A FULL-COOPERATIVE DIVERSITY BEAMFORMING SCHEME IN TWO-WAY AMPLIFIED-AND-FORWARD RELAY SYSTEMS

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ABSTRACT

Consider a simple two-way relaying channel in which two single-antenna sources exchange information via a multiple-antenna relay. For such a scenario, all the existing approaches that can achieve full cooperative diversity order are based on antenna/relay selection, for which the difficulty in designing the beamforming lies in the fact that a single beamformer needs to serve two destinations. In this paper, a new full-cooperative diversity beamforming scheme that ensures that the relay signals are coherently combined at both destinations is proposed. Both analytical and numerical results are provided to demonstrate the performance gains.

***Index Terms*—** Two-way relay systems, beamforming, network coding, amplify-and-forward, cooperative diversity

1. INTRODUCTION

Relay-assisted cooperative transmission is an efficient method to extend the coverage and improve the throughput of wireless systems. To characterize this improvement in transmission reliability, the definition of full cooperative diversity order is defined in [1]. To improve the spectrum efficiency of relaying without compromising reliability, the joint design of MIMO and network coding has drawn considerable attention [4–6], especially in MIMO two-way relay systems.

Despite a number of works on MIMO two-way relaying channels, how to design a full cooperative diversity relay beamformer is still an open problem to the best of the authors' knowledge. In this paper, we propose a solution to the addressed problem and the main contribution can be summarized as follows: *Firstly*, a full-cooperative diversity relay beamformer is designed for the scenario under consideration, where the relay signals are coherently combined due to the symmetry of the observation phases at both destinations. *Secondly*, to evaluate the performance of the proposed transmission scheme, the outage probability is analyzed in this paper. *Finally*, the numerical results are provided to verify the accuracy of analytical results, and show the performance gains.

Notation: Vectors are denoted as boldface small letters, i.e., $\mathbf{a}$, and a_m denotes the m-th element of $\mathbf{a}$ and $\mathbf{a}^T$ is the transpose of $\mathbf{a}$. $|c|$ is the Frobenius norm of c, and c can be either a vector or a number. $\mathbb{E}\{x\}$ is the expectation of x. $\Gamma(d)$ is the Gamma function, $\gamma(a,x)$ denotes the lower incomplete Gamma function, $K_\nu(z)$ is the modified Bessel function with imaginary argument, and all the referred special functions follows the form given in [9]. σ^2 is the variance of additive white Gaussian noise at each antenna.

2. SYSTEM MODEL AND PROTOCOL DESCRIPTION

Consider a two-way relay system, in which two sources S_1 and S_2 exchange messages via a relay R. Each source node is equipped with one antenna, while the relay is equipped with N antennas. For simplicity, all nodes are assumed to employ time division duplexing, where the incoming channel and the corresponding outgoing channel are symmetric. All the channels are modeled as quasi-static Rayleigh fading channels, and each node has access to full perfect source-relay channel state information (CSI).

The transmission can be accomplished in two phases by applying network coding. During the first phase, both sources transmit their own messages to the relay simultaneously, and a network coded observation at the relay can be expressed as

$$\mathbf{y}_R = \mathbf{h}x_1 + \mathbf{g}x_2 + \mathbf{n}_R, \tag{1}$$

where x_1 is a message transmitted by S_1, $\mathbb{E}\{x_1^T x_1\} = P_1$, $\mathbf{h}$ denotes an $N \times 1$ channel vector for the link from S_1 to R, x_2 and $\mathbf{g}$ are defined similarly for S_2, the corresponding transmit power is P_2, and $\mathbf{n}_R$ is additive Gaussian noise at the relay. Assuming that an amplify-and-forward (AF) strategy is applied, the relay broadcasts its network coding message after beamforming in the second phase. In particular, the forward message at the relay can be given as $\mathbf{t} = \beta\mathbf{Q}\mathbf{y}_R$, where $\mathbf{Q}$ is a $N \times N$ beamforming matrix at the relay, $\beta = \sqrt{\frac{P_R}{|\mathbf{Qh}|^2P_1+|\mathbf{Qg}|^2P_2+|\mathbf{Q}|^2\sigma^2}}$ is the power normalization factor, and P_R is the relay transmit power. Then the observations at S_1 and S_2 can be expressed respectively, as follows:

