RECEIVER-BASED BAYESIAN PAPR REDUCTION IN OFDM

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ABSTRACT

One of the main drawbacks of OFDM systems is the high peak-to-average-power ratio (PAPR). Most of the PAPR reduction techniques require transmitter-based processing. However, we propose a receiver-based low-complexity clipping signal recovery method. This method is able to $i)$ reduce PAPR via a simple clipping scheme, $ii)$ use a Bayesian recovery algorithm to reconstruct the distortion signal with high accuracy, and $iii)$ is energy efficient due to low complexity. The proposed method is robust against variation in noise and signal statistics. The method is enhanced by making use of all prior information such as, the locations and the phase of the non-zero elements of the clipping signal. Simulation results demonstrate the superiority of using the proposed algorithm over other recovery algorithms.

Index Terms— Sparse signal estimation, PAPR reduction, tone reservation, SABMP, OFDM

1. INTRODUCTION

Recently, orthogonal frequency-division multiplexing (OFDM) has been adopted for high-speed wireless communications due to its robustness against multipath fading. One of the major drawbacks of OFDM which leads to inefficient use of nonlinear high power amplifiers (HPA), is its high peak-to-average-power ratio (PAPR). Operation in nonlinear region of HPA is power efficient but causes distortion. Though one option is to back off and let the HPA operate in the linear region, it results in power inefficiency. A high PAPR signal requires power amplifiers with linear response over a wide range, hence, expensive transmitters. This is one of the main reasons why in LTE the use of OFDM in the uplink was avoided [1, 2].

Many transmitter-based techniques have been proposed to reduce the PAPR, including coding, partial transmit sequence (PTS), selected mapping (SLM), interleaving, tone reservation (TR), tone injection (TI) and active constellation extension (ACE) [2–5]. However, the drawback is increased transmitter complexity and the advantage of energy efficiency provided by PAPR reduction is lost. In many applications which have limited source of power (e.g. satellites, mobile phones etc.), the transmitter complexity is the bottleneck, and hence there is a need for energy efficient low complexity alternatives.

Recently, compressive sensing (CS) techniques have been proposed for PAPR reduction [6, 7]. These techniques require simple clipping scheme at the transmitter and relegate any distortion mitigation at the receiver side. A hybrid approach composed of transmitter-side and receiver-side CS is presented in [8]. However, these techniques are based on convex relaxation regularized ℓ_1-optimization and suffer from high complexity. In addition, they do not make full use of *a priori* information that could enhance the performance.

In this paper, we propose a Bayesian approach to receiver-based PAPR reduction which by its nature is energy efficient due to low complexity. While the approach is Bayesian (thus acknowledging the sparsity of sparse clipping signal and the Gaussianity of the additive noise), it is agnostic to the distribution of the sparse signal support and robust to uncertainty in the noise variance and the sparsity rate. The approach is enhanced by utilizing *a priori* information about the clipping signal from the received signal including the phase information and probable locations where clipping has occurred. Hence, on one hand, our approach is robust against the uncertainties of clipping signal statistics, while on the other hand, the approach utilizes the received signal to extract information that assists in a robust recovery of clipping signal. In addition, sparsity recovery is achieved via a greedy low-complexity Bayesian matching pursuit method.

The remainder of the paper is organized as follow. Section 2 introduces data model for OFDM signals with clipping. In Section 3 we present the proposed clipping signal recovery algorithm. In Section 4, we present the simulation results and the conclusion is presented in Section 5.

1.1. Notation

We use lower case, bold-face letters for time domain vectors (e.g. $\mathbf{x}$) and upper case bold-face letters (e.g. $\mathbf{X}$) for matrices. We denote Discrete Fourier Transform (DFT) of a vector $\mathbf{x}$ by $\boldsymbol{\mathcal{X}}$. Further, we use $\hat{\mathbf{x}}$, $\mathbf{x}^{\mathsf{T}}$, $\mathbf{x}^{\mathsf{H}}$ and $x(i)$, to denote the estimate, transpose, conjugate transpose (or Hermitian) and the i^{th} element of vector $\mathbf{x}$ respectively.

