PILOT ALLOCATION BY GENETIC ALGORITHMS FOR SPARSE CHANNEL ESTIMATION IN OFDM SYSTEMS

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ABSTRACT

The comb-type pilot-aided channel estimation in OFDM systems is here considered. The number of pilot subcarriers is generally tuned to tradeoff between performance and spectral efficiency. For time domain sparse channels, the application of the recent theory of Compressed Sensing has proved its efficiency to both ameliorate the estimation accuracy and reduce the transmitted overhead. The performance and uniqueness guarantees remain however closely related to the pilots placement, which are optimized prior to transmission. Few works have addressed the pilot allocation issues. To avoid the computationally prohibitive exhaustive search over all possible allocations, they proposed sub-optimal solutions. This contribution proposes an enhanced pilot design scheme based on Genetic Algorithms. First, we set the adopted problem formulation in the frame of Genetic Algorithms. Then, we propose two algorithms, tailored for the decomposition matrix coherence minimization. Simulation results prove the proposed schemes effectiveness and performance enhancement compared to former pilot design schemes.

***Index Terms*—** Sparse channel estimation, OFDM Systems, Pilot Allocation, Compressed Sensing, Coherence measure, Genetic Algorithms.

1. INTRODUCTION

High data rate transmissions implied in most of the present and future wireless communication standards are usually subject to severe frequency selective channels. Most of these standards, such as LTE, WiFi and WiMAX have adopted the OFDM modulation technique for its ability to avoid inter symbol interference and to drastically simplify the equalization task. For effective coherent demodulation, accurate channel estimation is however required [1]. Conventional pilot-assisted channel estimators usually tradeoff between estimation accuracy and spectral efficiency in the pilot subcarriers number choice. They also use uniform pilot allocation [2]. For channels exhibiting a sparse response in time domain, the Compressed Sensing (CS) theory application has proved its suitability to simultaneously reduce the number of, possibly not uniformly placed, pilots while enhancing the channel estimation accuracy [3]-[5]. In this paper, we address the problem of the sensing matrix optimization which corresponds to pilots pattern design.

By applying CS methods, a denoised estimate of the channel impulse response is obtained through a linear decomposition of the Channel Frequency Response (CFR) Least Squares estimate on pilot subcarriers over a partial DFT sensing matrix. The decomposition performance and uniqueness guarantees in terms of maximal non zero entries of the channel impulse response, are closely related to the decomposition matrix choice, related in this framework to the pilots placement. Some measures exist that are able to quantify the sensing matrix quality. The herein adopted quality measure of the sensing partial DFT matrix is that of coherence [6]. The coherence indeed characterizes the maximal correlation between the CS matrix pairwise columns. In [7], it is shown that minimizing the coherence is equivalent, for some specific pairs of (N, N_p), where N is the number of channels and N_p is the number of pilots, to exactly balance between all possible cyclic difference occurrences. For improper pairs (N, N_p), the exhaustive search over the huge set of allocation possibilities results in a computationally prohibitive complexity. Few works have addressed the pilot design issues and proposed sub-optimal computationally efficient solutions [7]-[11].

The Genetic Algorithms (GA) [12] offer a powerful optimization tool, especially suitable for nonlinear criteria and those where the solution belongs to a large dimensional space.

Thanks to their natural selection process, the GA have the potential to avoid local optima and to guide the criterion, to be optimized, known as fitness function, towards near-optimum solutions.

In this paper, the Genetic Algorithms approach is adopted, where two original schemes are proposed to optimize the DFT sensing matrices coherence measure.

The first scheme consists of a Constrained Iterative Processing (CIP), where the population individuals, evolving through generations, are forced, through random transformations, to lie within the solution candidates set. The second scheme avoids such random transformations and operates in a Forward-Backward Processing (FBP) fashion that insures the convergence to an element within the potential solutions set while optimizing the coherence measure.

The remainder of this paper is organized as follows. Section 2 formulates the DFT sensing matrix optimization in the frame of channel estimation and revisits the methods of [7] and [10] recently proposed to this aim. Section 3 first formulates the DFT submatrix coherence optimization in the Genetic Algorithms frame, then it details the two proposed schemes CIP and FBP. Section 4 is devoted to assess the new schemes relevance. Finally, some concluding remarks are drawn.