$$y_1 = \mathbf{h}^T\mathbf{t} + n_1 = \beta\mathbf{h}^T\mathbf{Q}\mathbf{h}x_1 + \beta\mathbf{h}^T\mathbf{Q}\mathbf{g}x_2 + \beta\mathbf{h}^T\mathbf{Q}\mathbf{n}_R + n_1, \quad (2)$$

$$y_2 = \mathbf{g}^T\mathbf{t} + n_2 = \beta\mathbf{g}^T\mathbf{Q}\mathbf{g}x_2 + \beta\mathbf{g}^T\mathbf{Q}\mathbf{h}x_1 + \beta\mathbf{g}^T\mathbf{Q}\mathbf{n}_R + n_2, \quad (3)$$

where n_i is additive Gaussian noise at S_i, $i = 1, 2$. By subtracting self-interference from observed network coded message, S_i can obtain the desired message.

2.1. Full-Cooperative Diversity Beamforming Design

To provide a full cooperative diversity gain scheme, the beamforming design is proposed in this subsection. Particularly, we further derive the definition of outage probability based on the results given in [1],

$$P^{\text{out}} = \Pr\{R < R_{\text{th}}\} = \Pr\{\text{SNR} < \gamma_{\text{th}}\}, \quad (4)$$

where R is the transmission data rate, R_{th} is set as a threshold for R, SNR is the receive signal-to-noise ratio (SNR) and γ_{th} is its threshold. Then the receive SNRs for S_1 and S_2 based on the observations given in (2) and (3), respectively,

$$\text{SNR}_1 = \frac{\lambda|\mathbf{h}^T\mathbf{Q}\mathbf{g}|^2}{(\lambda+1)|\mathbf{Q}\mathbf{h}|^2 + |\mathbf{Q}\mathbf{g}|^2 + |\mathbf{Q}|^2/\rho}\rho, \quad (5)$$

$$\text{SNR}_2 = \frac{\lambda|\mathbf{g}^T\mathbf{Q}\mathbf{h}|^2}{(\lambda+1)|\mathbf{Q}\mathbf{g}|^2 + |\mathbf{Q}\mathbf{h}|^2 + |\mathbf{Q}|^2/\rho}\rho, \quad (6)$$

where the transmit power is set as $P_1 = P_2 = P$ at S_1 and S_2, $P_R = \lambda P$ is the transmit power at the relay, and average SNR is defined as $\rho = P/\sigma^2$.

The key step to achieve full cooperative diversity gain is to design the relay beamforming matrix $\mathbf{Q}$. In particular, we focus on the signal parts of SNR_1 and SNR_2,

$$\mathbf{h}^T\mathbf{Q}\mathbf{g} = \sum_{m=1}^{N}\sum_{n=1}^{N} q_{mn}|h_m||g_n|e^{j(\phi_m+\theta_n)}, \quad (7)$$

$$\mathbf{g}^T\mathbf{Q}\mathbf{h} = \sum_{m=1}^{N}\sum_{n=1}^{N} q_{mn}|g_m||h_n|e^{j(\theta_m+\phi_n)}, \quad (8)$$

where ϕ_p denotes the argument of h_p and θ_q is defined similarly for g_q. By letting $\mathbf{Q}$ as a diagonal matrix with its diagonal elements as $e^{-j(\theta_m+\phi_n)}$, we can ensure that coherent combination at both sources. Then the beamforming matrix can be defined as follows:

$$\mathbf{Q} = (q_{mn})_{N\times N} = \left\{ \begin{array}{l} e^{-j(\phi_m+\theta_n)},\ m = n \\ 0,\ m \neq n \end{array} \right. . \quad (9)$$

Substituting (9) into (5) and (6), the SNRs at S_1 and S_2 can be further derived as

$$\text{SNR}_1 = \frac{\lambda\big(\sum_{m=1}^{N}|h_m||g_m|\big)^2}{(\lambda+1)|\mathbf{h}|^2 + |\mathbf{g}|^2 + N/\rho}\rho, \quad (10)$$

$$\text{SNR}_2 = \frac{\lambda\big(\sum_{m=1}^{N}|h_m||g_m|\big)^2}{(\lambda+1)|\mathbf{g}|^2 + |\mathbf{h}|^2 + N/\rho}\rho. \quad (11)$$

By combining the signal from both paths coherently, our proposed beamforming scheme can achieve full cooperative diversity gain, which is demonstrated in the next section.

