VALIDATION OF MAGNETO HYDRODYNAMICS MODEL

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ABSTRACT

This study validates the simulation of the magnetic field effect on the electrocardiogram (ECG), during cardiovascular magnetic resonance imaging (MRI). The validation corresponds to a theoretical part and another modeling & simulation part. First, we write and develop equations showing equivalence between vector analysis and Magneto hydrodynamics (MHD) theory, which resulted in a relation between the spatial variation of the cross product of velocity and the magnetic field B. Later, we apply these equations on a Simulink built model, and we achieve the time derivatives of different components. The simulation results show correspondence between output signals, which echoes theoretical proof. We validate our choices of using velocity field and considering the magnetic field as being the difference between MRI affected and unaffected ECG signals. An important continuation, aims MHD indicators.

***Index terms*– MHD, MRI, ECG, velocity, divergence.**

1. INTRODUCTION

Magneto hydrodynamics, or MHD, is a branch of the science of the dynamics of matter moving in an electromagnetic field, especially where currents established in the matter by induction modify the field, so that the field and dynamics equations are coupled. MHD treats, in particular, conducting fluids, in which certain simplifying postulates are accepted. A biomedical illustration is given by the flow of ions, during the cardiac cycle, which is affected by the magnetic field. Such application can be found during the use of magnetic resonance (MR) techniques that has increased to gain important insights into the functional and metabolic basis of heart disease. Plus, acquisitions of good quality MR images of the cardiovascular system need to be ECG gated [1, 2].

In general, increasing the static magnetic field strength B_0 increases the signal-to-noise ratio (SNR); and when combined with strong-fast switching gradient systems and fast data acquisition schemes, high spatial-resolution and high-temporal-resolution MR experiments can be performed. A specific interest is accorded to the effect of this high B_0, and its perturbations induced on the ECG. Indeed, the interaction between the blood flow and the magnetic field results in an induced electric field in the arteries [3]. Except for this MHD influence, all other artifacts have been observed, modeled and anticipated for real-time or delayed time adaptation and elimination [4]. This work corresponds to the first stage of verifying a developed modeling tool able to isolate and manipulate induced MHD related signals and components (conduction, diffusion, and coupling to Navier-Stokes fluid dynamics equation). This study, demonstrates soundness of realizing the magnetic field from different MR related ECG signals. This confirmation is fundamental for the inclusion of the effect of time rate of Magnetic field (dB/dt) with the pulsed gradient fields. This paper will begin with a brief description of MHD theory. Then, simulations of models are presented, compared and assessed.

2. METHODOLOGY

2.1. Requirements

The MHD mathematical model depends on dynamics equations based on blood velocity u, and Maxwell's equations relating magnetic field B to its intensity H. Indeed, the electric current density J is expressed in terms of curl of B, using nabla operator:

$$\vec{J} = \frac{1}{\mu_0}\left(\vec{\nabla} \times \vec{B}\right) \qquad (1)$$

The magnetic permeability is $\mu_0 = 4\pi 10^{-7}$ H/m. For a conducting fluid, such as blood, *Ohm's law* relates the current density J to the electric field E:

$$\vec{J} = \sigma\left(\vec{\mathrm{E}}\right) \qquad (2)$$

The blood conductivity is $\sigma = 4$ S/m. The Lorentz electromagnetic force is the summation of the electric and the magnetic force. Consequently:

$$\vec{J} = \sigma\left(\vec{\mathrm{E}} + \vec{u} \times \vec{B}\right) \qquad (3)$$

Then taking equation (1) ≡ equation (3) we get:

$$\frac{1}{\mu_0}\left(\vec{\nabla} \times \vec{B}\right) = \sigma\left(\vec{\mathrm{E}} + \vec{u} \times \vec{B}\right) \qquad (4)$$

Now apply curl on both sides of equation (4), we get:

$$\frac{1}{\mu_0}\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \sigma\left[\vec{\nabla} \times \vec{\mathrm{E}} + \vec{\nabla} \times \left(\vec{u} \times \vec{B}\right)\right] \qquad (5)$$