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2. DATA MODEL

Let us consider an OFDM system in which the incoming information bits are mapped into an L-ary QAM constellation and concatenated to form an N-dimensional frequency domain data symbols vector $\boldsymbol{\mathcal{X}}$. Its time domain counterpart $\mathbf{x}$ is obtained by performing IDFT i.e., $\mathbf{x} = \mathbf{F}^{\mathsf{H}}\boldsymbol{\mathcal{X}}$, where $\mathbf{F}$ is a unitary DFT matrix with its $(k,\ell)^{th}$ element $\mathbf{F}(k,\ell) = \frac{1}{\sqrt{N}}e^{-j2\pi k\ell/N}, \quad k,\ell \in \{0,1,...,N-1\}$. This time-domain signal has a high PAPR which can be reduced through a simple clipping scheme by subjecting the signal to a magnitude limiter as follows

$$x_p(i) = \begin{cases} \gamma e^{j\theta_{x(i)}} & \text{if} \quad |x(i)| > \gamma \\ x(i) & \quad \text{otherwise} \end{cases} \tag{1}$$

where $x_p(i)$ is the i^{th} element of the signal after clipping, γ is the limiting threshold and $\theta_{x(i)}$ is the phase of $x(i)$.

This clipping scheme assures that, i) the phase of $\mathbf{c}$ is exactly opposite to that of $\mathbf{x_p}$, and ii) no distortion to the phase of $\mathbf{x_p}$ is introduced. It is very important to make sure that the phase of the clipped signal is undistorted, as will be seen later.

This hard clipping in (1) is equivalent to adding a *sparse* signal $\mathbf{c}$ to the original time domain signal $\mathbf{x}$ with active elements only where clipping has occurred, i.e.,

$$\mathbf{x_p} = \mathbf{x} + \mathbf{c}, \tag{2}$$

where $\mathbf{c}$ represents the distorting signal due to clipping. This equation could be equivalently written as

$$\mathbf{x_p} = \mathbf{F}^{\mathsf{H}}\boldsymbol{\mathcal{X}} + \mathbf{c}. \tag{3}$$

Before OFDM signal's transmission, a cyclic prefix is appended to the time domain signal. This cyclic prefix portion is removed at the receiver, and FFT is performed for the remaining signal. The received signal is

$$\mathbf{y} = \mathbf{H}\mathbf{x_p} + \mathbf{z} \tag{4}$$

where $\mathbf{z}$ is a zero-mean i.i.d complex Gaussian noise with variance σ_n^2. $\mathbf{H}$ is the circulant channel matrix by virtue of cyclic prefix insertion and removal (the channel impulse response is assumed to be known). The received signal is now transformed into the frequency domain. Hence, at the receiver (the channel matrix can be decomposed as $\mathbf{H} = \mathbf{F}^{\mathsf{H}}\mathbf{\Lambda}\mathbf{F}$)

$$\boldsymbol{\mathcal{Y}} = \mathbf{\Lambda}\boldsymbol{\mathcal{X}}_\mathbf{p} + \boldsymbol{\mathcal{Z}} = \mathbf{\Lambda}(\boldsymbol{\mathcal{X}} + \boldsymbol{\mathcal{C}}) + \boldsymbol{\mathcal{Z}} \tag{5}$$

where $\boldsymbol{\mathcal{Z}} = \mathbf{F}\mathbf{z}$, and $\mathbf{\Lambda}$ is a diagonal matrix with channel impulse response in frequency domain being in the diagonal.

Let us assume that the OFDM system has N subcarriers (tones), out of which K carriers are used for data transmission and the remaining $M \ll K$ carriers[1] are reserved for sparse signal recovery at the receiver. Let, $\mathbf{S_c}$ denotes an $N \times N$ binary selection matrix with 1's only at M locations along the diagonal determined according to the reserved carriers.