3. PERFORMANCE ANALYSIS

In this section, the performance of the proposed full-cooperative diversity beamforming design is evaluated. Firstly, a tractable upper bound on outage probability is derived, and then a closed-form expression for it can be obtained. Next an asymptotic high-SNR expression for the derived upper bound is analyzed, which demonstrates the cooperative diversity gain of our proposed scheme.

3.1. Upper Bound of P^{out} for Proposed Scheme

To find a feasible method to evaluate the outage probability of proposed scheme, we first derive a tractable upper bound for the SNRs, and the following lemma is presented.

Lemma 1 *Denoting that* $\mathbf{a} = \big[a_1 \cdots a_N\big]^T$ *and* $\mathbf{b} = \big[b_1 \cdots b_N\big]^T$, *where* a_m *and* b_n *follow independent and identical zero-mean Gaussian distributions with variances* ν, *and* $\nu \leqslant 1$, *the random variable* $w = \frac{\left(\sum_{m=1}^{N}|a_m||b_m|\right)^2}{\frac{1}{N}\left(\sum_{m=1}^{N}|a_m|^2\right)\left(\sum_{n=1}^{N}|b_n|^2\right)}$ *can be bounded as follows with probability 1:*

$$\lim_{N\to\infty}\Pr\{w \geqslant 1\} = \lim_{N\to\infty}\Pr\left\{w = 1 + \frac{1}{2}(N-1)\pi^2\right\} = 1. \quad (12)$$

Proof: Due to the definition of convergence with probability 1, it is equivalent to prove the following equation for any given positive number ϵ:

$$\lim_{N\to\infty}\Pr\Bigg\{\bigg|\underbrace{w - \Big(1 + \frac{1}{2}(N-1)\pi^2\Big)}_{\mathcal{A}}\bigg| < \epsilon\Bigg\} = 1. \quad (13)$$

Denoting $u = \frac{1}{N\nu^2}\big(\sum_{m=1}^{N}|a_m||b_m|\big)^2$, $\Pr\big\{|\mathcal{A}| < \epsilon\big\}$ can be further developed as

$$\begin{aligned} & \Pr\big\{|\mathcal{A}| < \epsilon\big\} \\ = \ & \Pr\Bigg\{\bigg|(\underbrace{w-u}_{\mathcal{B}_1}) + \Big[\underbrace{u - \Big(1+\frac{1}{2}(N-1)\pi^2\Big)}_{\mathcal{B}_2}\Big]\bigg| < \epsilon\Bigg\}. \\ \geqslant \ & \Pr\Big\{|\mathcal{B}_1| < \frac{\epsilon}{2}, |\mathcal{B}_2| < \frac{\epsilon}{2}\Big\} \\ \overset{(a)}{\geqslant} \ & \Pr\Big\{|\mathcal{B}_1| < \frac{\epsilon}{2}\Big\} + \Pr\Big\{|\mathcal{B}_2| < \frac{\epsilon}{2}\Big\} - 1, \end{aligned} \quad (14)$$

where inequality (a) in (14) can be obtained directly by using Boole's inequality [10]. Then we need to demonstrate that $\Pr\left\{\left|\mathcal{B}_1\right| < \frac{\epsilon}{2}\right\}$ and $\Pr\left\{\left|\mathcal{B}_2\right| < \frac{\epsilon}{2}\right\}$ are both convergent with probability 1, respectively, which are given as follows.

The proof of $\lim_{N\to\infty} \Pr\left\{\left|\mathcal{B}_1\right| < \frac{\epsilon}{2}\right\} = 1$: $\Pr\left\{\left|\mathcal{B}_1\right| < \frac{\epsilon}{2}\right\}$ can be bounded as follows by using the Cauchy-Schwarz inequality $(\sum_{m=1}^{N}|a_m||b_m|)^2 \leqslant (\sum_{m=1}^{N}|a_m|^2)(\sum_{n=1}^{N}|b_n|^2)$,

$$\Pr\left\{\left|\mathcal{B}_1\right| < \frac{\epsilon}{2}\right\} \geqslant$$
$$\Pr\left\{\frac{N}{\nu^2}\left|\frac{1}{N^2}\left(\sum_{m=1}^{N}|a_m|^2\right)\left(\sum_{n=1}^{N}|b_n|^2\right) - \nu^2\right| < \frac{\epsilon}{2}\right\}. \tag{15}$$