Considering curl of E, being the time change of B, as given in *Faraday's law,* then equation (5) becomes:

$$\frac{1}{\mu_0}\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \sigma\left[-\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \left(\vec{u} \times \vec{B}\right)\right] \qquad (6)$$

From the properties of Laplace operator (Δ) and vector analysis we have:

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \vec{\nabla}\left(\vec{\nabla} * \vec{B}\right) - \Delta\vec{B} \qquad (7)$$

$$\vec{\nabla} \times (\vec{u} \times \vec{B}) = \vec{u}(\vec{\nabla} * \vec{B}) - \vec{B}(\vec{\nabla} * \vec{u}) + (\vec{B} * \vec{\nabla})\vec{u} - (\vec{u} * \vec{\nabla})\vec{B} \quad (8)$$

Recalling from Maxwell's equations that there is no isolated magnetic charge and that divergence of B is zero. Therefore, equation (6) can be stated as follows:

$$\frac{1}{\mu_0}(-\Delta\vec{B}) = \sigma\left[-\frac{\partial\vec{B}}{\partial t} + (-\vec{B}(\vec{\nabla} * \vec{u}) + B * \nabla u - u * \nabla B\right. \quad (9)$$

Since divergence of the velocity is zero, due to continuity, the above equation is rearranged as:

$$\frac{\partial\vec{H}}{\partial t} = \frac{1}{\mu_0}\frac{\partial\vec{B}}{\partial t} = \underbrace{(\vec{H} * \vec{\nabla})\vec{u}}_{C_1} - \underbrace{(\vec{u} * \vec{\nabla})\vec{H}}_{C_2} + \underbrace{\eta\Delta\vec{H}}_{D} \quad (10)$$

Hence, there are three terms C_1, C_2, and D that specify the variation of the magnetic field. Terms C_1 and C_2 are combined under the conduction constituent of MHD, made by the interaction between blood velocity and magnetic field. Accordingly, the motion at the center of the fluid could increase or decrease the magnetic field intensity B. Term D relates to the diffusion component of MHD, and it means that if the diffusivity η, $(1/\sigma\mu_0)$, is different than zero, then the magnetic field fluctuations are dissipated according to Joule's effect.

2.2. Assumptions

The magnetic field lines are straight and parallel to the axis of symmetry. In order to extract the magnetic field *B,* and its intensity *H*, we deduct a clean ECG, S_C, recorded in absence of magnetic field, from another noisy ECG, S_N, taken under MR machine field B_0, with no imaging sequence. Therefore, the magnetic field depends on the following formula:
Magnetic field signal = noisy ECG – clean ECG. And so, when considering the link between B and H through the permeability μ_0, we write:

$$\frac{\partial\vec{H}}{\partial t} = \frac{1}{\mu_0}\frac{\partial\vec{B}}{\partial t} = \frac{1}{\mu_0}\left[\frac{\partial(S_C - S_N)}{\partial t}\right] \quad (11)$$

Similarly the velocity is considered as a field that is a function of space directions: u= (u_x, u_y, u_z).
The consideration of MR machine sequence gradient, while studying MHD effect under unique influence of the magnetic field, yields to count the inhomogeneities of the magnetic field in time and space. Relations (10) and (11) will be modeled based on single z direction of B and H. The development of these relations shows that velocity is given, similar to B and H, only in z direction. Where:

$$u_z = -0.08 * z * \cos(0.1 * z) \quad (12)$$

The z coordinates are obtained after dividing the field of view, FOV_z, in the magnetic field direction, z, by the spatial resolution dz. For FOV_z = 3mm and dz= 0.001 mm, a number of steps for z-direction coordinates is given by: N_z= FOV_z /dz= 3000. The used ECG signal during this study is collected from a healthy mouse ECG as described in [5].

