker model both reduces the number of parameters and allows for more tractable analysis.

The Kronecker MIMO channel model has the following form for channel correlation matrix [14].

$$\mathbf{R}_H = \frac{1}{tr\{\mathbf{R}_{Rx}\}}\mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx}$$

where $\otimes$ is the Kronecker product, $\mathbf{R}_{Tx}$ and $\mathbf{R}_{Rx}$ are the $n_T \times n_T$ transmit and $n_R \times n_R$ receive correlation matrices, respectively. These matrices are defined as follows.

$$\mathbf{R}_{Tx} \equiv E[\mathbf{H}^T(t)\mathbf{H}^*(t)] \text{ and } \mathbf{R}_{Tx} \equiv E[\mathbf{H}(t)\mathbf{H}^H(t)]$$

The major disadvantage of this simplified model is the low accuracy in describing a real MIMO channel, particularly if the number of antennas increases. However, this model is sufficient for non-stationarity evaluation [15].

For a non-stationary MIMO channel, the correlation function described in Eq. (2) is time dependent. Hence, it takes the form $\mathbf{R}_H(t)$. If we assume that the spatial correlation matrix $\mathbf{R}_H(t)$ does not change (i.e. the channel is stationary) within some small averaging time interval T_v, then $\mathbf{R}_H(t)$ can be rewritten in a discrete-time function as follows [5][16],

$$\mathbf{R}_H(n) = E_t[\mathrm{vec}\{\mathbf{H}(t)\}\mathrm{vec}\{\mathbf{H}(t)\}^H] \text{ for } t \in [nT_v, (n+1)T_v]$$

The spatial stationarity region of a non-stationary MIMO channel $\mathbf{H}(t)$ can be defined as the region (time or distance in meters for moving subjects) at which the correlation matrix described by $\mathbf{R}_H(n)$ stays relatively constant. That is,

$$\mathbf{R}_H(n_1) \approx \mathbf{R}_H(n_2) \quad \text{for } |n_1 - n_2|T_v < T_s$$

$\mathbf{R}_H(n_1)$ and $\mathbf{R}_H(n_2)$ are correlation matrices that correspond to two different regions. For non-stationarity measurement, we need a matrix metric denoted by $M(\mathbf{R}_H(n_1), \mathbf{R}_H(n_2))$, that compares $\mathbf{R}_H(n_1)$ and $\mathbf{R}_H(n_2)$ and produces a value that measures the dissimilarity between these two matrices. It is desirable that $M(\mathbf{R}_H(n_1), \mathbf{R}_H(n_2))$ has a value that ranges from zero to one, where the later means totally different matrices.

4. DISSIMILARITY METRICS

In this section, we introduce the CMD method and show its pitfalls in measuring the variation in the MIMO spatial correlation matrix. Then, we propose two remedies for these pitfalls; the NCMD and CLE methods.

4.1. Correlation Matrix Distance (CMD)

The CMD method proposed in [5] measures the dissimilarity between two correlation matrices using inner product operation. That is, if $\mathbf{R}(k)$ and $\mathbf{R}(l)$ are two different matrices of size ($n\times n$), then the CMD is defined as

$$CMD(\mathbf{R}(k),\mathbf{R}(l)) = 1-\frac{\mathrm{tr}\{\mathbf{R}(k)\mathbf{R}(l)\}}{\|\mathbf{R}(k)\|_F\|\mathbf{R}(l)\|_F} \tag{3}$$

where $\|\cdot\|_F$ is the Frobenius norm. The CMD can range from zero to one. If the vectorization of the two matrices are orthogonal, the inner product will be zero and the CMD will have the value of one, indicating that the two matrices are totally different. On the other hand, if the vectorization of the two matrices are parallel, the normalized inner product will be one so the CMD will be zero, indicating that the two matrices are equal. However, the CMD method is always below one for full rank correlation matrices (even if the two matrices are as different as they possibly can be). Also it considers all eigenmodes even if only a few are used in the transmission. Therefore, it is the objective of the next two subsections to present two new metrics, the first of which (NCMD) achieves one when the distance between two full rank Hermitian positive semi-definite matrices is at the maximum value possible. The second metric (DES) is formulated so that it only makes use of dominant eigenmodes considered for transmission in poor scattering environments.