We proceed by projecting $\boldsymbol{\mathcal{Y}}$ onto the reserved carriers subspace ($\mathbf{S_c}$). This gives us

$$\begin{aligned} \mathbf{S_c}\boldsymbol{\mathcal{Y}} &= \mathbf{S_c}\left(\mathbf{\Lambda}(\boldsymbol{\mathcal{X}} + \boldsymbol{\mathcal{C}}) + \boldsymbol{\mathcal{Z}}\right) \\ &= \mathbf{S_c}\mathbf{\Lambda}\boldsymbol{\mathcal{C}} + \mathbf{S_c}\boldsymbol{\mathcal{Z}} \quad \text{, or} \\ \boldsymbol{\mathcal{Y}}' &= \mathbf{\Psi}\mathbf{c} + \boldsymbol{\mathcal{Z}}', \end{aligned} \tag{6}$$

where $\boldsymbol{\mathcal{Y}}' = \mathbf{S_c}\boldsymbol{\mathcal{Y}}$, $\mathbf{\Psi} = \mathbf{S_c}\mathbf{\Lambda}\mathbf{F}$, and $\boldsymbol{\mathcal{Z}}' = \mathbf{S_c}\mathbf{F}\mathbf{z}$. Note that, projecting $\boldsymbol{\mathcal{X}}$ onto $\mathbf{S_c}$ results in a zero vector. The only unknown in the resulting equation is $\mathbf{c}$. Note that, since $\mathbf{S_c}$ is a diagonal selection matrix, it contains ($K = N - M$) zero rows corresponding to data subspace. Therefore, there are K zero entries in $\boldsymbol{\mathcal{Y}}'$ which we remove to get a new M-dimensional vector $\boldsymbol{\mathcal{Y}}'_m$. Similarly the new $M \times N$ measurements matrix is $\mathbf{\Psi}_m$ and $\boldsymbol{\mathcal{Z}}'_m$ is white Gaussian noise with $\boldsymbol{\mathcal{Z}}'_m \sim \boldsymbol{\mathcal{CN}}(0, \sigma_n^2\mathbf{I})$. Therefore (6) becomes

$$\boldsymbol{\mathcal{Y}}'_m = \mathbf{\Psi}_m\mathbf{c} + \boldsymbol{\mathcal{Z}}'_m, \tag{7}$$

which is a set of M equations and thus represents projection of the N-dimensional sparse signal onto a basis of dimension much smaller ($M \ll N$).

Recall that for clipping at the i^{th} entry, $c(i) = -(x(i) - \gamma)$. This implies that the phase of the nonzero elements of $\mathbf{c}$ is always opposite to that of the corresponding elements of the clipped signal $\mathbf{x_p}$. Since the phase can be deduced from $\mathbf{x_p}$, we need to estimate only the magnitudes and locations of the non-zero elements of the sparse signal. Therefore the modified model becomes (note that, $\mathbf{c}$ is now just a vector of magnitudes)

$$\boldsymbol{\mathcal{Y}}'_m = \mathbf{\Psi}_m\mathbf{\Theta}_c\mathbf{c} + \boldsymbol{\mathcal{Z}}'_m, \tag{8}$$

where $\mathbf{\Theta_c}$ is a matrix containing the anti-phases of signal $\mathbf{x_p}$ along the diagonal, i.e.

$$\mathbf{\Theta_c} = -\mathbf{\Theta}_{\mathbf{x_p}} = -\mathsf{diag}\{e^{j\theta_{x_p(1)}}, e^{j\theta_{x_p(2)}}, ..., e^{j\theta_{x_p(N)}}\}.$$

where $\mathsf{diag}\{a,b\} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$.

Combining $\mathbf{\Psi}_m$ and $\mathbf{\Theta_c}$ we get

$$\boldsymbol{\mathcal{Y}}'_m = \mathbf{\Phi}_m\mathbf{c} + \boldsymbol{\mathcal{Z}}'_m, \tag{9}$$

where $\mathbf{\Phi}_m = \mathbf{\Psi}_m\mathbf{\Theta_c}$. Since all parameters except $\mathbf{c}$ are complex in the above equation, we can split the complex equation into real and imaginary parts as follows