2.3. Design and Simulations:

To simulate equations (10) and (11) we use Simulink, the dynamic system modeling extension of Matlab. The main model system is presented in figure 1, and it enables implementation of both equations. Its purpose is to compare between two magnetic field effect signals. The first signal results from simple time derivation of magnetic field, B, as in equation (11) when measured during ECG acquisition and it is called "B_time rate_ Extracted". The second one corresponds to time variation of magnetic field intensity, H, made by conduction & diffusion MHD effects of equation (10). This signal is "H_time rate_MHD generating". Furthermore, the model includes three principal modules. First, a part containing signals to be studied referred as "Signals and inputs" subsystem in figure 1. It consists of five inputs; the noisy ECG S_N, the normal ECG S_C, the magnetic field B, its intensity H, as given in equation (11) and the z-component of velocity, u_z, from equation (12). All of these inputs are obtained using the structure bloc, which allows communicating to signals present at the workspace of Matlab. Then a bloc processes and executes the model engendering MHD constituents. It is the core of the subsystem entitled "MHD Conduction & Diffusion Effects", and detailed in figure 2. The last module displays ECG inputs and magnetic field derivation results outputs into the visualization scope.

As for the subsystem of MHD components, it implements MHD conduction and diffusion mathematical model of equation (10), with inputs being u_z and H, while outputs are equivalent to the three terms C_1, C_2, and D, as well as, their summation shown in equation (10) right and left side, respectively. As shown in the upper part of figure 2 the connections of the three terms C_1, C_2, and D blocs are linked to the adder bloc, (+-+), leading to the MHD related magnetic effect signal. This is the subsystem's first output, del H/ del t, which, according to the relation between time derivative and partial derivative, it corresponds to half of material derivative of the field intensity dH/dt; the second and third outputs relate to the space variation of the magnetic field intensity and velocity field; H and u_z, summarized in "(H*Nabla)u" and "(u*Nabla)H". The second order derivative output corresponds to the output of magnetic field intensity's Laplacian operator subsystem, or "ηΔH", which implementation is illustrated in the lower part of figure 2.

We check no available solution of partial derivatives and their applications under Simulink. Hence, we propose a new method using partial derivative del(f)/del(x) for the elaboration of nabla operator and resulting vector analysis subsystems. "del z", representing the partial differentiation, is obtained from z coordinates vector, N_z, derived from assumptions, calculations and correspondence to displacement flow direction, (z). The Diff bloc, shown in figure 2, represents the difference between adjacent vector elements leading to the approximate direction of the studied variable.

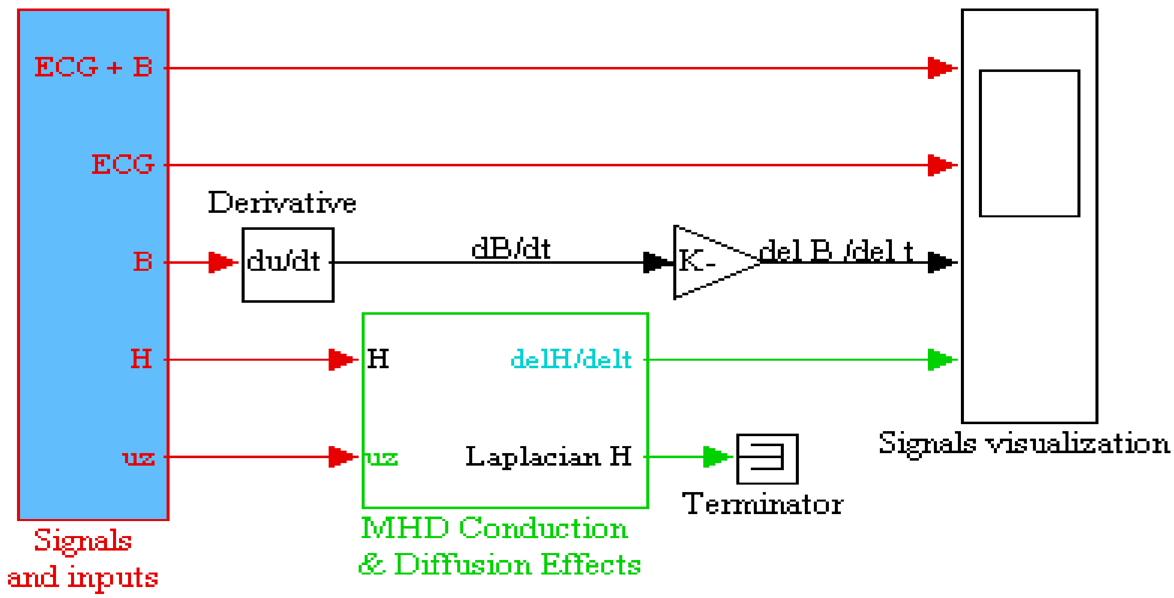

Figure.1: Simulink model of MHD coupled to the time change of magnetic field B.

