OPTIMAL BIT AND POWER ALLOCATION FOR RATE-CONSTRAINED DECENTRALIZED DETECTION AND ESTIMATION

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ABSTRACT

In decentralized wireless sensor networks, the decision taken at the fusion center depends on the quality of the signals received from the sensing nodes. The typical sources of errors are observation noise, quantization errors due to source encoding, and communication noise, i.e. errors in the transmission from the sensing nodes to the sink node. In this work we propose a method for allocating quantization bits and transmission powers in each node in order to satisfy alternative optimization criteria in either detection or estimation. In both cases, to limit transmission errors, we enforce the additional constraint that the number of coding bits on each sensor is less than the channel capacity over the channel that connects each sensor to the sink node.

***Index Terms*—** Decentralized estimation, decentralized detection, wireless sensor networks

1. INTRODUCTION

Wireless sensor networks (WSN) enable the distributed monitoring of the environment through the fusion of the information gathered by several sensors. A recent interesting development of WSN's is in cognitive radios, as a decentralized tool to perform spectrum sensing and detection of spectral opportunities for secondary users. The approach is also of interest for small cell networks, as a way to endow the network with self-organization capabilities, in terms of radio resource allocation. In such a case, the cooperation among different sensing nodes is fundamental to prevent detection errors (either false alarms or misdetections) due to local shadowing effects. The distinguishing feature of decentralized decision, either detection or estimation, systems is that the sensing and communication steps are strictly intertwined with each other. In particular, the radio resource allocation should be performed in order to meet the requirements of either the estimation accuracy or the probability of detection. This calls for a *joint* optimization of source coding and radio resource allocation. Several works have already considered the decentralized estimation problem, see e.g. [1], [2] and the decentralized detection problem [3]. Many works in literature have already tackled the problem of distributed estimation and detection incorporating realistic channels. In [4] the authors proposed the minimization of the Euclidean norm of the transmit power vector under the constraints that the estimation variance is lower than a given quantity and that the number of bits per symbol is less than the channel capacity. In [5], it has been proposed an optimal design of quantization bits and power allocation under the criterion of minimizing the distortion using linear minimum mean square error estimation rule at the fusion center, for a given total network power consumption. A decentralized estimation strategy over a bandwidth-constraint network has been considered in [6] where each sensor, before sending its measured values to the fusion center, compresses the measurement to a small number of bits without any knowledge of the noise distribution function.

As far as distributed detection is concerned [3], the problem of taking into account both channel capacity and number of bits to quantize the local observations, becomes more complicated. In [7] it was shown that binary quantization is optimal for the problem of detecting deterministic signals in Gaussian noise and for detecting signal in Gaussian noise using a square-low detector. In [8] the authors propose a universal decentralized detector with a distributed sensor network where each sensor is restricted to send a one-bit message to the fusion center.

In this paper we propose an optimization strategy to find the optimal number of coding bits and transmission powers in each node in order to satisfy alternative optimization criteria in either detection or estimation networks over fading communication links. To make the effect of transmission errors negligible, we constrain the number of coding bits per sample to be less than channel capacity, expressed in bits per symbol, in order to make the error probability on the transmitted codeword arbitrarily small. More specifically, in the estimation case, we find the optimal resource (power/bit) allocation that minimizes the transmit power, while enforcing an upper bound on the estimation variance. In the detection case, we

This work was funded by the European Community 7th Framework Programme Project ICT-TROPIC, under grant nr. 318784.

derive the optimal transmit powers and bit allocation in order to maximize the detection probability, under the constraints of satisfying a given false alarm rate and a maximum transmit power budget.