Based on the law of large numbers [10], we can obtain that

$$\lim_{N\to\infty}\Pr\left\{\left|\mathcal{B}_1\right| < \frac{\epsilon}{2}\right\} \geqslant \lim_{N\to\infty}\Pr\left\{\left|\frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N}|a_m|^2 \times|b_n|^2 - \nu^2\right| < \frac{\nu^2}{2N}\epsilon\right\} = 1. \tag{16}$$

The proof of $\lim_{N\to\infty} \Pr\left\{\left|\mathcal{B}_2\right| < \frac{\epsilon}{2}\right\} = 1$: u can be expanded as follows:

$$u = \frac{1}{N\nu^2}\sum_{m=1}^{N}|a_m|^2|b_m|^2 + \frac{1}{N\nu^2}\sum_{m\neq n}2|a_m||b_m||a_n||b_n|. \tag{17}$$

Due to the law of large numbers, both expanded parts of u in (17) also approach constants as follows when N is large:

$$\lim_{N\to\infty}\Pr\left\{\underbrace{\left|\frac{1}{N\nu^2}\sum_{m=1}^{N}|a_m|^2|b_m|^2 - 1\right|}_{\mathcal{C}_1} < \frac{\epsilon}{4}\right\} = 1, \tag{18}$$

$$\lim_{N\to\infty}\Pr\left\{\left|\mathcal{C}_2\right| < \frac{\epsilon}{4}\right\} = 1, \tag{19}$$

where $\mathcal{C}_2 = \frac{1}{N\nu^2}\sum_{m\neq n}2|a_m||b_m||a_n||b_n| - \frac{1}{2}(N-1)\pi^2$ and (19) follows the fact that $|a_p|$ and $|b_q|$ are independent Rayleigh distribution variables. Then the convergence of $\Pr\left\{\left|\mathcal{B}_2\right| < \frac{\epsilon}{2}\right\}$ can be proved by following (14):

$$\lim_{N\to\infty}\Pr\left\{\left|\mathcal{B}_2\right| < \frac{\epsilon}{2}\right\} \geqslant \lim_{N\to\infty}\Pr\left\{\left|\mathcal{C}_1\right| < \frac{\epsilon}{4}\right\} + \lim_{N\to\infty}\Pr\left\{\left|\mathcal{C}_2\right| < \frac{\epsilon}{4}\right\} - 1 = 1, \tag{20}$$

where inequalities (a) and (b) in (20) can be obtained by following (14).

Based on (14), (16) and (20), we can prove that the inequality $w \geqslant 1$ can be almost surely (a.s.) established, and the lemma has been proved. □

Based on Lemma 1, the lower bound of the receive SNRs can be given as follows:

$$\mathrm{SNR}_1 \overset{\text{a.s.}}{\geqslant} \frac{\lambda|\mathbf{h}|^2|\mathbf{g}|^2}{N(\mu|\mathbf{h}|^2 + |\mathbf{g}|^2 + N/\rho)}\rho, \tag{21}$$

$$\mathrm{SNR}_2 \overset{\text{a.s.}}{\geqslant} \frac{\lambda|\mathbf{h}|^2|\mathbf{g}|^2}{N(\mu|\mathbf{g}|^2 + |\mathbf{h}|^2 + N/\rho)}\rho, \tag{22}$$

where $\mu = \lambda+1$. Then an upper bound of P^{out} can be derived in closed-form, which is presented in the next theorem.

Theorem 2 *An upper bound on outage probability for proposed scheme in Section II is given by* (23), *where* $a = \frac{\gamma_{\text{th}}}{\rho}$ *and* $b = N^2\gamma_{\text{th}}(\mu\gamma_{\text{th}} + \lambda)$.