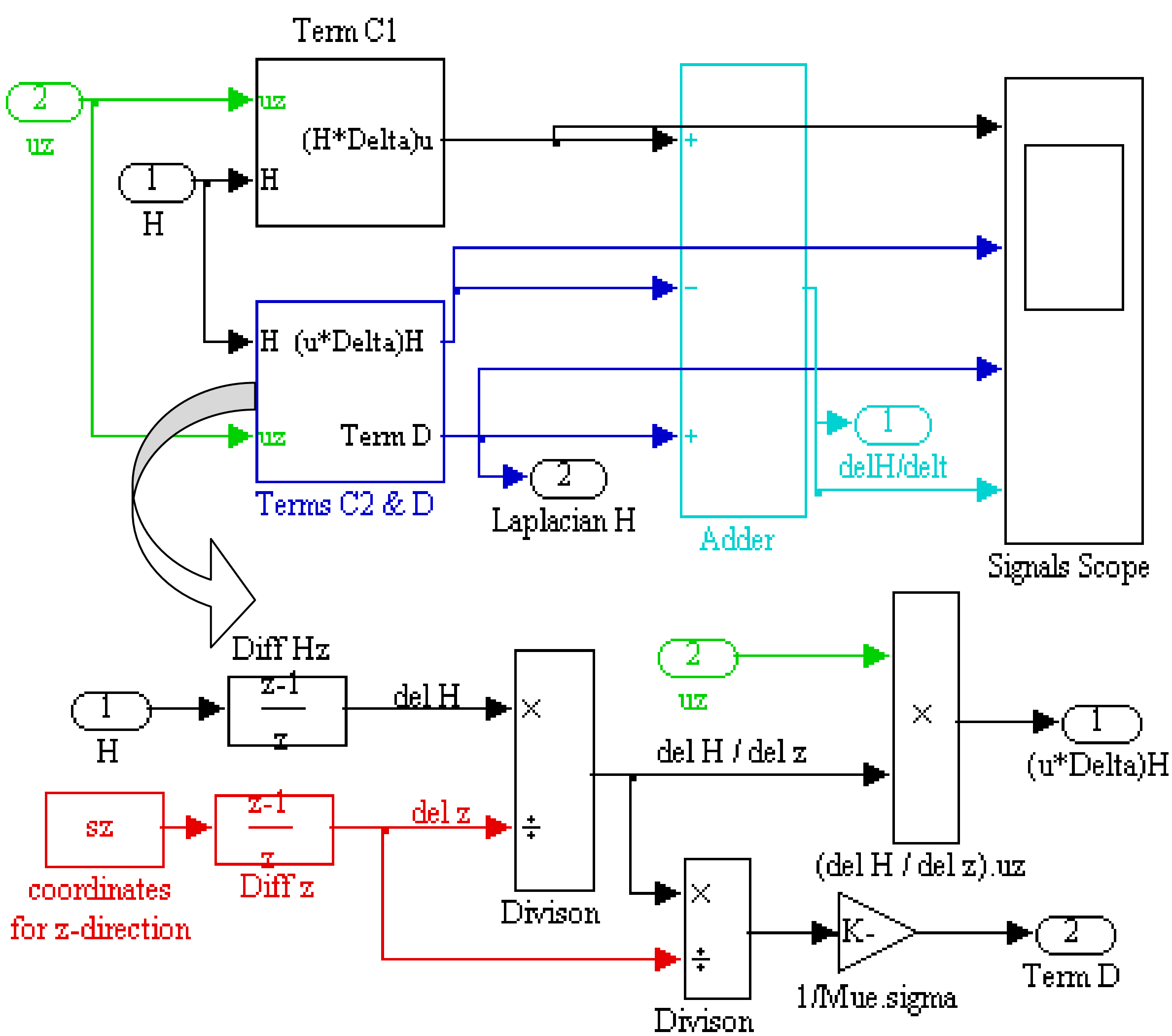

Figure.2: MHD components subsystem; Upper part relates to the subsystem, lower part details diffusion.