$$\begin{aligned} \begin{bmatrix} \mathfrak{Re}(\boldsymbol{\mathcal{Y}}'_m) \\ \mathfrak{Im}(\boldsymbol{\mathcal{Y}}'_m) \end{bmatrix} &= \begin{bmatrix} \mathfrak{Re}(\mathbf{\Phi}_m) \\ \mathfrak{Im}(\mathbf{\Phi}_m) \end{bmatrix}\mathbf{c} + \begin{bmatrix} \mathfrak{Re}(\boldsymbol{\mathcal{Z}}'_m) \\ \mathfrak{Im}(\boldsymbol{\mathcal{Z}}'_m) \end{bmatrix} \text{, or} \\ \bar{\boldsymbol{\mathcal{Y}}} &= \bar{\mathbf{\Phi}}\mathbf{c} + \bar{\boldsymbol{\mathcal{Z}}}. \end{aligned} \tag{10}$$

where $\mathfrak{Re}(a), \mathfrak{Im}(a)$ denote operations of extracting, respectively, the real and imaginary components of complex valued a.

[1] These few frequencies are enough to estimate $\mathbf{c}$. For more details in sparse signal reconstruction from incomplete frequency information, see [9] [10].

Unlike (9) which is a system of M equations, we now have $2M$ equations to estimate a *real* unknown vector $\mathbf{c}$ of dimension N. This results in a better estimation. We aim to recover $\mathbf{c}$ using this underdetermined system and to do so we pursue a minimum mean-square error (MMSE) estimate which is discussed in Section 3.

Once we evaluate an estimate of $\mathbf{c}$ (i.e., $\hat{\mathbf{c}}$), then using phase information, we can subtract it from the estimated clipped signal $\hat{\mathbf{x}}_\mathbf{p}$ to get an estimate of the original signal $\mathbf{x}$, see (2). The time domain clipped signal can be estimated by equalizing the channel effect. Thus,

$$\begin{aligned}\hat{\mathbf{x}}_\mathbf{p} &= \mathbf{F}^\mathsf{H}(\boldsymbol{\mathcal{X}}_\mathbf{p} + \boldsymbol{\Lambda}^{-1}\boldsymbol{\mathcal{Z}}) \\ &= \mathbf{x}_\mathbf{p} + \mathbf{e}, \end{aligned} \tag{11}$$

where $\mathbf{e} = \mathbf{F}^\mathsf{H}\boldsymbol{\Lambda}^{-1}\boldsymbol{\mathcal{Z}}$ represents the error in estimating $\mathbf{x}_\mathbf{p}$.

This explains the process of recovering the transmitted signal while avoiding the high PAPR problem. Now, we present the algorithm and its proposed modified version for the recovery of sparse signal $\mathbf{c}$.

3. RECOVERY ALGORITHM

A tractable Bayesian approach to recover the magnitude of $\mathbf{c}$ from (10) would normally impose an assumption that the active elements of $\mathbf{c}$ are drawn from a Gaussian distribution. However, this is not the the case for the clipping signal. Recall from (1) and (2) that $\mathbf{c}$ is composed of the difference between a constant value γ and a Rayleigh distributed signal (the amplitude of $\mathbf{x}$ follows Rayleigh distribution [3]). Hence, the nonzero elements of $\mathbf{c}$ are certainly not Gaussian. Therefore, we pursue a Bayesian approach for the estimation of $\mathbf{c}$ which does not make any assumption about the statistics of the nonzero elements of $\mathbf{c}$.

We proceed by finding a MMSE estimate of $\mathbf{c}$ given $\bar{\boldsymbol{\mathcal{Y}}}$ as follows:

$$\hat{\mathbf{c}}_\mathbf{mmse} \stackrel{\triangle}{=} \mathbb{E}[\mathbf{c}|\bar{\boldsymbol{\mathcal{Y}}}] = \sum_{\mathcal{S}} p(\mathcal{S}|\bar{\boldsymbol{\mathcal{Y}}})\mathbb{E}[\mathbf{c}|\bar{\boldsymbol{\mathcal{Y}}}, \mathcal{S}], \tag{12}$$

where the sum is executed over all 2^N possible support sets $\mathcal{S}$ of $\mathbf{c}$. If we know the actual support $\mathcal{S}$, the linear model in (10) becomes,

$$\bar{\boldsymbol{\mathcal{Y}}} = \bar{\boldsymbol{\Phi}}_\mathcal{S}\mathbf{c}_\mathcal{S} + \bar{\boldsymbol{\mathcal{Z}}},$$