4.2. Normalized Correlation Matrix Distance (NCMD)

In rich scattering environments, the channel correlation matrix $\mathbf{R}_H(t)$ is full rank, and all eigenvalues of $\mathbf{R}_H(t)$ are larger than zero. Since the correlation matrix $\mathbf{R}_H(t)$ is always positive semi-definite Hermitian matrix, the maximum possible CMD value for full rank correlation matrices is less than one, as will be shown below. In this case, we propose to adjust the CMD metric by a normalization factor so that the maximum value of dissimilarity between two correlation matrices can achieve one.

Let $\mathbf{R}(k)$ be a given $n\times n$ correlation matrix and $\mathbf{R}(l)$ be an arbitrary correlation matrix. The maximum possible value of the CMD is then

$$\begin{aligned}&\max_{\|\mathbf{R}(l)\|_F\neq 0} CMD(\mathbf{R}(k),\mathbf{R}(l))\\ &=\max_{\|\mathbf{R}(l)\|_F\neq 0} 1-\frac{\mathrm{tr}\{\mathbf{R}(k)\mathbf{R}(l)\}}{\|\mathbf{R}(k)\|_F\|\mathbf{R}(l)\|_F}\end{aligned} \tag{4}$$

where the worst case $\mathbf{R}(l)$ equivalently can be found from

$$\min_{\|\mathbf{R}(l)\|_F=1} \mathrm{tr}\{\mathbf{R}(k)\mathbf{R}(l)\}$$

Introducing the eigenvalue decomposition $\mathbf{R}(k)\equiv\mathbf{U}\mathbf{\Lambda}(k)\mathbf{U}^H$, with the eigenvalues sorted in decreasing order, we obtain

$$\mathrm{tr}\{\mathbf{R}(k)\mathbf{R}(l)\} = \mathrm{tr}\{\mathbf{\Lambda}(k)\underbrace{\mathbf{U}^H\mathbf{R}(l)\mathbf{U}}_{\widetilde{\mathbf{R}}(l)}\} = \sum\nolimits_{i=1}^{n}\lambda_i(k)\widetilde{\mathbf{R}}(l)_{ii}$$

which clearly is minimized by setting all elements of $\widetilde{\mathbf{R}}(l)$ to zero, except the lower right corner element which is set to one.

Inserting this solution back in Eq. (4) gives that the maximum value of the CMD takes the form:

$$\begin{aligned}\max_{\|\mathbf{R}(l)\|_F\neq 0} CMD(\mathbf{R}(k),\mathbf{R}(l)) &= 1-\frac{\lambda_n(k)}{\|\mathbf{R}(k)\|_F}\\ &=1-\frac{\lambda_n(k)}{\sqrt{\sum_{i=1}^{n}\lambda_i^2(k)}}\end{aligned}$$

For instance, if $\mathbf{R}(k)$ has equal eigenvalues then the maximum CMD value would be $(1-1/\sqrt{n})$. We therefore propose a new metric[1] called the Normalized CMD (NCMD), which is defined as

$$\begin{aligned}NCMD(\mathbf{R}(k),\mathbf{R}(l)) &= \frac{CMD(\mathbf{R}(k),\mathbf{R}(l))}{1-\frac{\lambda_n(k)}{\sqrt{\sum_{i=1}^{n}\lambda_i^2(k)}}}\\ &=\frac{CMD(\mathbf{R}(k),\mathbf{R}(l))}{K}\end{aligned} \tag{5}$$

where K is a normalization factor. If the smallest eigenvalue of $\mathbf{R}(k)$ is zero, then the normalization factor will be one, and hence the NCMD will be identical to the CMD. On the other hand, if $\mathbf{R}(k)$ has equal eigenvalues, then the minimum value of K is achieved, and the NCMD value would be one which is $1/(1-1/\sqrt{n})$ times the CMD value.

4.3. Correlation between Largest Eigen Modes (CLE)

In weak scattering environments, where the correlation matrix is low rank, only few eigenvalues are large enough to be used in the transmission. In case of low SNR, only the strongest eigenvectors are useful when transmitting data. Therefore, we are interested in measuring the non-stationarity of the channel with respect to the used eigenvalues. In this method, we extract the eigenvectors of the two correlation matrices and take only the first k eigenvectors that correspond to the largest eigenvalues. To illustrate this, let $\mathbf{R}(p)$ and $\mathbf{R}(q)$ be two correlation matrices to be compared. Accordingly, $\mathbf{R}(p)$ can be written as

$$\mathbf{R}(p)=\sum\nolimits_{i=1}^{n}\lambda_i(p)\mathbf{u}_i(p)\mathbf{u}_i^H(p)$$

where $\lambda_i(p)$ and $\mathbf{u}_i(p)$ are the ith largest eigenvalue and eigenvector of $\mathbf{R}(p)$, respectively. We define $\mathbf{U}_k(p)$, $k<n$, as

$$\mathbf{U}_k(p)=[\mathbf{u}_1(p)\,\mathbf{u}_2(p)\cdots\mathbf{u}_k(p)]$$

$\mathbf{U}_k(p)$ and $\mathbf{U}_k(q)$ are then compared using the DES method presented in [13, Sect. 2.6.3]. The value of k can be chosen depending on the transmission scenario.