2. CENTRALIZED ESTIMATION WITH DECENTRALIZED OBSERVATIONS

Let us consider N sensors observing a common deterministic parameter ϑ corrupted by additive noise. The observation x_k of sensor k is described by

$$x_k = \vartheta + n_k, \qquad k = 1, \ldots, N \tag{1}$$

where the noise variables n_k are supposed to be zero mean spatially uncorrelated random variables with variance σ_{nk}^2. The nodes send their measurements to the fusion center that carries out the estimation of the unknown parameter ϑ based on the received messages. Since the system is controlled by the fusion center, we assume that orthogonal channels are assigned to the links from different nodes so as to avoid interference. We assume that the estimation criterion adopted by the fusion center is the minimization of the mean square error $\mathrm{MSE}(\hat{\vartheta}) = E[(\hat{\vartheta} - \vartheta)^2]$. According to the observation model in (1), under an ideal transmission scenario where the observations are unquantized and received by the fusion center without errors, the best linear unbiased estimator having minimum variance (BLUE) is given by [9]:

$$\hat{\vartheta} = \left(\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2} \right)^{-1} \sum_{k=1}^{N} \frac{x_k}{\sigma_{nk}^2} \tag{2}$$

with $\mathrm{MSE}(\hat{\vartheta}) = \left(\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2} \right)^{-1}$. Under the assumption that the noise variables are jointly Gaussian and uncorrelated, the BLUE estimator coincides with the maximum likelihood estimator. Consider now the realistic case where each sensor quantizes its observation x_k to generate a discrete message m_k of q_k bits. In the case of error-free transmission, the fusion center is able to find an estimate $\hat{\vartheta}$ of the true parameter by linearly combining the messages m_k transmitted by all the nodes. More specifically, assuming that the sensors employ the uniform quantizer, let us suppose that the unknown signal to be estimated belongs to the interval $[-A, A]$ and each sensor divides the range $[-A, A]$ uniformly into 2^{q_k} intervals of length $\Delta_k = 2A/2^{q_k}$, rounding x_k to the midpoint of the interval into which it belongs. Hence, the quantized value m_k of x_k at the k-th sensor can be modeled as the observation plus a quantization noise v_k, i.e.

$$m_k = \vartheta + n_k + v_k, \tag{3}$$

where the quantization noise v_k can be assumed to be independent of n_k. By denoting with σ_{qk}^2 the quantization noise variance given by

$$\sigma_{qk}^2 = \frac{\Delta_k^2}{12} = \frac{A^2}{3 \cdot 2^{2q_k}}, \tag{4}$$

a linear unbiased estimator of ϑ is [9]

$$\hat{\vartheta} = \left(\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2 + \sigma_{qk}^2} \right)^{-1} \sum_{k=1}^{N} \frac{m_k}{\sigma_{nk}^2 + \sigma_{qk}^2} \tag{5}$$

and the resulting MSE is

$$E[(\hat{\vartheta} - \vartheta)^2] \le \left(\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2 + \sigma_{qk}^2} \right)^{-1}. \tag{6}$$

As mentioned before, this formulation assumes that the transmission links are error-free. To make this property as close to reality as possible, we enforce the transmission rate of sensor k to be less than the channel capacity from that sensor to the fusion center. This poses a strict constraint between the maximum number of bits per sample q_k and the channel capacity, expressed in bits/symbol. Denoting with p_k the transmit power of sensor k, with h_k the channel coefficient between sensor k and the sink node and with N_0 the noise variance at the sink receiver, the constraint is

$$q_k \le \frac{1}{2} \log_2 \left(1 + \frac{p_k h_k^2}{N_0} \right). \tag{7}$$

The problem is then how to allocate power and number of bits over each channel in order to fulfil some optimality criterion dictated by the estimation problem, while respecting (7). The optimization problem proposed here is the minimization of the total transmit power subject to the constraint that the final MSE is upper bounded by a given quantity $\epsilon > 0$. Note that, from (7), defining $a_k = \frac{h_k^2}{N_0}$, we can express the number of quantization levels as a function of the transmit power[1]

$$2^{2q_k} = 1 + p_k a_k . \tag{8}$$

Our aim is to minimize the sum of powers transmitted by all the sensors under the constraint

$$\left(\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2 + \sigma_{qk}^2} \right)^{-1} \le \epsilon \tag{9}$$

while guaranteeing (7). This formulation is similar to the one considered in [4] except that we minimize the sum of powers, whereas in [4] is the Euclidean norm of the power vector to be minimized. Hence, substituting (4) in (9) and denoting with

[1] To simplify the problem and derive closed form expressions, we neglect here the discretization of q_k.