Notations: $\mathbf{M}_i$ *indicates the* ith *column of* $\mathbf{M}$. $Card(\bullet)$ *denotes the argument cardinal.* $\mathbf{1}_N$ *is a ones vector of length* N. $\oplus$ *stands for the addition modulo 2 (logical exclusive OR: Xor) and* $\odot$ *denotes the element-wise product (logical And).*

2. DFT SENSING MATRICES FOR CHANNEL ESTIMATION IN OFDM SYSTEMS

The application of CS theory to sparse channel estimation has a double benefit. It first allows to reduce the number of, possibly non uniformly placed pilots, below the minimal required number of pilots which coincides for arbitrary (not sparse channels) to the channel memory length. These constraints relaxation allows to achieve a higher spectral efficiency. CS theory further allows, by exploiting the sparsity property, inherent to some channels structure, to obtain enhanced solutions compared to those not accounting for the sparsity features.

By sparse channels, we indeed refer to channels whose equivalent sampled impulse response presents a reduced number of significant energy (nonzero and non near zero valued) coefficients with respect to the channel memory length.

In the frame of OFDM pilot-assisted sparse channel estimation, the CS matrix design step, is carried prior to transmission, and aims to optimize the N_p pilot subcarriers pattern, among a total number of N subcarriers. This is realized through the DFT submatrix $\mathbf{F_p}$ choice, where $\mathbf{F_p}$ is obtained from the $N \times N$ DFT matrix by the N_p rows corresponding to pilots positions selection. $\mathbf{F_p}$ is chosen to optimize a given measure, corresponding in this work to minimize the coherence, which is expressed as

$$\mu(\mathbf{F_p}) = \max_{1 \leq i \neq j \leq N} \frac{|\langle \mathbf{F_p}_i, \mathbf{F_p}_j \rangle|}{\|\mathbf{F_p}_i\|_2 \|\mathbf{F_p}_j\|_2}, \tag{1}$$

where $\mathbf{F_p}_i$ is the matrix $\mathbf{F_p}$ ith column.

Noisy outputs on pilot subcarriers at the receiver side can be expressed as [7]-[10]

$$\mathbf{y_p} = \mathbf{A_p}\mathbf{F_p}\mathbf{h} + \mathbf{n}, \tag{2}$$

where $\mathbf{A_p}$ is an $N_p \times N_p$ diagonal matrix made of the pilot symbols, $\mathbf{F_p}$ is a DFT submatrix with rows corresponding to the N_p pilot tones positions, $\mathbf{h}$ is the equivalent discrete channel impulse response and $\mathbf{n}$ is a complex normally distributed noise. The channel estimation corresponds to recover the sparse impulse response $\mathbf{h}$ from $\mathbf{y_p}$, and its performance is sensitive to the CS matrix $\mathbf{F_p}$ choice.

In Cyclic Prefixed (CP) OFDM systems, the CP length N_g is generally chosen larger than the channel memory. Incorporating these considerations, the channel estimation is reduced to recover $\mathbf{h}$ of length N_g, with $K \leq N_g$ nonzero entries, from $\mathbf{y_p}$. The sensing matrix $\mathbf{F_p}$ is in this way reduced to a selection of N_p rows from the block submatrix formed by the first N_g columns of the $N \times N$ DFT matrix.

Let $\mathcal{S}$ denote the set of possible choices of N_p subcarriers among N, then its cardinality is given by $Card(\mathcal{S}) = C_N^{N_p} = \frac{N!}{N_p!(N-N_p)!}$, where $n! = \prod_{i=1}^{n} i$, and $\mathbf{F_p}$ with size $N_p \times N_g$ should be chosen within $\mathcal{S}$ such that its coherence is minimized.

2.1. Scheme of Pakrooh $et\ al.$

In [7], it is shown that non uniformly spaced pilots based on Cyclic Difference Sets (CDS) are optimal in the sense of minimizing the DFT submatrix coherence. For improper pairs of (N, N_p), for which no CDS exist, a sub-optimal iterative scheme with $Np - 1$ steps is proposed, where at each step/iteration a new pilot is positioned with the firstly chosen ones according to a specific criterion. Denoting by a_i the number of occurrences of pilots positions difference i where $i \in \{1, 2, \cdots, N-1\}$, the CDS are shown to verify the equality between all $\{a_i\}$. Following this, at each step, the proposed sub-optimal scheme chooses the additional pilot position which minimizes the variance of $\{a_i\}$. In [7], no considerations exploit the CP length knowledge and the CS matrix is of size $N_p \times N$.

2.2. Scheme of Qi $et\ al.$

In [10], a backward tree-based scheme iteratively constructs the CS matrix by eliminating from the initial $N \times N_g$ DFT submatrix one row at each of the $N - N_p$ iterations. At each step/iteration, the q_B minimal coherence submatrices are selected as candidates to be considered for the next iteration.

