RANDOM MATRIX THEORY APPLIED TO LOW RANK STAP DETECTION

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ABSTRACT

The paper addresses the problem of target detection embedded in a disturbance composed of a low rank Gaussian clutter and a white Gaussian noise. In this context, it is interesting to use an adaptive version of the Low Rank Normalized Matched Filter detector, denoted LR-ANMF, which is a function of the estimation of the projector onto the clutter subspace. In this paper, we show that the LR-ANMF detector based on the sample covariance matrix is consistent when the number of secondary data K tends to infinity for a fixed data dimension m but not consistent when m and K both tend to infinity at the same rate, i.e. $m/K \to c \in (0, \infty)$. Using the results of random matrix theory, we then propose a new version of the LR-ANMF which is consistent in both cases. The application of our new detector on STAP (Space Time Adaptive Processing) data shows the interest of our approach.

***Index Terms*—** Low rank detection, Random matrix theory, G-MUSIC estimator, STAP processing, Adaptive Normalized Matched Filter.

1. INTRODUCTION

In the context of target detection in a noise composed of a low rank Gaussian clutter and an AWGN (Additive White Gaussian Noise), one could use the Low Rank Normalized Matched Filter (LR-NMF) detector [1] in order to exploit this low rank structure. Although its full rank version (NMF detector [2]) depends on the covariance matrix, the LR-NMF detector only requires the projector onto the clutter subspace. In practice, this projector and the covariance matrix are unknown and it is necessary to estimate them using K secondary data which share the same properties as the tested data. It is well known that the adaptive low rank version of the detector needs much less secondary data as its classical version (ANMF detector [3]) for equivalent performances [4, 5].

Nevertheless, for high dimensional data, the performances of the LR-ANMF detector can suffer of this reality. Furthermore, the LR-ANMF detector is composed of 3 quadratic forms and, relying on [6], it is known that, although these quadratic forms are consistent when the number of secondary data K tends to infinity for a fixed length m of the data vector, they are no more consistent when the data length m also tends to infinity. Hence, we show in this paper that the LR-ANMF detector is consistent when $K \to \infty$ with a fixed m but it is no more consistent when $m, K \to \infty$ at the same rate, i.e. $m/K \to c \in (0, \infty)$. By using the random matrix theory and the Girko's estimators [7], we propose to develop a new LR-ANMF detector based on estimates of the quadratic forms. These estimates are built from the G-MUSIC estimator [8] which is designed to be consistent when the number of secondary data K and the data dimension m both tend to infinity at the same rate. In this paper, we study the consistency of this new LR-ANMF detector.

As an illustration of the interest of the proposed detector, the STAP application [9] is studied as it is well known that the disturbance is then composed of a low rank Gaussian clutter plus a white Gaussian noise. Moreover, the dimension of the STAP data is often large with respect to the available number of secondary data.

The paper is organized as follows: Section 2 presents the problem statement and the definition of the LR-NMF detector. Section 3 contains the demonstration outline of the inconsistency of the classical LR-ANMF detector when $m/K \to c \in (0, \infty)$. Then, the new version of the LR-ANMF detector is presented and we study its consistency. Finally, Section 4 shows the STAP application which illustrates the obtained results.

Notations: An italic letter stands for a scalar quantity, boldface lowercase (uppercase) characters stand for vectors (matrices) and $(.)^H$ stands for the conjugate transpose. $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\mathrm{tr}(.)$ denotes the trace operator and $\mathrm{diag}(.)$ denotes the diagonalization operator such as $(\mathbf{A})_{i,i} = (\mathrm{diag}(\mathbf{a}))_{i,i} = (\mathbf{a})_{i,i}$ and equal to zero otherwise.