$\boldsymbol{p} = (p_1, \ldots, p_N)$ the power vector, our optimization problem can be formulated as follows

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{k=1}^{N} p_k \\ \text{s.t.} \quad & \sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2 + \frac{A^2}{3 \cdot 2^{2q_k}}} \geq \frac{1}{\epsilon} \\ & p_k \geq 0 \qquad \forall\, k = 1, \ldots, N \end{aligned} \tag{10}$$

where q_k is a function of p_k, as in (7). Note that the values q_k are integer so that searching for the optimal integer values q_k leads to an integer programming problem. To relax the problem, here we suppose that the variables q_k are real. Then, by exploiting (8), the optimization problem in (10) can be reformulated as

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{k=1}^{N} p_k \\ \text{s.t.} \quad & \sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2 + \frac{A^2}{3(1 + p_k a_k)}} \geq \frac{1}{\epsilon} \qquad [\mathcal{P}] \\ & \boldsymbol{p} \geq \boldsymbol{0}. \end{aligned} \tag{11}$$

Problem $[\mathcal{P}]$ is indeed a convex optimization problem and it is feasible if $\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2} > \frac{1}{\epsilon}$. Hence the optimal solution of the convex problem $[\mathcal{P}]$ can be found by solving its dual problem, i.e. by imposing the following Karush-Kuhn-Tucker (KKT) conditions

$$\begin{aligned} & 1 - \mu_k - \lambda \frac{3A^2 a_k}{[3\sigma_{nk}^2(1 + p_k a_k) + A^2]^2} = 0 \quad \forall\, k = 1, \ldots, N \\ & 0 \leq \lambda \perp \sum_{k=1}^{N} \frac{3(1 + p_k a_k)}{3\sigma_{nk}^2(1 + p_k a_k) + A^2} - \frac{1}{\epsilon} \geq 0 \\ & 0 \leq \mu_k \perp p_k \geq 0 \quad \forall\, k = 1, \ldots, N \end{aligned} \tag{12}$$

where λ and μ_k denote the Lagrangian multipliers associated to the $N + 1$ constraints. Interestingly, the optimal powers can be expressed in closed form as

$$p_k^* = \left[\frac{1}{\sigma_{nk}^2} \sqrt{\frac{\lambda A^2}{3a_k}} - \frac{1}{a_k} - \frac{A^2}{3a_k \sigma_{nk}^2} \right]^+ \tag{13}$$

where $[x]^+ := \max(0, x)$ and $\lambda > 0$ is found by imposing the MSE constraint (9) to be valid with equality.

Some numerical results are shown next to assess the effectiveness of our approach. In our example, the number of sensors is $N = 10$ and $N_0 = 1$. To guarantee the existence of a solution, we set the bound $\epsilon = \eta \epsilon_{min}$ with $\eta > 1$ and $\epsilon_{min} = \left(\sum_{k=1}^{N} \frac{1}{\sigma_{nk}^2} \right)^{-1}$. In Fig. 1 we report an example of optimal power allocation (bottom plot) obtained by solving the optimization problem $[\mathcal{P}]$, corresponding to a given channel realization (top plot), assuming a constant observation noise variance $\sigma_{nk}^2 = 0.1$. It can be noticed that the optimal solution tends to let only the nodes with the stronger channel coefficients to transmit. Furthermore, in Fig. 2 we plot the sum of the optimal transmit powers versus the number of sensors N, for different values of η. We can see that the optimal transmit power decreases as N increases, as expected, and that, as η gets closer to one, more power is needed because we are requiring the system to behave closer and closer to the ideal communication system.