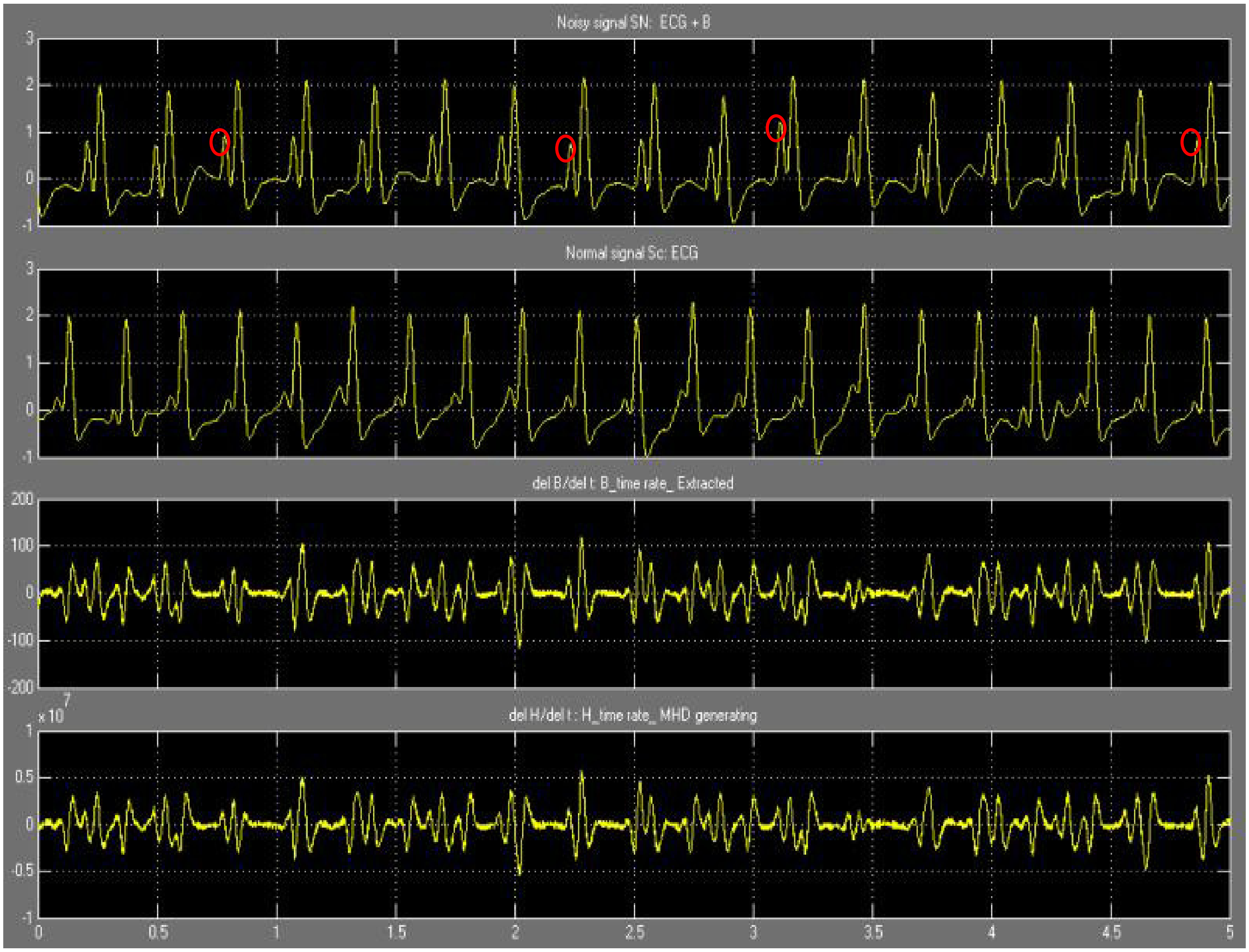

Figure.3: Display of main model outputs. S_C: Clean ECG, S_N: Noisy ECG, del B/del t: B_time rate_ Extracted, and del H/del t: H_time rate_ MHD generating; Red circles illustrate the added B effect potential.