By exploiting the CP length knowledge, the sensing matrix can be reduced to its first N_g columns. This size reduction not only decreases the computational burden for coherence evaluation but also guarantees that the chosen pilots minimize the coherence of the effective part of the sensing matrix.

3. PROPOSED GA APPROACH-BASED ALGORITHMS

The GA are based on a generic structure that should be specifically reformulated and adapted to each problem. They iteratively operate along several generations to optimize an objective function or criterion, known as fitness function, until a pre-fixed maximal number of generations or a stop condition is reached. During the evolutionary process, at each iteration, corresponding to one generation, after evaluating the fitness function for each individual, a portion of the best ranked is chosen through the *selection* process to transit into the next generation. To preserve good individuals properties, a *crossing* is operated, which corresponds to cross some pairs of individuals, which are called parents. This is achieved by mixing their corresponding chromosomes, which characterize their elementary genetic properties. The so-obtained new individuals, called children, are then included within the next generation population. To avoid the criteria critical local optima, some new individuals are introduced through both the *mutation* process, which introduces random changes in some individuals chromosomes, and through random individuals insertion in each new generation population.

3.1. Problem formulation in the Genetic Algorithms frame

We begin by providing the basic implied entities definitions and formulate the different processings involved in the CIP and FBP herein proposed GA schemes.

- **Generation/ Population/ individuals/ chromosomes:** in each generation, a population which is a set of individuals is generated. Each individual is coded as a binary vector of length N whose entries correspond to the chromosomes. The ith chromosome value 1 or 0 indicates if the ith subcarrier, among N, is dedicated respectively to carry pilot or data.

- **Fitness function:** it computes, for every individual, the coherence of the DFT submatrix with rows positions indicated by 1 valued chromosomes.

- **Crossing:** we here adopt a multi-point crossing where for each pair of parents $\mathbf{I_1}$ and $\mathbf{I_2}$ (2 individuals), we define a binary mask $\mathbf{m}$ of length N and its complementary (modulo 2) mask $\mathbf{m_c} = \mathbf{m} \oplus \mathbf{1}_N$, then the children are obtained as

$$\mathbf{c_1} = \mathbf{m} \odot \mathbf{I_1} + \mathbf{m_c} \odot \mathbf{I_2}, \text{ and} \tag{3}$$
$$\mathbf{c_2} = \mathbf{m_c} \odot \mathbf{I_1} + \mathbf{m} \odot \mathbf{I_2}. \tag{4}$$

  Note that the addition in $\mathbf{c_1}$ and $\mathbf{c_2}$ construction is naturally modulo 2 since $\mathbf{m}$ and $\mathbf{m_c}$ are originally chosen to be complementary (modulo 2).

- **Mutation:** for each individual $\mathbf{I}$ to be muted, a random binary mask $\mathbf{m}$ is generated, then the muted version is

$$\mathbf{I}' = \mathbf{I} \oplus \mathbf{m}. \tag{5}$$

3.2. Constrained Iterative Processing CIP

In this scheme, the initial population and new individuals to introduce within each generation are taken as binary sequences of length N where the occurrence of 1 is exactly N_p, thus corresponding to an element of the solution candidates set $\mathcal{S}$.

The above pilot allocation problem formulation in the GA frame implies multiple operations among which the crossing and mutation processing. Following the adopted procedures, it is obvious that both the crossing and mutation may produce individuals not contained within $\mathcal{S}$, even when operating over individuals taken from the potential solutions set. The proposed CIP scheme constrains the so-evolving solutions to lie within $\mathcal{S}$.

For each new individual, obtained after either crossing or mutation, let n_1 denote the number of 1 valued chromosomes. If $n_1 = 1$, to classify the individual, the fitness function is directly evaluated. Otherwise if $n_1 \neq N_p$, before fitness function evaluation a transformation is carried. If $n_1 > N_p$, then a set of $n_1 - N_p$ randomly chosen 1 valued chromosomes are transformed into 0 and respectively if $n_1 < N_p$, a set of $N_p - n_1$ randomly chosen 0 valued chromosomes are transformed into 1, insuring in this way the transformed individual to lie within $\mathcal{S}$.

3.3. Forward-Backward Processing FBP

In this scheme, the population is generated randomly from all the possible binary vectors of length N. Then, for each individual evaluation, if the number of 1 valued chromosomes n_1 verifies $n_1 = N_p$, the coherence is directly evaluated. Otherwise, if $n_1 < N_p$ (or resp. $n_1 > N_p$), a forward (resp. backward) processing is operated to bring the number of 1 valued chromosomes to N_p.