Proof: Without loss of the generality, the upper bound on outage probability for S_1 is derived. Denoting that $x_1 = |\mathbf{h}|^2$ and $x_2 = |\mathbf{g}|^2$, it can be written as

$$P^{\text{out-up}} = \Pr\left\{x_2 < \frac{\mu N a}{\lambda}\right\} + \Pr\left\{0 < x_1 < \frac{N\gamma_{\text{th}}x_2 + N^2 a}{\lambda\rho x_2 - \mu N\gamma_{\text{th}}}, x_2 > \frac{\mu N a}{\lambda}\right\} = \mathcal{K}_1 + \mathcal{K}_2. \tag{24}$$

As introduced previously, x_1 and x_2 are two independent chi-squared random variables, $\mathcal{K}_1$ and $\mathcal{K}_2$ can be obtained,

$$\mathcal{K}_1 = \int_0^{\frac{\mu N a}{\lambda}}\frac{x_2^{N-1}e^{-x_2}}{\Gamma(N)}dx_2 = \frac{1}{\Gamma(N)}\gamma\left(N, \frac{\mu N a}{\lambda}\right), \tag{25}$$

$$\mathcal{K}_2 = \frac{1}{[\Gamma(N)]^2}\int_{\frac{\mu N a}{\lambda}}^{\infty}\gamma\left(N, \frac{N\gamma_{\text{th}}x_2 + N^2 a}{\lambda\rho x_2 - \mu N\gamma_{\text{th}}}\right)x_2^{N-1}e^{-x_2}dx_2. \tag{26}$$

To obtain the closed-form expression, (26) can be expanded as follows by denoting $z = \frac{N\gamma_{\text{th}}x_2 + N^2 a}{\lambda\rho x_2 - \mu N\gamma_{\text{th}}}$,

$$\begin{aligned}\mathcal{K}_2 &= \frac{b}{\Gamma(N)}\int_{\frac{Na}{\lambda}}^{\infty}\frac{\left(\mu N\gamma_{\text{th}}z + N^2 a\right)^{N-1}}{\left(\lambda\rho z - N\gamma_{\text{th}}\right)^{N+1}}e^{-\frac{\mu N\gamma_{\text{th}}z + N^2 a}{\lambda\rho z - N\gamma_{\text{th}}}}dz \\ &-\frac{b}{\Gamma(N)}\sum_{i=0}^{N-1}\int_{\frac{Na}{\lambda}}^{\infty}\frac{z^i\left(\mu N\gamma_{\text{th}}z + N^2 a\right)^{N-1}}{\Gamma(i)\left(\lambda\rho z - N\gamma_{\text{th}}\right)^{N+1}}\times \\ &e^{-\left(\frac{\mu N\gamma_{\text{th}}z + N^2 a}{\lambda\rho z - N\gamma_{\text{th}}} + z\right)}dz = \mathcal{L}_1 - \mathcal{L}_2,\end{aligned} \tag{27}$$

where $\mathcal{K}_2$ is simplified by applying $\gamma(N,z) = \Gamma(N)\left[1 - e^{-z}\left(\sum_{i=0}^{N-1}\frac{z^i}{\Gamma(i)}\right)\right]$ in [9]. Then we focus on deriving the closed-form expressions for $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively. Particularly denoting $t = \lambda\rho z - N\gamma_{\text{th}}$, $\mathcal{L}_1$ can be derived as

$$\begin{aligned}\mathcal{L}_1 &= \frac{be^{-\frac{\mu N a}{\lambda}}}{\Gamma(N)\lambda^N\rho^N}\int_0^{\infty}\frac{(\mu N\gamma_{\text{th}}t + b)^{N-1}}{t^{N+1}}e^{-\frac{b}{\lambda\rho t}}dt \\ &= \sum_{j=0}^{N-1}\binom{N-1}{j}\frac{\Gamma(N-j-1)(\mu N\gamma_{\text{th}})^j\lambda^{N-j}}{\Gamma(N)}\frac{e^{-\frac{\mu N a}{\lambda}}}{\rho^j},\end{aligned} \tag{28}$$