3. RESULTS AND DISCUSSION

Figure 3 shows the four outputs of the main model of figure1. Signals have duration of 5 seconds. The shape, form, and sampling of the different parts of the figure indicate on the validity of the proposed method, since the third signal found from the MR magnet, "B_time rate_ Extracted" taken from the difference between the first (Noisy) and second (clean) ECGs, is similar to the fourth signal derived from MHD theory, "H_time rate_ MHD generating". Compared to the clean signal of the figure 3 second row, the noisy ECG has elevated amplitude of ST segment as shown with red circles on figure 3 first row, due to the MR machine magnetic field B effect. The variation in the amplitude scale between third and fourth signals is due to two main reasons. Firstly, the magnetic permeability μ_0, is present in case of H and it leads to a factor of 10^7; secondly the amplification effect of the simulated divergence.

Figure 4 which corresponds to a 1 second simulation of the second subsystem, proves that the magnetic field intensity H leads to a potential signal very close to time variation of the field B. Plus, we notice the equality between the third and fourth signals. This is explained by the fact, that the products of divergence of velocity and magnetic fields; "(H*Nabla)u" and "(u*Nabla)H" are negligible comparing to the "$\eta\Delta H$" component. This resonates with literature showing that these products tend toward zero. Indeed, the difference between the two signals representing these divergence products results into a very weak signal added as high-frequency on the final output, entitled "H_time rate_ MHD generating".

Simulation and configuration parameters refer to a variable step, or sampling period that corresponds to the available different coordinate steps for z direction. A continuous state simulation method, is applied, for the numerical solution of ordinary differential equations using 4 iterations; known as: (ode23**Bogacki–Shampine**) analogous to the Runge- Kutta fourth order RK4. The latter offers optimum cost to time ratio.

The presented multi inputs multi outputs, (MIMO), configuration, offers the chance of a larger range of operation related to application and objectives based on selecting, manipulating and comparing or isolating MHD constituents.