where $\bar{\boldsymbol{\Phi}}_\mathcal{S}$ is a matrix formed by selecting columns of $\bar{\boldsymbol{\Phi}}$ indexed by the support $\mathcal{S}$, while $\mathbf{c}_\mathcal{S}$ is formed by selecting entries of $\mathbf{c}$ indexed by support $\mathcal{S}$. Since the distribution of $\mathbf{c}$ is unknown, computing $\mathbb{E}[\mathbf{c}|\bar{\boldsymbol{\mathcal{Y}}}, \mathcal{S}]$ is impossible. Therefore, instead we use the best linear unbiased estimate (BLUE) as follows

$$\mathbb{E}[\mathbf{c}|\bar{\boldsymbol{\mathcal{Y}}}, \mathcal{S}] = (\bar{\boldsymbol{\Phi}}_\mathcal{S}^\mathsf{H}\bar{\boldsymbol{\Phi}}_\mathcal{S})^{-1}\bar{\boldsymbol{\Phi}}_\mathcal{S}^\mathsf{H}\bar{\boldsymbol{\mathcal{Y}}}.$$

It remains to evaluate the posterior $p(\mathcal{S}|\bar{\boldsymbol{\mathcal{Y}}})$ and the sum in (12). Using Bayes rule we can write

$$p(\mathcal{S}|\bar{\boldsymbol{\mathcal{Y}}}) = \frac{p(\bar{\boldsymbol{\mathcal{Y}}}|\mathcal{S})p(\mathcal{S})}{p(\bar{\boldsymbol{\mathcal{Y}}})}, \tag{13}$$

where $p(\bar{\boldsymbol{\mathcal{Y}}})$ is common to all posteriors, and therefore, could be ignored. A normal Bayesian approach would consider that the elements of $\mathbf{c}$ are activated according to a Bernoulli distribution with success probability ρ. However, note that, the closer the clipped signal $\mathbf{x}_\mathbf{p}$ to the threshold the more probable it is to be clipped. Therefore, we see that using ρ as success probability for each location is not a good idea. Thus, we modify this approach such that the probability of success of some entries are enhanced over the others. To do so, we define $\mathbf{w}$ as the difference between the amplitude of estimated clipped signal $\hat{\mathbf{x}}_\mathbf{p}$ and threshold γ, i.e., $\mathbf{w} = \gamma - |\hat{\mathbf{x}}_\mathbf{p}|$, and use it as a weighted vector. Therefore, it is obvious that we assign higher weights to locations where the abovementioned difference is small. Hence, we have [2].

$$p(\mathcal{S}) = \prod_i^N p_i \ \ , \text{ for all } \ i = 1, 2, \dots, N \tag{14}$$

where $p_i = \rho\, e^{-w(i)}$ (note that, p_i's are normalized such that the maximum value is 1).

By this modification, we increased the probability of those elements of $\mathbf{c}$ where $\hat{\mathbf{x}}_\mathbf{p}$ is close to the threshold γ. Note that, ρ represents the probability of an occurrence of a non-zero value at a location in $\mathbf{c}$ which in our case translates to the probability of a clipping occurrence.

We are left with the likelihood $p(\bar{\boldsymbol{\mathcal{Y}}}|\mathcal{S})$. Since, $\mathbf{c}_\mathcal{S}$ is not Gaussian, determining $p(\bar{\boldsymbol{\mathcal{Y}}}|\mathcal{S})$ is in general very difficult. To go around this, we note that $\bar{\boldsymbol{\mathcal{Y}}}$ is formed by a vector in the subspace spanned by the columns of $\bar{\boldsymbol{\Phi}}_\mathcal{S}$ plus a Gaussian noise vector, $\bar{\boldsymbol{\mathcal{Z}}}$. This motivates us to eliminate the non-Gaussian component by projecting $\bar{\boldsymbol{\mathcal{Y}}}$ onto the orthogonal complement space of $\bar{\boldsymbol{\Phi}}_\mathcal{S}$. This is done by multiplying $\bar{\boldsymbol{\mathcal{Y}}}$ by the projection matrix $\mathbf{P}_\mathcal{S}^\perp = \mathbf{I} - \mathbf{P}_\mathcal{S} = \mathbf{I} - \bar{\boldsymbol{\Phi}}_\mathcal{S}\left(\bar{\boldsymbol{\Phi}}_\mathcal{S}^\mathsf{H}\bar{\boldsymbol{\Phi}}_\mathcal{S}\right)^{-1}\bar{\boldsymbol{\Phi}}_\mathcal{S}^\mathsf{H}$. This leaves us with $\mathbf{P}_\mathcal{S}^\perp\bar{\boldsymbol{\mathcal{Y}}} = \mathbf{P}_\mathcal{S}^\perp\bar{\boldsymbol{\mathcal{Z}}}$, which is Gaussian with a zero mean and covariance