Contrarily to CIP, which constrains the individuals to lie in $\mathcal{S}$ by randomly inverting a random subset of chromosomes values, the FBP optimizes the sequence choice from an initial one, not in $\mathcal{S}$, to an other one within $\mathcal{S}$ in such a way to guide it towards the lowest coherence possible solution. This is achieved by using the greedy principles of the forward (add one pilot per step as in [7]) or backward (remove one pilot per step as in [10]), where for both schemes the herein optimized criterion is that of the coherence measure.

For each individual verifying $n_1 > N_p$, a backward processing of $n_1 - N_p$ iterations of the scheme of [10] is operated where in each iterations only the lowest allocations coherence are kept as possible candidates for the next iteration. For each individual such that $n_1 < N_p$, $N_p - n_1$ iterations of a forward processing like in [7] is operated where the coherence is minimized rather than the the cyclic difference occurences variance.

In both of our adopted forward and backward processing, we envisage a generalized tree-based structure, in the manner of [10] where, rather than choosing only the best (least coher-

Population size n_p	Maximal nb. of generations n_{max}	Selection ratio	Crossing ratio	Mutation ratio
40	100	0.2	0.2	0.2

Table 1. Proposed CIP and FBP Genetic Algorithms parameters.

	$N_p\mu(\mathbf{F_p})$	Optimized Pilot Placement
Forward	6.1937	1,2,4,8,13,42, 57,77,96,109,119,127
Backward	4.1612	6,26,58,63,67,75,83,87,91,103,111,122
CIP	3.9529	2,6,10, 34,41,66,70,80,84,88,103,123
FBP (1,1)	3.2803	4,10,14,29,35,39,55,64,77,85,123,127
FBP (2,2)	3.1642	7,11,22,43,69,74,78,95,110,114,119,124

Table 2. Optimized pilot allocations coherence performance.

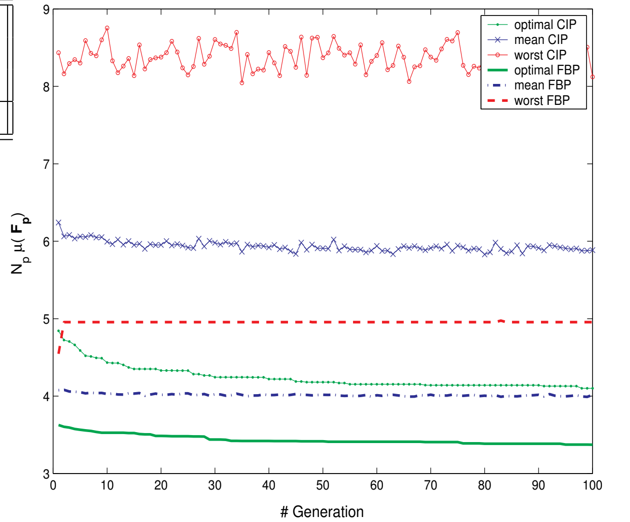

Fig. 1. CIP and FBP (1,1) Fitness best, mean and worst values. Average over 10 Monte Carlo trials.

ence) solution in each iteration, we select the q_F best solutions in the forward processing (case $n_1 < N_p$) resp. q_B best solutions (case $n_1 > N_p$) in the backward processing, to be considered in the next iteration of the forward (or resp. backward) processing, until reaching an element within $\mathcal{S}$ for which $n_1 = N_p$.

4. APPLICATION TO COMB-TYPE CHANNEL ESTIMATION IN OFDM SYSTEMS

In this section, we evaluate the proposed schemes performance first in terms of coherence minimization capacity, then in terms of channel estimation performance.
The channel estimation quality is evaluated through the channel frequency response (CFR) normalized MSE (NMSE), and the Symbol Error Rate (SER).
We consider an OFDM system with $N = 128$ subcarriers where $N_p = 12$ pilot subcarriers are used for channel estimation purpose. A sparse multipath of length $L = N_g = 25$ is generated where $K = 3$ nonzero coefficients are randomly positioned within the CP. The channel is Rayleigh distributed with independent impulse response coefficients $\mathbf{h}_i$ verifying $\mathbf{h}_i\mathcal{CN}(0, \sigma_i^2)$ where σ_i^2 exhibits an exponentially decaying power-delay profile, with decaying speed $\beta = \frac{2}{N_g}$.
The GA corresponding to CIP and FBP use the parameters specified in table 1. These parameters were adjusted by trial and error method. The optimized coherence by the different schemes is given in table 2 with the corresponding pilots patterns, FBP (m,n) corresponds to FBP for the choice of $q_F = m$ and $q_B = n$ tree-based schemes.