2. LOW RANK (LR) DETECTION

2.1. Problem formulation

The aim of the problem is to detect a complex signal $\boldsymbol{d}$ in an additive noise $\boldsymbol{c} + \boldsymbol{n}$ in the observation vector $\boldsymbol{x} \in \mathbb{C}^{m \times 1}$. Hence, one can define the detection problem with the following binary hypothesis test:

$$\begin{cases} H_0 : \boldsymbol{x} = \boldsymbol{c} + \boldsymbol{n} & \boldsymbol{x}_k = \boldsymbol{c}_k + \boldsymbol{n}_k, \quad k \in [\![1, K]\!] \\ H_1 : \boldsymbol{x} = \boldsymbol{d} + \boldsymbol{c} + \boldsymbol{n} & \boldsymbol{x}_k = \boldsymbol{c}_k + \boldsymbol{n}_k, \quad k \in [\![1, K]\!] \end{cases} \tag{1}$$

where K is the number of secondary data, $\boldsymbol{x}_k \in \mathbb{C}^{m\times 1}$ are the secondary data (learning data used for the estimation of the total noise covariance matrix) and $\boldsymbol{n} \in \mathbb{C}^{m\times 1}$ (or $\boldsymbol{n}_k$) $\sim \mathcal{CN}(0, \sigma^2\mathbf{I}_m)$ is the AWGN (Additive White Gaussian Noise) complex vector. $\boldsymbol{d}$ is the target response and is equal to $\alpha\boldsymbol{a}(\boldsymbol{\Theta})$ where α is an unknown deterministic parameter, $\boldsymbol{a}(\boldsymbol{\Theta})$ is the steering vector and $\boldsymbol{\Theta}$ is an unknown deterministic vector containing the localization parameters of the target. The clutter $\boldsymbol{c} \in \mathbb{C}^{m\times 1}$ is modeled by a random centered complex Gaussian vector with a covariance matrix $\mathbf{C}$ ($\boldsymbol{c}$ or $\boldsymbol{c}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{C})$). The covariance matrix is normalized as $\mathrm{tr}(\mathbf{C}) = m$. Consequently, the covariance matrix of the secondary data can be written as $\mathbf{R} = \mathbf{C} + \sigma^2\mathbf{I}_m \in \mathbb{C}^{m\times m}$. Moreover, the clutter is considered of low rank r (as in a STAP application according to Brennan's formula [10]). Hence, $\mathrm{rank}\,(\mathbf{C}) = r \ll m$ and one could write the eigendecomposition of $\mathbf{C}$ and define:

$$\mathbf{C} = \sum_{i=1}^{r} \gamma_i \boldsymbol{u}_i \boldsymbol{u}_i^H \tag{2}$$

where γ_i and $\boldsymbol{u}_i$, $i \in [\![1; r]\!]$ are respectively the non zero eigenvalues and the associated eigenvectors of $\mathbf{C}$, unknown in practice. The covariance matrix $\mathbf{R}$ of the secondary data can be decomposed as:

$$\mathbf{R} = [\mathbf{U}_r\ \mathbf{U}_0] \begin{bmatrix} \boldsymbol{\Delta}_r & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Delta}_0 = \sigma^2\mathbf{I}_{m-r} \end{bmatrix} [\mathbf{U}_r\ \mathbf{U}_0]^H \tag{3}$$

where $\boldsymbol{\Delta}_r$ and $\boldsymbol{\Delta}_0$ are the diagonal matrices respectively composed of the clutter and the noise eigenvalues, $\lambda_1 = \gamma_1 + \sigma^2 > \cdots > \lambda_r = \gamma_r + \sigma^2 > \lambda_{r+1} = \cdots = \lambda_m = \sigma^2$. Then, we define the projector onto the clutter subspace $\boldsymbol{\Pi}_{\mathrm{c}}$ and the projector onto the orthogonal subspace to the clutter subspace $\boldsymbol{\Pi}_{\mathrm{c}}^{\perp}$:

$$\begin{array}{lllll} \boldsymbol{\Pi}_{\mathrm{c}} & = & \mathbf{U}_r\mathbf{U}_r^H & = & \sum_{i=1}^{r} \boldsymbol{u}_i\boldsymbol{u}_i^H \\ \boldsymbol{\Pi}_{\mathrm{c}}^{\perp} & = & \mathbf{U}_0\mathbf{U}_0^H & = & \mathbf{I}_m - \boldsymbol{\Pi}_{\mathrm{c}} = \sum_{i=r+1}^{m} \boldsymbol{u}_i\boldsymbol{u}_i^H \end{array} \tag{4}$$