[1] Strictly speaking, this is not a metric, since it is asymmetric in the two arguments. However, this asymmetry corresponds to the intended application, where a possibly outdated covariance matrix is used.

5. MEASUREMENTS AND DATA PROCESSING

In this section, we consider real 4x4 MIMO channels measured by Ericsson AB for the purpose of LTE systems studies. The measurements that include LOS and NLOS routes were performed in Kista, a suburb of Stockholm city in Sweden by installing 4 receive antennas on a moving car. Table 1 summarizes the measurements parameters.

The channel correlation matrix is assumed to be stationary within 10 λ_c =1.15 meters. Therefore, the averaging time T_v is calculated to be 10 λ_c / S_v = 188.2 ms, where S_v is the average speed of the car. Moreover, the spatial structure is assumed to be stationary within the entire channel bandwidth since the bandwidth of the channel is much smaller than the center frequency. In [17], it is shown that these choices satisfy the Doubly Underspread (DU) condition i.e. the number of independent fading realizations is sufficiently large [6]. The discrete correlation matrix at the transmit side $\mathbf{R}_{TX}(n)$ can be estimated as:

$$\mathbf{R}_{TX}(k) = \frac{1}{N_f}\frac{1}{N_t}\sum_{m}^{N_f}\sum_{n}^{N_t}\mathbf{H}^T(m,n)\mathbf{H}^*(m,n)$$

Where $\mathbf{H}(m, n)$ is the sampled version of the time-frequency channel $\mathbf{H}(t, f)$, N_f are N_t are the number of frequency samples, the number of time samples, respectively.

Table 1. Measurements Parameters

Location	Kista, Stockholm, Sweden
Scenario	Suburban, Driving car
Transmit antennas	4 antennas , at base station
Receive antennas	4 antennas, at moving car
Center frequency f_c	2.6 GHz
Wavelength	0.115 m
Bandwidth B	20 MHz
Frequency sample spacing	123 KHz
Number of frequency samples N_f	162
Time sample spacing T_p	5.33 ms
Average receiver speed S_v	22 Km/hr

5.1. Spatial stationary MIMO channel

In this section, we illustrate the performance of the dissimilarity metrics for three seconds measurement while the car was non-moving. This is to ensure proper functionality of the dissimilarity metric functions. Fig. 1(a) shows the dissimilarity metrics values of the transmit correlation matrix with respect to the first instant $\mathbf{R}_{Tx}$ (t=0). Here, the CLE method is used with a rank k=1. Apparently from Fig. 1a, all the dissimilarity metrics stay below 0.08 which gives an indication of spatial stationarity.

5.2. LOS MIMO channel non-stationarity

Initially, the dissimilarity metrics for the starting point of LOS route were evaluated using the estimated correlation matrix $\mathbf{R}_{Tx}$ as a function of distance, d. Fig. 1(b) shows the variation of the dissimilarity metrics with travel distance when comparing $\mathbf{R}_{Tx}(0)$ with the entire route.

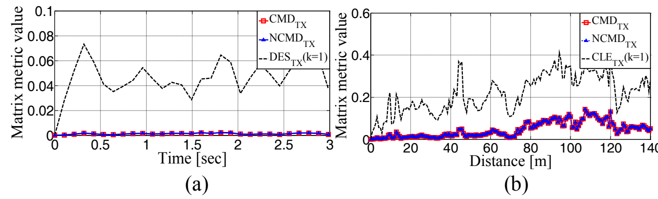

Figure 1: CMD, NCMD and CLE of (a) spatial stationary channel with respect to R_{TX} (t = 0), (b) of the LOS route with respect to R_{TX} (d = 0).