$$\begin{aligned}\mathbf{K} &= \mathbb{E}[(\mathbf{P}_\mathcal{S}^\perp\bar{\boldsymbol{\mathcal{Z}}})(\mathbf{P}_\mathcal{S}^\perp\bar{\boldsymbol{\mathcal{Z}}})^\mathsf{H}] \\ &= \mathbf{P}_\mathcal{S}^\perp\mathbb{E}[\bar{\boldsymbol{\mathcal{Z}}}\bar{\boldsymbol{\mathcal{Z}}}^\mathsf{H}]\mathbf{P}_\mathcal{S}^{\perp\,\mathsf{H}} = \mathbf{P}_\mathcal{S}^\perp\sigma_n^2\mathbf{P}_\mathcal{S}^{\perp\,\mathsf{H}} \\ &= \sigma_n^2\mathbf{P}_\mathcal{S}^\perp. \end{aligned} \tag{15}$$

Thus we have,

$$p(\bar{\boldsymbol{\mathcal{Y}}}|\mathcal{S}) \simeq \frac{1}{\sqrt{(2\pi\sigma_n^2)^M}}\exp\left(-\frac{1}{2}\left(\mathbf{P}_\mathcal{S}^\perp\bar{\boldsymbol{\mathcal{Y}}}\right)^\mathsf{H}\mathbf{K}^{-1}\left(\mathbf{P}_\mathcal{S}^\perp\bar{\boldsymbol{\mathcal{Y}}}\right)\right). \tag{16}$$

[2] For example, $p(\mathcal{S})$ of coefficients 1 and 2 being the only active elements is $p(\mathcal{S}) = p_1p_2\prod_{k\neq 1,2}^N(1-p_k)$, while in the i.i.d case it is $p(\mathcal{S}) = \rho^2(1-\rho)^{N-2}$.

Simplifying and dropping the pre-exponential factor yields,

$$p(\bar{\boldsymbol{\mathcal{Y}}}|\mathcal{S}) \simeq \exp\left(-\frac{1}{2\sigma_n^2}\left\|\mathbf{P}_{\mathcal{S}}^{\perp}\bar{\boldsymbol{\mathcal{Y}}}\right\|^2\right). \tag{17}$$

Substituting (14) and (17) into (13) finally yields an expression for the posterior probability. In this way, we have all the ingredients to compute the sum in (12). Computing this sum is a challenging task when N is large because the number of support sets can be extremely large and the computational complexity can become unrealistic. To have a computationally feasible solution, this sum can be computed over a few support sets corresponding to significant posteriors. Let $\mathcal{S}_d$ be the set of supports for which the posteriors are significant. Hence, we arrive at an approximation to the MMSE estimate given by,

$$\hat{\mathbf{c}}_{\mathbf{ammse}} = \mathbb{E}[\mathbf{c}|\bar{\boldsymbol{\mathcal{Y}}}] = \sum_{\mathcal{S}\in\mathcal{S}_{\mathrm{d}}} p(\mathcal{S}|\bar{\boldsymbol{\mathcal{Y}}})\mathbb{E}[\mathbf{c}|\bar{\boldsymbol{\mathcal{Y}}},\mathcal{S}] \tag{18}$$

We follow the method mentioned in [11, 12] where they devised a greedy algorithm to find a subset of the dominant support $\mathcal{S}_{\mathrm{d}}$. Noteworthy, the weighted version of $p(\mathcal{S})$ in (14) also helps to find the dominant support faster than the unweighed version.