2.2. LR-NMF detector

A filtering preprocessing on the observation vector $\boldsymbol{x}$ is first done in order to remove the clutter, and we retrieve a complex signal detection problem defined by the following binary hypothesis test:

$$\begin{cases} H_0 : \boldsymbol{r} = \mathbf{U}_0^H\boldsymbol{x} & = \boldsymbol{n}_0 \\ H_1 : \boldsymbol{r} = \mathbf{U}_0^H\boldsymbol{x} & = \boldsymbol{d}_0 + \boldsymbol{n}_0 \end{cases} \tag{5}$$

The detection problem is solved considering the white noise power $\boldsymbol{n}_0$ unknown. The used detection test corresponds to the Normalized Matched Filter in its low rank version, denoted by LR-NMF (Low Rank Normalized Matched Filter [1]):

$$\Lambda_{\mathrm{LR-NMF}}(\boldsymbol{\Theta}) = \frac{|\boldsymbol{a}(\boldsymbol{\Theta})^H\boldsymbol{\Pi}_{\mathrm{c}}^{\perp}\boldsymbol{x}|^2}{(\boldsymbol{a}(\boldsymbol{\Theta})^H\boldsymbol{\Pi}_{\mathrm{c}}^{\perp}\boldsymbol{a}(\boldsymbol{\Theta}))(\boldsymbol{x}^H\boldsymbol{\Pi}_{\mathrm{c}}^{\perp}\boldsymbol{x})} \underset{H_0}{\overset{H_1}{\gtrless}} \delta_{\mathrm{NMF}} \tag{6}$$

where $\underset{H_0}{\overset{H_1}{\gtrless}} \delta_{\mathrm{NMF}}$ means that the H_1 hypothesis (respectively H_0) is decided if the test $\Lambda_{\mathrm{LR-NMF}}(\boldsymbol{\Theta})$ is over (respectively under) the threshold δ_{NMF}.

3. LOW RANK DETECTOR FROM RANDOM MATRIX THEORY

In this section, we will show the consistency of the LR-SCM detector when $K \to \infty$ with a fixed m and its inconsistency when $m, K \to \infty$ at the same rate. Then, we will present another estimator of quadratic form, named G-MUSIC, which compose the detectors and coming from random matrix theory. We will apply it to the LR-ANMF detector and, thus, propose a new detector, named LR-GSCM, consistent when $K \to \infty$ with a fixed m and when $m, K \to \infty$ at the same rate.

3.1. Inconsistency of the LR-SCM detector

The traditional estimation of the total noise covariance matrix $\mathbf{R}$ and the projector $\boldsymbol{\Pi}_{\mathrm{c}}^{\perp}$ orthogonal to the clutter subspace are first presented as, in practice, they are unknown. The estimation is based on the Sample Covariance Matrix (SCM) which is computed from the K secondary data and can be written as:

$$\begin{aligned} \hat{\mathbf{R}}_{\mathrm{SCM}} &= \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{x}_k\boldsymbol{x}_k^H = \sum_{i=1}^{r} \hat{\lambda}_i\hat{\boldsymbol{u}}_i\hat{\boldsymbol{u}}_i^H + \sum_{i=r+1}^{m} \hat{\lambda}_i\hat{\boldsymbol{u}}_i\hat{\boldsymbol{u}}_i^H \\ &= \left[\hat{\mathbf{U}}_r\ \hat{\mathbf{U}}_0\right] \begin{bmatrix} \hat{\boldsymbol{\Delta}}_r & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\Delta}}_0 \end{bmatrix} \left[\hat{\mathbf{U}}_r\ \hat{\mathbf{U}}_0\right]^H \end{aligned} \tag{7}$$