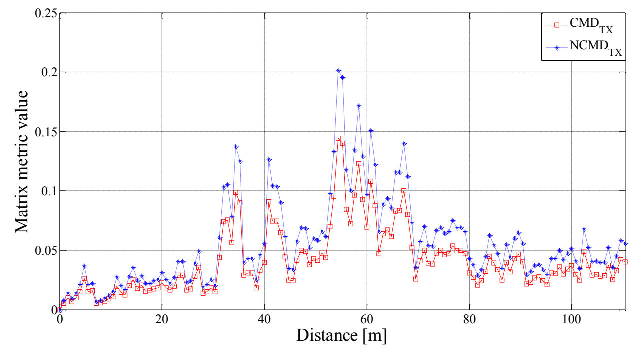

Figure 2: CMD and NCMD of the NLOS route with respect to R_{TX} (d = 0).

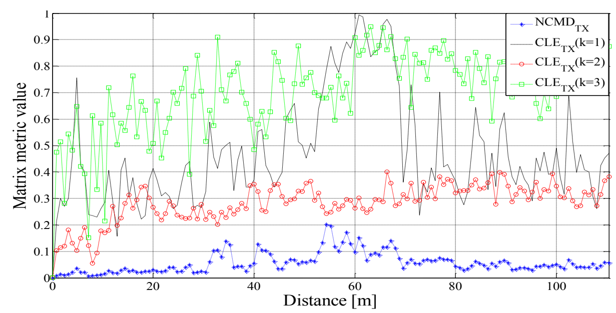

Figure 3: NCMD and CLE with k = 1, 2 and 3 of the NLOS route with respect to R_{TX} (d = 0).

As clearly observed from Fig. 1b, the NCMD method performance is very close to CMD method. In fact, the normalization factor was very close to one (0.99 for $\mathbf{R}_{Tx}$ (0)) due to the large ratio between the maximum and minimum eigenvalues of the correlation matrix. However, the CLE with respect to the first eigenmode (k=1) appears to be more sensitive to the spatial variations, which is relevant in this case since the remaining eigenmodes were found to be relatively very small.

5.3. NLOS MIMO channel non-stationarity

Fig. 2 shows the variation of the CMD and NCMD with distance of the NLOS channel when $\mathbf{R}_{Tx}$ (0) is compared with the entire route. Here, the difference between the CMD and NCMD is relatively large compared to the LOS case (the normalization factor is 0.66; that is, the CMD underestimates the non-stationarity by 34%).

For NLOS environments, the channel changes rapidly with distance and shows higher rank compared to the LOS environments. In this case, the CLE method could be applied with rank k=1, 2, or 3 depending on the number of utilized eigenmodes. The performance of the CLE method with different ranks at the transmit side is shown in Fig. 3. In this figure, it can be seen that different rank produces different metric values depending on the employed MIMO transmission scenario.

5.4. MIMO channel stationarity distance

The local stationarity distance can be defined as the distance at which the dissimilarity metric is lower than a certain threshold value c as defined in [16]. For example, the CLE($\mathbf{R}_{Tx}(0)$, $\mathbf{R}_{Tx}(d)$) of the LOS route shown in Fig. 1b crosses the threshold c=0.2 at around 10 meters. This means that the stationarity distance is 10 meters with respect to d=0.

The average stationarity distance of the LOS and NLOS channels (for c=0.2 and d = 0 to 140m) is shown in Table 2. For the LOS route, if only the largest eigenmode is used, both the CMD and the NCMD indicate higher average stationarity distance compared to the CLE with k=1. When the NLOS route is considered, the CMD indicates higher average stationarity distance compared to the NCMD method. If only few eigenmodes are used (k=1, 2 or 3) the CLE metric becomes more appropriate. However, for full spatial multiplexing scenario in NLOS channel where all the eigenmodes are often close to each other and the channel is of high rank, the NCMD metric will be the appropriate choice.

Table 2. Average stationarity distance

Dissimilarity Metric	Av. St. Dis. LOS	Av. St. Dis. NLOS
CMD	115 m	120 m
NCMD	115 m	82 m
CLE (k=1)	9 m	0.63 m
CLE (k=2)	-	4 m
CLE (k=3)	-	0.5 m

6. CONCLUSION

This paper has considered the CMD method and proposed new metrics for proper evaluation of non-stationarity of MIMO channels. The new metrics include the NCMD and CLE, each of which is well-suited for a specific transmission scheme/scenario.

Based on the results obtained from the measured MIMO channels, the CMD method give higher estimation to the stationarity regions compared to the NCMD particularly under the NLOS routes. For poor scattering environments, the CLE metric can be used to evaluate the non-stationarity with respect to the employed eigenmodes. The CLE metric will be more relevant in this case.

7. ACKNOWLEDGMENT

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