Note that, the Bayesian approach, discussed above, requires information about the sparsity rate (ρ), the noise variance (σ_n^2) and the threshold (γ), to recover $\mathbf{c}$. However, exact values of these parameters may not be available at the receiver. Therefore, there should be a way to recover $\mathbf{c}$ perfectly even in the presence of rough estimates. We start with initial estimates of required parameters and estimate $\mathbf{c}$. This estimate is in turn used to refine the abovementioned parameters which are then used to get a better estimate of $\mathbf{c}$ again. This process is repeated a number of times. Specifically, the refinement process is continued until the percentage change in the two consecutive estimates of sparsity rate ($\hat{\rho}$) becomes less than 2%. An algorithmic description of the recovery process that we follow is provided in Table 1. A discussion about the computational complexity of the proposed method can be seen in [12].

1) **estimate** $\hat{\mathbf{x}}_{\mathbf{p}} = \mathbf{F}^{\mathsf{H}}\boldsymbol{\Lambda}^{-1}\mathbf{F}\mathbf{y}$
2) $\hat{\gamma} = \max(\hat{\mathbf{x}}_{\mathbf{p}})$.
3) $\hat{\sigma}_n^2 = \mathrm{var}(\bar{\mathcal{Y}})$.
4) $\mathbf{w} = \hat{\gamma} - |\hat{\mathbf{x}}_{\mathbf{p}}|$.
5) $\hat{\rho}_o = \mathsf{Q}\left(\frac{\hat{\gamma}-\mu}{\sigma}\right)$, an initial estimate, where μ and σ are the mean and standard deviation of $\hat{\mathbf{x}}_{\mathbf{p}}$, respectively.
6) $i = 0$, **repeat**
7) $p_k = \hat{\rho}_i\ e^{-w(k)},\ k = 1, 2, \dots, N.$
8) **Compute** $\hat{\mathbf{c}}_{\mathrm{ammse}}$ and $\hat{\rho}_{i+1}$ using the technique discussed in [12]
9) **until** $\left(\frac{|\hat{\rho}_i - \hat{\rho}_{i-1}|}{\hat{\rho}_{i-1}} < 0.02\right)$
10) $\hat{\mathbf{c}} = \boldsymbol{\Theta}_{\mathbf{c}}|\hat{\mathbf{c}}_{\mathrm{ammse}}|$
11) $\hat{\mathbf{x}} = \hat{\mathbf{x}}_{\mathbf{p}} - \hat{\mathbf{c}}$

Table 1: Proposed signal recovery algorithm

4. SIMULATION RESULTS

For numerical implementation $N = 512$ subcarriers were used, where 20% of them were reserved randomly for clipping signal recovery ($M = 20\%$ of N). It has been shown in [13, 14] a small number of random measurements can recover a sparse signal with high accuracy from a fixed incoherent basis. Data is generated from a 64-QAM constellation ($L = 64$) and i.i.d additive white Gaussian noise (AWGN) is generated with 30dB SNR. We consider a frequency-selective fading channel, with 7-taps, which is assumed to be known at the receiver. The two performance metrics considered are bit error rate (BER) and average run-time. All results are obtained by averaging 200 independent realizations.

We used two Bayesian matching pursuit algorithms for sparse signal recovery, the fast Bayesian matching pursuit (FBMP) [11] and the support agnostic Bayesian matching pursuit (SABMP) [12]. In addition, we also estimated $\mathbf{c}$ using ℓ_1-optimization using CVX, a package for specifying and solving convex programs [15]. All these techniques are used to estimate the sparse signal from the system of equations given in (7). We compared the performance of these methods with the performance of our enhanced version (i.e., weighted and phase augmented (WPA)-SABMP), which is able to utilize the phase and clipping probability to solve for $\mathbf{c}$. We also compare the performance of these methods to oracle-LS where the receiver knows the actual support of $\mathbf{c}$ and least-square estimate is used.