The obtained results show the advantage of the GA-based proposed schemes over both the schemes of [7] and [10]. Figure 1 depicts the fitness worst, best and mean values behavior through generations, obtained by CIP and FBP(1,1) schemes, averaged over 10 Monte Carlo trials. It shows the more important dynamics of the fitness function realized by CIP compared to the FBP through generations, which can be attributed to its more random evolution of the individuals, compared to FBP. It also shows the decrease of the best solutions coherence through generations. Figure 2 superimposes the NMSE on CFR, obtained by the Orthogonal Matching Pursuit (OMP) algorithm [13] for different pilots allocations, and the Least Squares solution adopting the true nonzero coefficients positions (Oracle estimator). Figure 3 exhibits the SER in the case of data taken from a QPSK constellation. The obtained results show the effectiveness of the proposed schemes with a clear advantage of the FBP over CIP. This advantage can be attributed to the association of GA natural selection process with the Forward-Backward coherence optimization, which allows for relatively small populations to converge to effective solutions in few generations. For optimized pilot pattern computation, the number of tested individuals is here $n_p \times n_{max} = 40 \times 100 = 4000$ which is very small compared to $Card(\mathcal{S}) = 2.3726\ 10^{16}$. Analyzing the impact of the parameters q_F and q_B shows that no sufficiently significant improvement is obtained by increasing their values, through a FBP tree-based processing, compared to the case $q_F = q_B = 1$.

5. CONCLUSION

In this paper, the problem of pilot allocation optimization for channel estimation in OFDM systems is considered. In the compressed sensing frame, this is equivalent to optimizing the sensing partial DFT matrix coherence measure. We proposed two optimization tailored schemes in the Genetic Algorithms framework. The first scheme forces the individuals to

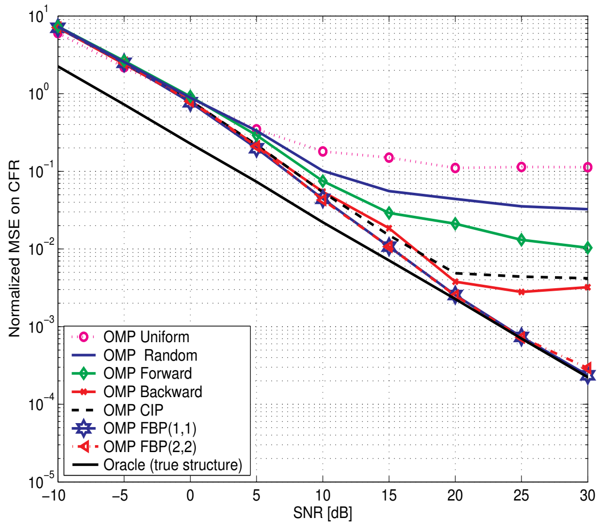

Fig. 2. Normalized MSE on Channel Frequency Response.

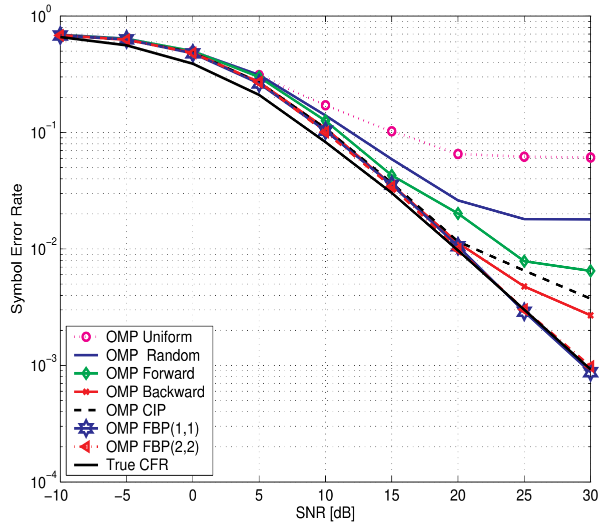

Fig. 3. Symbol Error Rate over data subcarriers from QPSK.

remain within the solution candidates set by imposing random changes within the individuals chromosomes before their fitness evaluation. The second scheme however operates, for each individual, through a combined forward and backward processing to converge to a solution within the candidates set while optimizing the sensing matrix coherence. This solution is then classified within its generation. The application to sparse channel estimation in OFDM systems shows the efficiency of the Forward-Backward Processing proposed scheme and the performance improvement, compared to former pilot pattern optimization schemes.

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