$$P^{\text{out-up}} = \frac{1}{\Gamma(N)}\gamma\left(N, \frac{\mu N a}{\lambda}\right) + \sum_{j=0}^{N-1}\binom{N-1}{j}\frac{\Gamma(N-j-1)(\mu N\gamma_{\text{th}})^j\lambda^{N-j}}{\Gamma(N)}\frac{e^{-\frac{\mu N a}{\lambda}}}{\rho^j} - \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1} 2\binom{i}{k}\binom{N-1}{l}\frac{\mu^l N^{i+l-k} b^{\frac{N+k-l}{2}}\gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)\lambda^{N+i}} K_{k+l-N}\left(\frac{2}{\lambda\rho}\sqrt{b}\right)\frac{e^{-\frac{(\mu+1)Na}{\lambda}}}{\rho^{N+i}} \quad (23)$$

$$\begin{aligned}\mathcal{L}_2 &= \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1}\binom{i}{k}\binom{N-1}{l}\frac{\mu^l N^{i+l-k} b^{N-l}\gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)\lambda^{N+i}}\frac{e^{-\frac{(\mu+1)Na}{\lambda}}}{\rho^{N+i}}\int_0^\infty t^{k+l-N-1}e^{-\left(\frac{b}{\lambda\rho t}+\frac{t}{\lambda\rho}\right)}dt \\ &= \sum_{i=0}^{N-1}\sum_{k=0}^{i}\sum_{l=0}^{N-1} 2\binom{i}{k}\binom{N-1}{l}\frac{\mu^l N^{i+l-k} b^{\frac{N+k-l}{2}}\gamma_{\text{th}}^{i+l-k}}{\Gamma(N)\Gamma(i)\lambda^{N+i}} K_{k+l-N}\left(\frac{2}{\lambda\rho}\sqrt{b}\right)\frac{e^{-\frac{(\mu+1)Na}{\lambda}}}{\rho^{N+i}} \quad (29)\end{aligned}$$

where $\mathcal{L}_1$ is simplified by using the binomial theorem. Following the notation in (28), a closed-form expression for $\mathcal{L}_2$ can be obtained in a similar way, which is given by (29). Then the proof is complete. □

3.2. Analysis of Cooperative diversity Gain

To derive the achievable cooperative diversity gain of the proposed scheme, an asymptotic analysis of $P^{\text{out-up}}$ is given.

3.2.1. High SNR Approximation of $\mathcal{K}_1$ in (24)

In the high SNR region, $\frac{\mu N a}{\lambda}$ approaches 0, and the lower incomplete gamma function in $\mathcal{K}_1$ achieves the following asymptotic form:

$$\gamma\left(N, \frac{\mu N a}{\lambda}\right) \to \frac{(\mu N\gamma_{\text{th}})^N}{N}\frac{1}{\rho^N}, \quad (30)$$

and (30) can be obtained directly by using L'Hôpital's rule on the definition of lower incomplete gamma function given by Eq. (8.350.1) in [9] when its argument approaches zero. Then a high SNR approximation to $\mathcal{K}_1$ can be derived as

$$\mathcal{K}_1^\infty = \frac{(\mu N\gamma_{\text{th}})^N}{N\Gamma(N)}\frac{1}{\rho^N}. \quad (31)$$

3.2.2. High SNR Approximation of $\mathcal{K}_2$ in (24)

To derive an accurate approximation, we begin with the expression for $\mathcal{K}_2$ given in (26). When ρ is in high SNR region, the argument of the lower incomplete gamma function in (26) approaches

$$\frac{N\gamma_{\text{th}}x_2 + N^2 a}{\lambda\rho x_2 - \mu N\gamma_{\text{th}}} = \frac{N\gamma_{\text{th}}x_2 + (N^2\gamma_{\text{th}}/\rho)}{\rho(\lambda x_2 - (\mu N\gamma_{\text{th}}/\rho))} \to \frac{N\gamma_{\text{th}}}{\lambda\rho}. \quad (32)$$

Then the approximation of $\mathcal{K}_2$ can be derived as follows by substituting (32) into (26):

$$\begin{aligned}\mathcal{K}_2^\infty &= \frac{1}{[\Gamma(N)]^2}\gamma\left(N, \frac{N\gamma_{\text{th}}}{\lambda\rho}\right)\int_{\frac{\mu N a}{\lambda}}^\infty x_2^{N-1}e^{-x_2}dx_2 \\ &= \frac{1}{[\Gamma(N)]^2}\gamma\left(N, \frac{N\gamma_{\text{th}}}{\lambda\rho}\right)\left[1-\gamma\left(N, \frac{\mu N a}{\lambda}\right)\right]. \quad (33)\end{aligned}$$