where $\hat{\lambda}_i$, $\hat{\boldsymbol{u}}_i$, $\hat{\mathbf{U}}_r$, $\hat{\mathbf{U}}_0$, $\hat{\boldsymbol{\Delta}}_r$ and $\hat{\boldsymbol{\Delta}}_0$ are respectively the eigenvalues, the eigenvectors, the clutter subspace, the noise subspace and the diagonal matrices of the sample eigenvalues and eigenvectors. Finally, the estimated projectors are:

$$\hat{\boldsymbol{\Pi}}_{\mathrm{c,SCM}} = \hat{\mathbf{U}}_r\hat{\mathbf{U}}_r^H = \sum_{i=1}^{r} \hat{\boldsymbol{u}}_i\hat{\boldsymbol{u}}_i^H \tag{8}$$

$$\hat{\boldsymbol{\Pi}}_{\mathrm{c,SCM}}^{\perp} = \hat{\mathbf{U}}_0\hat{\mathbf{U}}_0^H = \mathbf{I}_m - \hat{\boldsymbol{\Pi}}_{\mathrm{c,SCM}} = \sum_{i=r+1}^{m} \hat{\boldsymbol{u}}_i\hat{\boldsymbol{u}}_i^H \tag{9}$$

Then, using the SCM, the estimated LR-SCM detector can be written as:

$$\hat{\Lambda}_{\mathrm{LR-SCM}}(\boldsymbol{\Theta}) = \Lambda_{\mathrm{LR-NMF}}(\boldsymbol{\Theta})\,|_{\boldsymbol{\Pi}_{\mathrm{c}}^{\perp} = \hat{\boldsymbol{\Pi}}_{\mathrm{c,SCM}}^{\perp}} \underset{H_0}{\overset{H_1}{\gtrless}} \delta_{\mathrm{SCM}} \tag{10}$$

Observing the LR-ANMF detector at Eq. (10) and (6), one could note that each element is a quadratic form ($\mathbf{s}_1^H\mathbf{A}\mathbf{s}_2$) which we will study their asymptotic consistence when $K \to \infty$ and m fixed and when $m \to \infty$ and $K \to \infty$ at the same rate $\frac{m}{K} \to c$.

The convergence results are here presented under the following assumptions:

(**As1**) The covariance matrix $\mathbf{R}$ has uniformly bounded spectral norm for all m.

(**As2**) The two deterministic vectors $\mathbf{s}_1$, $\mathbf{s}_2$ have uniformly bounded norm for all m[1].

(**As3**) The SCM can take the form $\hat{\mathbf{R}}_{\text{SCM}} = \mathbf{R}^{1/2}\mathbf{V}\mathbf{V}^H\mathbf{R}^{1/2}$, where $\mathbf{R}^{1/2}$ is a $m \times m$ Hermitian positive definite square root of the true covariance matrix, and $\mathbf{V}$ is a $m \times K$ matrix with i.i.d. absolutely continuous random entries, each one of them having i.i.d. centered real and imaginary parts with variance $1/(2K)$, and finite eighth order moments.

(**As4**) The ratio m/K is chosen such as $m/K < \xi$ where ξ is defined at Eq.(20) of [6].

According to [6]-[8] and under the assumptions (**As1-As4**):

$$\begin{cases} \eta_{\text{trad}} = \mathbf{s}_1^H \hat{\mathbf{\Pi}}_{\text{c,SCM}}^{\perp} \mathbf{s}_2 \underset{\substack{K\to\infty \\ m<\infty}}{\overset{\text{a.s.}}{\longrightarrow}} \eta_{\text{th}} = \mathbf{s}_1^H \mathbf{\Pi}_{\text{c}}^{\perp} \mathbf{s}_2 \\ \eta_{\text{trad}} = \mathbf{s}_1^H \hat{\mathbf{\Pi}}_{\text{c,SCM}}^{\perp} \mathbf{s}_2 \underset{\substack{m,K\to\infty \\ m/K\to c<\infty}}{\overset{\text{a.s.}}{\longrightarrow}} \bar{\eta} = \mathbf{s}_1^H \bar{\mathbf{\Pi}}_{\text{c}}^{\perp} \mathbf{s}_2 \neq \eta_{\text{th}} \end{cases} \tag{11}$$