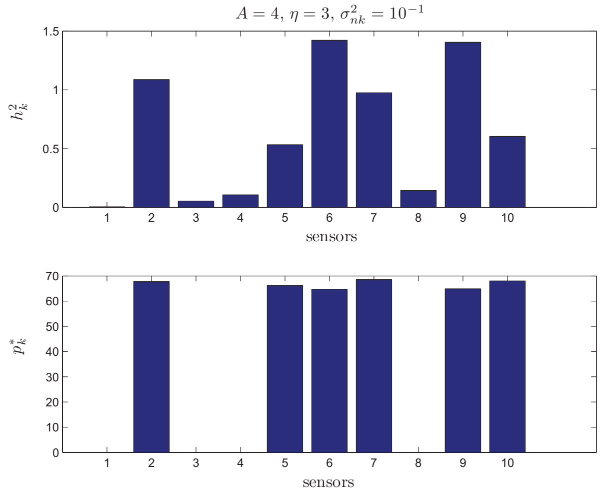

Fig. 1. Optimal power allocation of the sensors for problem $[\mathcal{P}]$ by fixing the per-node observation noise variance.

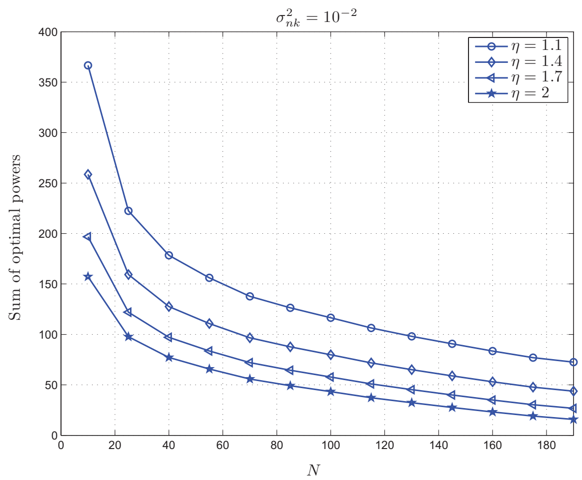

Fig. 2. Sum of optimal powers versus N for several values of η.

3. CENTRALIZED DETECTION OF DETERMINISTIC SIGNAL WITH DECENTRALIZED OBSERVATIONS

We turn now our attention to the decentralized detection of a known deterministic signal $\boldsymbol{s}$ embedded in additive noise. Again, after local quantization, the sensors observations are sent to a fusion center which takes the decision on the basis of the collected data. Let us denote with $\boldsymbol{x} = (x_1, \ldots, x_N)$ the observation vector where x_k represents the observation of node k. Hence, denoting with $\mathcal{H}_0$ and $\mathcal{H}_1$ the two alternative hypotheses, i.e. absence or presence of the signal $\boldsymbol{s}$, the observation model is

$$\boldsymbol{x} \sim \begin{cases} \boldsymbol{n} + \boldsymbol{v} & \text{under } \mathcal{H}_0 \\ \boldsymbol{s} + \boldsymbol{n} + \boldsymbol{v} & \text{under } \mathcal{H}_1 \end{cases}, \tag{14}$$

where $\boldsymbol{s}$ is the useful signal vector, $\boldsymbol{n}$ is the background noise, and $\boldsymbol{v}$ is the quantization noise. Let us assume the noise vector to be Gaussian with zero mean and spatial covariance matrix $\boldsymbol{C}_n$, i.e. $\boldsymbol{n} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{C}_n)$. Additionally, let us consider a dithered quantization so that the quantization error can be modeled as a random process statistically independent of noise. After dithering, the quantization noise variables over different sensors can be assumed to be statistically independent. Hence, we can state that the quantization noise vector $\boldsymbol{v}$ has zero mean and a diagonal covariance matrix $\boldsymbol{C}_q = \mathrm{diag}(\sigma_{q1}^2, \ldots, \sigma_{qN}^2)$. If we suppose, as in the previous section, that the amplitude range of the useful signal is $[-A, A]$ and the number of bits used by node k is q_k, the quantization noise variance at node k is

$$\sigma_{qk}^2 = \frac{\Delta_k^2}{12} = \frac{A^2}{3 \cdot 2^{2q_k}}. \tag{15}$$

Hence, the overall noise $\boldsymbol{n} + \boldsymbol{v}$ has a zero mean and covariance matrix $\boldsymbol{C} = \boldsymbol{C}_n + \boldsymbol{C}_q$.