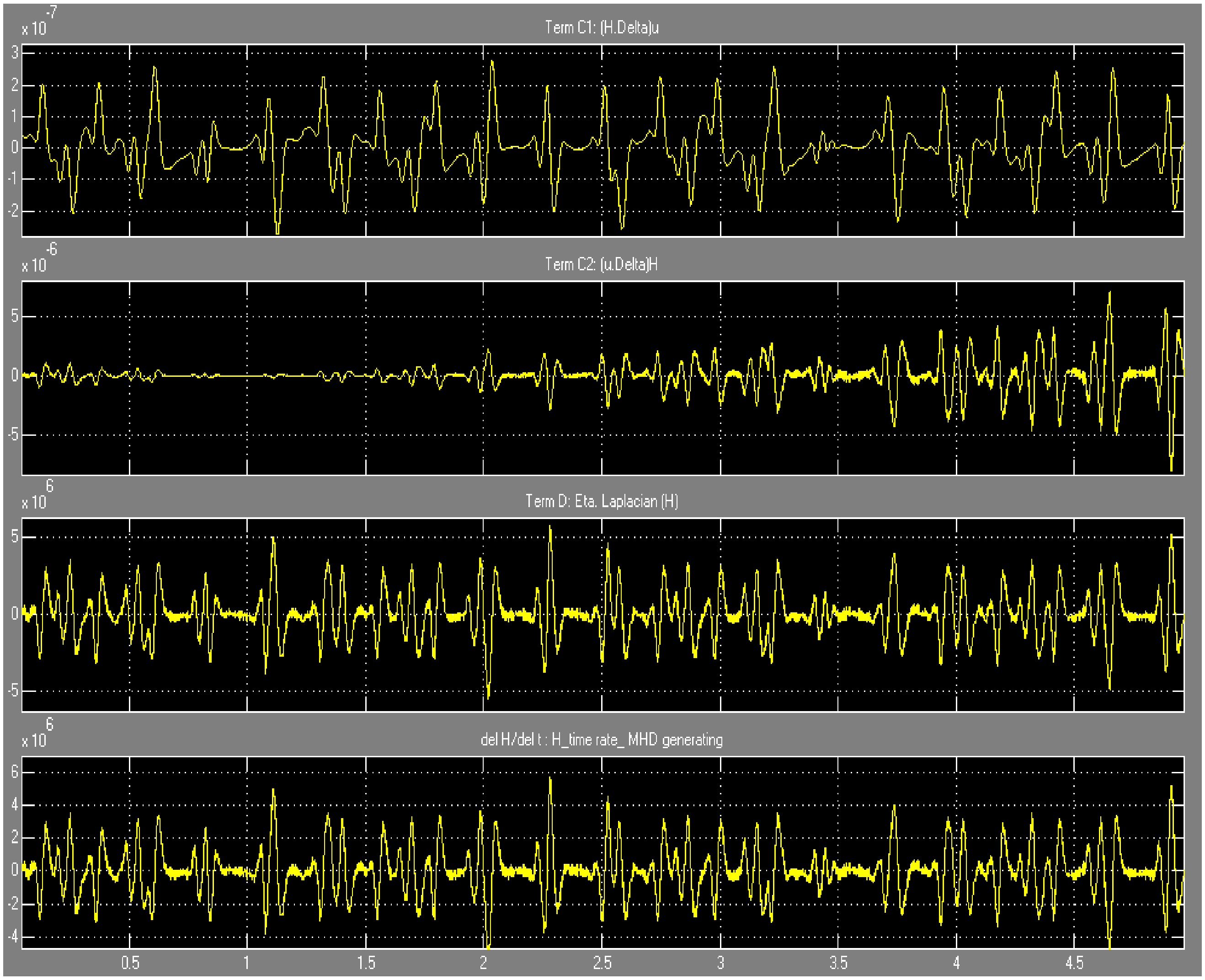

Figure.4: MHD components subsystem outputs: Term C_1: (H*Nabla)u, Term C_1: (u*Nabla)H, Term D: $\eta\Delta$H and final output: del H/del t.

4. CONCLUSION

Thus, a first step of a completed model for different MHD parts validation is presented. The proposed method of extracting magnetic field from the noisy signals generated under MR magnet is successfully tested using analysis vector and divergence model implementations and simulations. The space variation related diffusion and conduction effects of MHD are coupled, to the time rate of magnetic field change. The MHD theory is necessary, to show vector analysis based relation between magnetic field variation and velocity. The velocity choice rightness is verified. The equivalence between the two signals is shown; same data from two different origins and reasons behind generation, and the predetermined objective is met. This should lead to further demonstration like calculating MHD indicators and parameters such as magnetic pressure, magnetic Reynolds and Hartmann numbers.

5. REFERENCES

[1] Cassidy PJ, Schneider JE, Grieve SM, Lygate C, Neubauer S, Clarke K., *Assessment of Motion Gating Strategies for Mouse Magnetic Resonance at High Magnetic Fields*, J. Magn Reson Imaging, pp. 19:229-237, 2004.

[2] Fischer SE, Wickline SA, and Lorenz CH. "Novel real-time Rwave detection algorithm based on the vectorcardiogram for accurate gated magnetic resonance acquisitions," Magn Reson Med 42: pp. 361-370, 1999.

[3] Martin V, Drochon Agnès, Fokapu O, Gerbeau JF *"MagnetoHemoDynamics in the Aorta and Electrocardiograms"*, Physics in Medicine and Biology 57, 10 pp. 3177-3195, 2012.

[4] Sabbah M, Diab M, Moslem B, *"Magnetohydrodynamics: Application in Cardiovascular MRI"*, ISMA'12 American University of Sharjah, UAE April 10-12, 2012.

[5] Sabbah, M., Alsaid, H., Fakri-Bouchet, L., Pasquier, C., Briguet, A., Canet-Soulas, E., and Fokapu, O. "Real-time gating system for mouse cardiovascular MR imaging" Magn Reson Med 57: pp.29-39, 2007.