In addition, $\bar{\mathbf{\Pi}}_{\text{c}}^{\perp} = \sum_{i=1}^{m} w(i) \boldsymbol{u}_i \boldsymbol{u}_i^H$ [6], with:

$$w(i) = \begin{cases} 1 - \frac{1}{m-r-1} \sum_{n=1}^{r} \left(\frac{\sigma^2}{\lambda_n - \sigma^2} - \frac{\mu_m}{\lambda_n - \mu_m} \right), & \text{si } i > r \\ \frac{\sigma^2}{\lambda_i - \sigma^2} - \frac{\mu_m}{\lambda_i - \mu_m}, & \text{si } i \leqslant r \end{cases} \tag{12}$$

where $\mu_1 \geqslant \cdots \geqslant \mu_m$ are the eigenvalues of $\text{diag}(\boldsymbol{\lambda}) - \frac{1}{m}\sqrt{\boldsymbol{\lambda}}\sqrt{\boldsymbol{\lambda}}^T$ and $\boldsymbol{\lambda} = [\lambda_1, \cdots, \lambda_m]^T$. Consequently, each detector element is consistent when $K \to \infty$ and m fixed and inconsistent when $m \to \infty$ and $K \to \infty$ at the same rate $\frac{m}{K} \to c$. Furthermore, according to Slutsky's theorem, the ratio of these quantities and the LR-SCM detector are consistent when $K \to \infty$ with a fixed m but inconsistent when $m, K \to \infty$ at the same rate.

3.2. Quadratic form from random matrix theory

In order to overcome this problem, we suggest using a consistent estimator of our quadratic forms when $m, K \to \infty$ at the same rate. It is named G-MUSIC [8] and we have, under the assumptions (**As1-As4**):

$$\eta_{\text{GSCM}} = \mathbf{s}_1^H \hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp} \mathbf{s}_2 \xrightarrow[m,K\to\infty, m/K\to c]{\text{a.s.}} \eta_{\text{th}} \tag{13}$$

where $\hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp}$ is a pseudo-projector taking into account all the estimated eigenvectors from the SCM and leading to a better estimation of the quadratic form $\mathbf{s}_1^H \mathbf{A} \mathbf{s}_2$. It can be written as $\hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp} = \sum_{i=1}^{m} \phi(i) \hat{\boldsymbol{u}}_i \hat{\boldsymbol{u}}_i^H$ [6], with:

$$\phi(i) = \begin{cases} 1 + \sum_{n=1}^{r} \left(\frac{\hat{\lambda}_n}{\hat{\lambda}_i - \hat{\lambda}_n} - \frac{\hat{\mu}_n}{\hat{\lambda}_i - \hat{\mu}_n} \right), & \text{si } i > r \\ - \sum_{n=r+1}^{m} \left(\frac{\hat{\lambda}_n}{\hat{\lambda}_i - \hat{\lambda}_n} - \frac{\hat{\mu}_n}{\hat{\lambda}_i - \hat{\mu}_n} \right), & \text{si } i \leqslant r \end{cases} \tag{14}$$

where $\hat{\mu}_1 \geqslant \cdots \geqslant \hat{\mu}_m$ are the eigenvalues of $\text{diag}(\hat{\boldsymbol{\lambda}}) - \frac{1}{m}\sqrt{\hat{\boldsymbol{\lambda}}}\sqrt{\hat{\boldsymbol{\lambda}}}^T$ and $\hat{\boldsymbol{\lambda}} = [\hat{\lambda}_1, \cdots, \hat{\lambda}_m]^T$. When $c > 1$, $\hat{\mu}_m = \cdots = \hat{\mu}_K = 0$.