Similar to the approximation of $\mathcal{K}_1$, the asymptotic form of $\mathcal{K}_2$ can be given as follows:

$$\begin{aligned}\mathcal{K}_2^\infty &= \frac{N^{N-1}\gamma_{\text{th}}^N}{\lambda^N[\Gamma(N)]^2}\left[1-\frac{(\mu N\gamma_{\text{th}})^N}{N\lambda^N}\frac{1}{\rho^N}\right]\frac{1}{\rho^N} \\ &= \frac{N^{N-1}\gamma_{\text{th}}^N}{\lambda^N[\Gamma(N)]^2}\frac{1}{\rho^N} + o\left(\frac{1}{\rho^N}\right). \quad (34)\end{aligned}$$

Based on the high SNR approximation of $\mathcal{K}_1$ and $\mathcal{K}_2$, an asymptotic form of $P^{\text{out-up}}$ is given as

$$P_\infty^{\text{out-up}} = \frac{(\mu N\gamma_{\text{th}})^N}{N\Gamma(N)}\frac{1}{\rho^N} + \frac{N^{N-1}\gamma_{\text{th}}^N}{\lambda^N[\Gamma(N)]^2}\frac{1}{\rho^N} + o\left(\frac{1}{\rho^N}\right). \quad (35)$$

Such a result demonstrates that the proposed scheme can achieve full cooperative diversity gain.

4. NUMERICAL RESULTS

In this section, numerical results are provided, where the numbers of relay antennas are given as $N = 2, 3, 4$, respectively, and $r_{\text{th}} = 1$. Fig. 1 gives numerical results for the derived upper bound on the outage probability and the corresponding asymptotic analysis. The simulation results show that the derived upper bound is quite tight in the high SNR region, especially when N is small.

To further evaluate the outage performance of the proposed scheme, a comparison of the antenna selection scheme

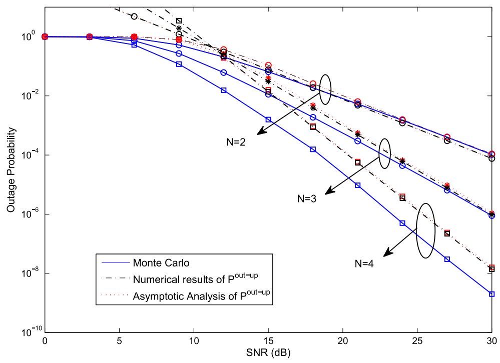

Fig. 1. The derived upper bound on the outage probability, $N = 2, 3, 4$.

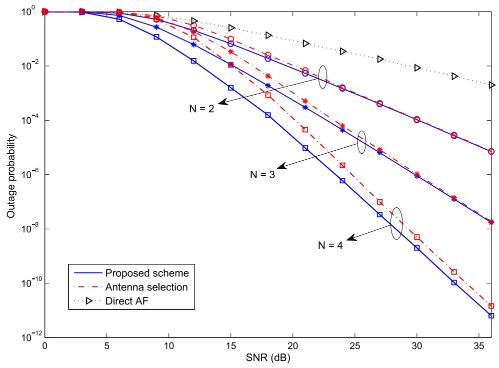

Fig. 2. The outage probability performance, $N = 2, 3, 4$. bit/s/Hz.

and the direct AF scheme for the studied scenario is provided in Fig. 2. As shown in the figure, the proposed scheme can achieve better performance than the antenna selection scheme, especially in the low SNR region. The performance gain achieved by the proposed beamforming scheme is analog to the advantage of maximal ratio combining (MRC) over selection combining in conventional single-input multiple-output systems. It has been verified in [8] that MRC can achieve full-diversity, and has better performance than selection combining, which is consistent with our results.

5. CONCLUSION

To achieve full-cooperative diversity gain, a joint beamforming and network coding scheme has been proposed for two-way relay systems. By combining the messages from different paths, transmission reliability can be improved. A closed-form upper bound on the outage probability has been derived, and a high SNR asymptotic analysis has also been given to demonstrate the achieved cooperative diversity gain. Simulation results have been provided to verify the derived analytical results, which also show that our proposed scheme outperforms the antenna selection scheme in the studied scenario.

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