4.1. Experiment 1

In this experiment, we study the performance of the abovementioned algorithms with respect to the varying clipping threshold γ. Specifically, we plot BER versus γ. In this experiment, exact values of the required parameters (i.e., sparsity rate ρ, noise variance and signal variance) were provided to the Bayesian estimation algorithms (SABMP and WPA-SABMP do not require signal variance).

Fig. 1 shows the superior performance of WPA-SABMP over other algorithms. We can also see that, for high γ (i.e., high sparsity) all algorithms have similar performance. Please note that, in all of the figures as γ increased from 10.13 to 11.43, sparsity rate decreased from 0.086 to 0.045.

4.2. Experiment 2

The performance of the algorithms, when the exact parameter values are unknown are studied in this experiment. The initial estimates provided to the Bayesian algorithms are chosen significantly away from the true values. The proposed WPA-SABMP algorithm is capable of refining these estimates in an iterative manner as mentioned in Table 1. The BER performance of WPA-SABMP (without refinement), (with refinement) and ℓ_1-optimization are plotted in Fig. 2.

As expected, WPA-SABMP (refined) performed better than its non-refined version. We also note that even with rough initial estimates, the non-refined WPA-SABMP

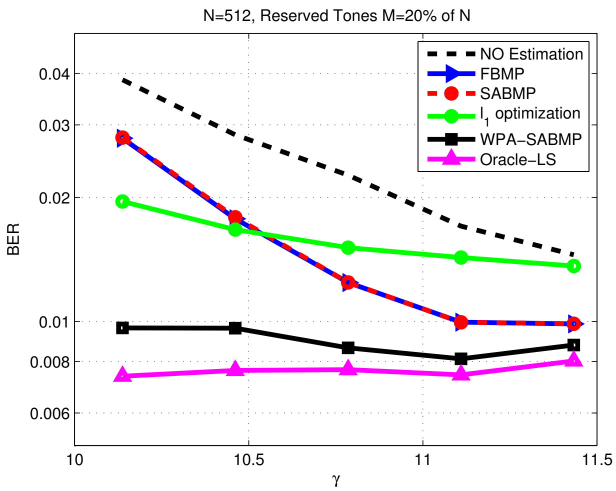

Fig. 1: BER versus γ when using exact parameter values

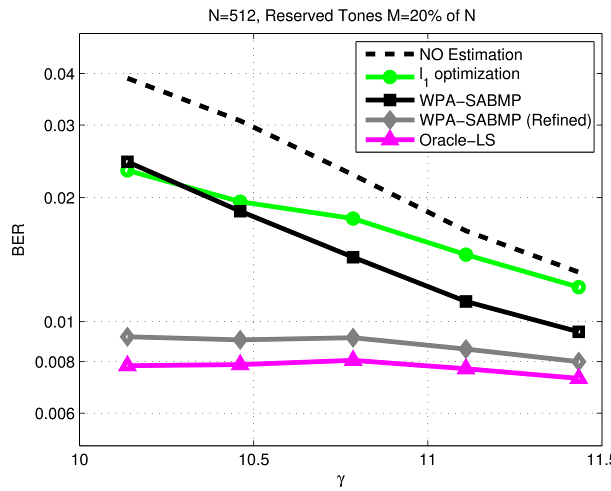

Fig. 2: BER versus γ when using rough initial estimates of parameters

	WPA-SABMP	WPA-SABMP (refined)	ℓ_1-Opt.
Run time (s)	0.0101	0.3339	1.6338

Table 2: Average run-time when using rough estimated parameters in experiment 2.

performed better than ℓ_1-optimization. As compared to ℓ_1-optimization programming, the WPA-SABMP (refined) algorithm requires much less time for estimation. It is worth mentioning that, by the virtue of the weighted $p(S)$, the WPA-SABMP algorithm requires less time than plain SABMP as it is able to find the correct support quickly (see the discussion after (18)).

5. CONCLUSION

In this paper, we present a robust Bayesian algorithm for clipping signal recovery to reduce PAPR in OFDM. The proposed method requires no information at the receiver about the noise variance, clipping signal statistics and the clipping threshold and can estimate these required parameters. In addition, the algorithm utilizes and exploits the prior information of clipping signal which can also be obtained from the received signal. The clipping signal estimates can be improved further by refining the estimates of parameters.

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