3.3. LR-GSCM detector

Then, the new LR-GSCM detector is defined using Eq.(13):

$$\hat{\Lambda}_{\text{LR}-\text{GSCM}}(\mathbf{\Theta}) = \Lambda_{\text{LR}-\text{NMF}}(\mathbf{\Theta}) \,|_{\mathbf{\Pi}_{\text{c}}^{\perp} = \hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp}} \underset{H_0}{\overset{H_1}{\gtrless}} \delta_{\text{GSCM}} \tag{15}$$

The expression of δ_{GSCM} is not determined. One could point out that the notation $\mathbf{\Pi}_{\text{c}}^{\perp} = \hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp}$ is not rigorous. Indeed, $\hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp}$ is not a projector but a pseudo-projector (function of all the estimated eigenvectors instead of the eigenvectors only corresponding to the desired subspace for a traditionnal projector). We replace $\mathbf{\Pi}_{\text{c}}^{\perp}$ by $\hat{\mathbf{\Pi}}_{\text{c,GSCM}}^{\perp}$ with the intention of benefiting of a better estimation of the quadratic forms. Next, following the same reasoning as for the LR-SCM detector, we conclude from Section 3.2 that each element is consistent when $m, K \to \infty$ at the same rate. As a consequence, according to Slutsky's theorem, the ratio of these quantities and the LR-GSCM detector are consistent when $m, K \to \infty$ at the same rate:

$$\hat{\Lambda}_{\text{LR}-\text{GSCM}}(\mathbf{\Theta}) \underset{\substack{m,K\to\infty \\ m/K\to c<\infty}}{\overset{\mathbb{P}}{\longrightarrow}} \Lambda_{\text{LR}-\text{NMF}}(\mathbf{\Theta}) \tag{16}$$

4. APPLICATION TO STAP PROCESSING

As an illustration of the interest of the LR-GSCM detector (from the random matrix theory), the STAP processing application is chosen. The purpose of STAP is to detect a moving target thanks to a uniform linear antenna composed of N sensors receiving M pulses. In this section, we choose $N = 4$ and $M = 16$ in order to have a large number for the data dimension $m = NM = 64$. In STAP application, $\mathbf{\Theta} = (\theta, v)$ where θ is the DoA (Direction of Arrival) and v the object relative speed. The target DoA and its relative velocity with respect to the airborne (radar platform) velocity are $\theta_d = 0°$ and $v_d = 35\text{m.s}^{-1}$. According to Brennan's formula, the clutter rank is equal to $r = N + (M-1)\beta = 19$, with $\beta = 1$ in our configuration. Then, the signal wavelength is $l_0 = 0.667$m, the AWGN power is $\sigma^2 = 1$, the signal to noise ratio is $SNR = 20$dB and the clutter to noise ratio is $CNR = 40$dB.

[1] In this paper, $\mathbf{s}_1$ and $\mathbf{s}_2$ take the values of $\boldsymbol{a}(\mathbf{\Theta})$ or $\boldsymbol{x}$ (linear combinaison of steering vectors $\boldsymbol{a}(\mathbf{\Theta})$ and a white noise) which are independant of the secondary data and consequently considered as deterministic vectors.

We first compare two typical maps of the LR-SCM and the LR-GSCM detector values as a function of the DoA and the relative velocity in Fig. 1 and Fig. 2, for a given number of secondary data $K = 2r$. We observe that, for both detectors, the target is well detected. However, we also note a high diagonal in the LR-GSCM detector corresponding to the clutter noise and due to the better estimation of the quadratic form $(\boldsymbol{a}(\theta, f)^H \boldsymbol{\Pi}_c^{\perp} \boldsymbol{a}(\theta, f))$. This phenomenon is also present on the LR-SCM detector as illustrated in Fig. 3 where we observe a section of the detectors for $\theta = \theta_d$ for a given number of secondary data $K = 2r$. Consequently, this problem could obstruct the detection as the values on this diagonal can be higher than the value on the target. Since we know that it is impossible to detect a target inside the clutter in both detectors, we propose an alternative solution which forces the detector elements corresponding to the clutter to be zero:

$$\begin{cases} \hat{\Lambda}_{\mathrm{LR-ANMF-SCM}}(\theta, f(\beta)\sin(\theta)) = 0 \\ \hat{\Lambda}_{\mathrm{LR-ANMF-GSCM}}(\theta, f(\beta)\sin(\theta)) = 0 \end{cases} \tag{17}$$

where β is a parameter of the radar system. We also apply the same process to the LR-SCM detector for a fair comparison between the detectors.