In the case where the quantization noise can be neglected, the Neyman-Pearson criterion applied to this case leads to the following linear detector [10]

$$\Gamma(\boldsymbol{x}) = \mathcal{R}\{\boldsymbol{s}^H \boldsymbol{C}^{-1} \boldsymbol{x}\} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \tag{16}$$

where $\mathcal{R}(x)$ denotes the real part of x and the detection threshold γ is chosen in order to guarantee the desired false alarm probability P_{fa}. However, when the quantization error is not negligible, the detection rule in (16) is no longer optimal because the composite noise $\boldsymbol{n} + \boldsymbol{v}$ is not Gaussian. Nevertheless, the rule used in (16) is still of interest as it maximizes the signal to noise ratio (SNR). Hence, it is meaningful to evaluate the performance of this detector in the presence of quantization noise. Since the computation of the pdf of $\Gamma(\boldsymbol{x})$ cannot be easily derived, the exact computation of the detection probability in closed form is not easy. Nevertheless, as the number of nodes becomes sufficiently high (an order of a few tens can be sufficient to get a good approximation), we can invoke the central limit theorem to state that $\Gamma(\boldsymbol{x})$ is approximately Gaussian. Using this approximation, we can derive the detection probability in closed form for any fixed P_{fa} as [10]

$$P_d = Q\left[Q^{-1}(P_{fa}) - \sqrt{\boldsymbol{s}^H (\boldsymbol{C}_n + \boldsymbol{C}_q)^{-1} \boldsymbol{s}}\right]. \tag{17}$$

This formula establishes a useful relation between the detection probability and the number of bits allocated to each link. At the same time, we can also use (17) to find out the bit allocation that maximizes the detection probability. Proceeding as in the previous section, we place an upper bound on the maximum number of coding bits per signal sample equal to the capacity of the channel between each sensor and the fusion center. The problem we wish to solve is the maximization of the detection probability, for a given false alarm rate and a maximum global transmit power, under the constraint that the number of bits per sample be less than channel capacity. Denoting with p_k the power transmitted by user k and assuming flat fading channel, with channel coefficient h_k^2, and under the assumption, for simplicity, of spatially uncorrelated noise so that $\boldsymbol{C}_n = \mathrm{diag}(\sigma_{n1}^2, \ldots, \sigma_{nN}^2)$, the capacity is given by (7). From (17), maximizing P_d is equivalent to maximizing $\boldsymbol{s}^H (\boldsymbol{C}_n + \boldsymbol{C}_q)^{-1} \boldsymbol{s}$. Hence, by fixing P_{fa} and using (15), the maximum P_d, for a given global transmit power P_t, can be achieved by finding the power vector $\boldsymbol{p}$ that solves the following constrained problem

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \sum_{k=1}^{N} |s_k|^2 \left(\sigma_{nk}^2 + \frac{A^2}{3(1 + a_k p_k)}\right)^{-1} \\ \text{s.t.} \quad & \sum_{k=1}^{N} p_k \leq P_t, \;\; p_k \geq 0, k = 1, \ldots, N. \end{aligned} \tag{18}$$

It is straightforward to prove that this is a convex problem. Hence, imposing the KKT conditions, we can express the optimal powers in closed form as:

$$p_k = \left[\frac{1}{\sqrt{\lambda}} \sqrt{\frac{|s_k|^2 A^2}{3 a_k \sigma_{nk}^4}} - \frac{A^2}{3 a_k \sigma_{nk}^2} - \frac{1}{a_k}\right]^+ \tag{19}$$

where the Lagrange multiplier λ associated to the sum-power constraint can be determined by enforcing $\sum_{k=1}^{N} p_k = P_T$.