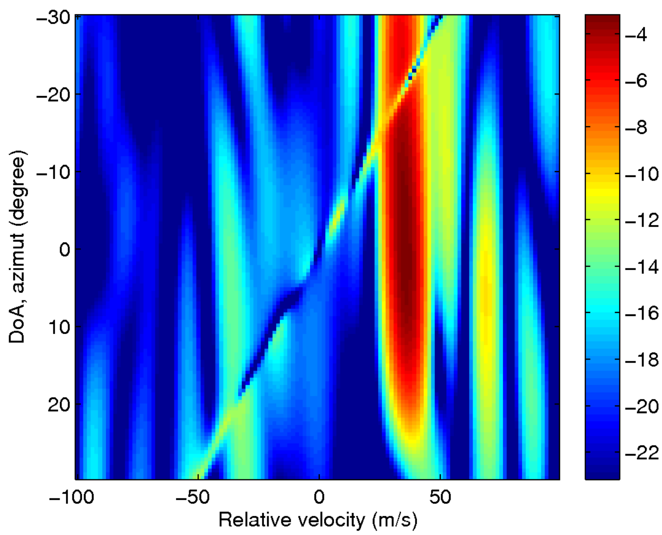

Fig. 1. LR-SCM detector value (dB) as a function of the DoA and the relative velocity of the reflector object.

Now, we present some results with the proposed solution of zero on the clutter diagonal. We first illustrate in Fig. 4 the detection gain through the section of the detectors for $\theta = \theta_d$ for a given number of secondary data $K = 2r$. We remark in Fig. 4 that the LR-GSCM detector leads to a gain of 0.15 in term of detector value with quite similar values outside the main lobe.

Then, we illustrate the estimation gain with the MSE between the LR-NMF detector and the estimated one (LR-SCM or LR-GSCM) in $\theta = \theta_d$ and $v = v_d$ in Fig. 5 as a function of the number of secondary data K. The MSE is measured over 10,000 realizations. We note that the MSE between the LR-NMF and the LR-GSCM detectors is lower than the MSE between the LR-NMF and the LR-SCM detectors, for all K. In consequence, the LR-NMF detector is better estimated with the proposed estimator than with the traditional one.

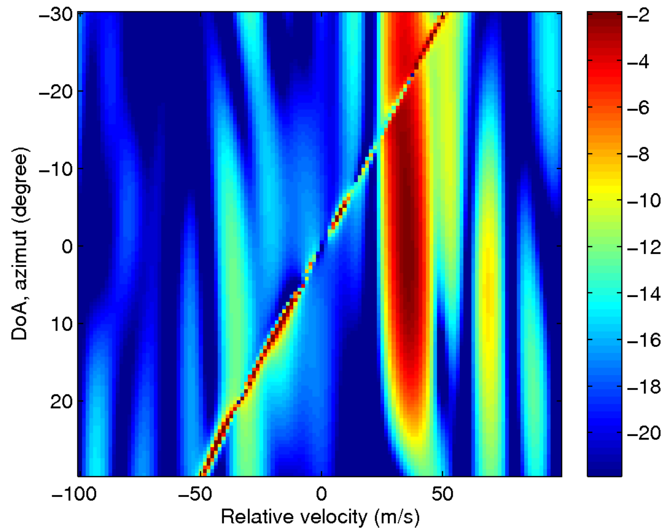

Fig. 2. LR-GSCM detector value (dB) as a function of the DoA and the relative velocity of the reflector object.

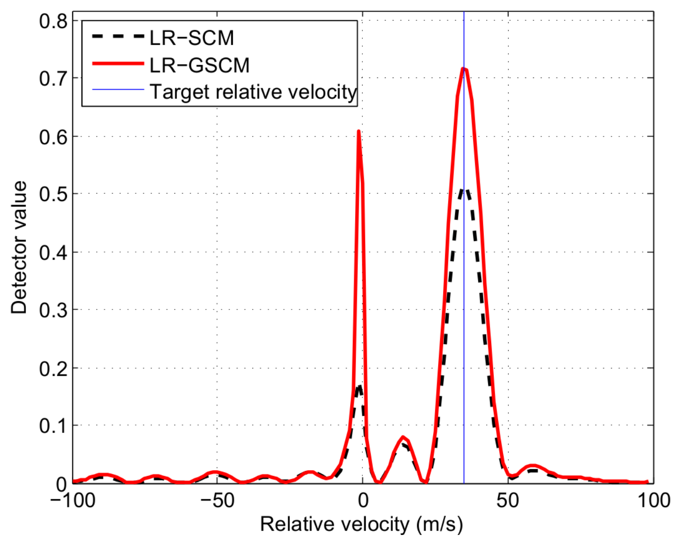

Fig. 3. Value of the LR-SCM and the LR-GSCM detectors for $\theta = \theta_d$ as a function of the relative velocity of the reflector object.

5. CONCLUSION

In this paper we developed a new adaptive low rank detector based on random matrix theory and more precisely on the

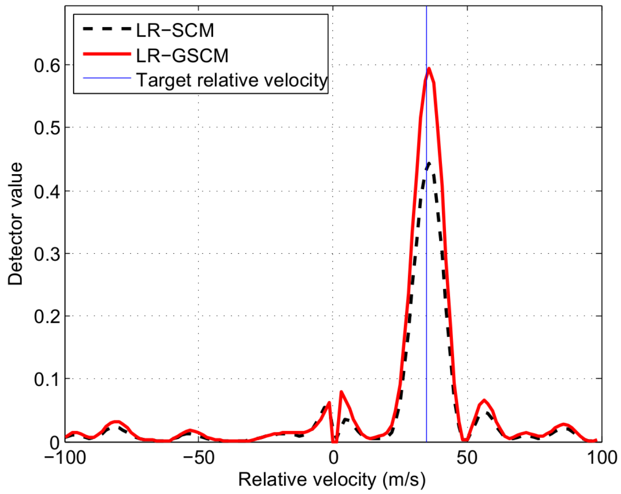

Fig. 4. Value of the modified LR-SCM and the modified LR-GSCM detectors for $\theta = \theta_d$ as a function of the relative velocity of the reflector object.

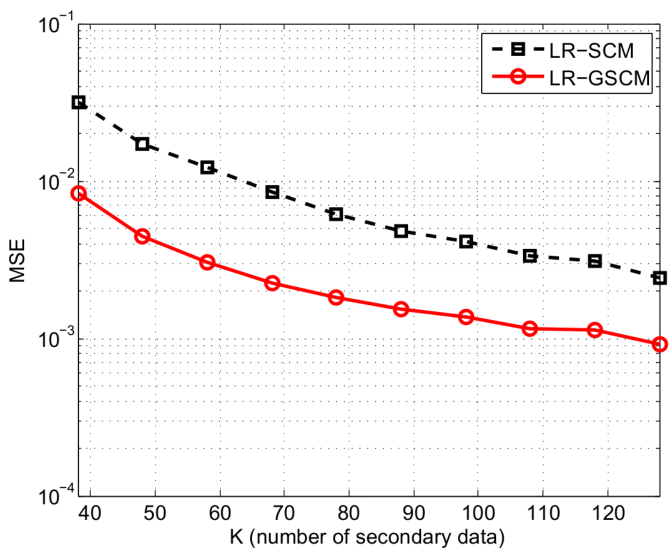

Fig. 5. MSE between the LR-NMF detector and the estimated one at the target DoA and velocity.

G-MUSIC estimator. Indeed, the traditional low rank (LR-SCM) detector, based on a simple eigendecomposition of the SCM in order to estimate the projector onto the clutter subspace, is shown inconsistent when the number of secondary data K and the data dimension m both tend to infinity at the same rate. On the contrary, our new LR-ANMF detector is consistent in this asymptotic regime. Moreover, we showed the interest of our approach in a STAP application in terms of MSE on localization parameters of the target.

The next future of this work is to compute the detection and the false alarm probability in order to measure the improvement reached by our new detector.

6. REFERENCES

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