A numerical example is useful to grasp some of the properties of the proposed algorithm. Let us consider a series of sensors placed along a bridge of length L. The goal of the network is to detect one possible spatial resonance, which we represent as the signal $s(z) = A\cos(\pi z/L)$, where $z \in [-L/2, L/2]$ denotes the spatial coordinate. We suppose that the sensors are uniformly spaced along the bridge, at positions $z_k = (k-1)L/N$, with $k = 1, \ldots, N$. The observation

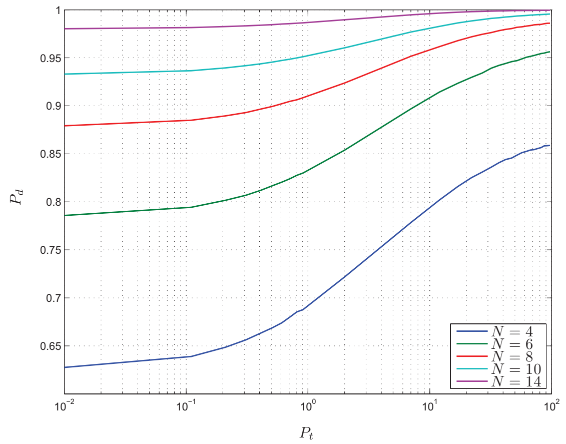

Fig. 3. Detection probability vs. P_t, for various numbers of sensors.

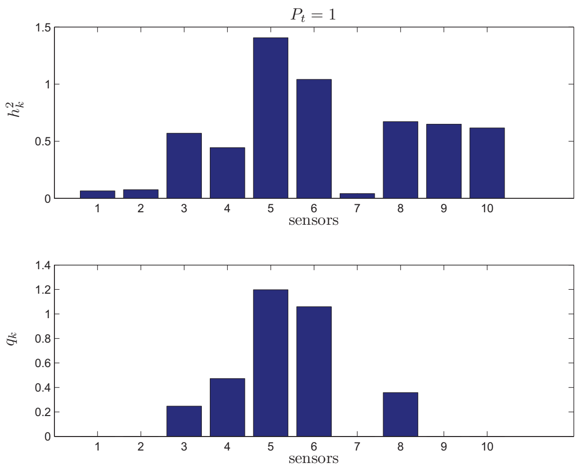

Fig. 4. Per sensor optimal bit allocation.

of each sensor is $x_k = s(z_k) + n_k$, affected by the error n_k. Before transmitting its own measurement to the fusion center, every sensor quantizes the measurement first. The optimal number of bits used by each sensor is found by solving problem (18) in order to get the optimal powers and then the optimal number of bits through (7). As an example, in Fig. 3 we plot the detection probability versus the power budget P_t available to the whole set of sensors, for different numbers N of sensors and $P_{fa} = 0.1$. We can observe that the optimal detection probability increases as the total power budget P_t increases because more bits per symbol can be transmitted so that the quantization error decreases. Additionally, we can notice how the detection probability improves as the number of sensors increases, for any given transmit power. Finally, in Fig. 4 we plot the optimal per channel bit allocation (bottom plot) corresponding to the channels profile h_k^2 shown in the top plot. Interestingly, we can see that, as expected, more bits are allocated to the sensors having the best channels and over the central elements of the array, where the useful signal assumes the largest variations.

4. CONCLUSION

In this paper we have proposed optimal strategies to allocate the number of coding bits for decentralized estimation and detection over rate-constrained transmission channels, in order to meet either an accuracy requirement or to maximize detection probability under a false alarm constraint. The interesting aspect of the proposed strategies is that, once we relax the constraint that the number of bits be an integer number, the resulting problems, in both detection and estimation, become convex problems that can be solved using efficient algorithms. Furthermore, the optimal solutions can be expressed in closed form. This facilitates the computation and the interpretation of the optimal strategies.

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