ON THE USE OF ZERO PADDING WITH DISCRETE COSINE TRANSFORM TYPE-II IN MULTICARRIER COMMUNICATIONS

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ABSTRACT

In this work, the problem of applying Zero Padding (ZP) as redundancy in multicarrier communications is addressed. To this goal, a general matrix formulation to recover the transmitted symbol when ZP is used, is provided for any kind of discrete transform employed at both the transmitter and the receiver. The obtained result not only generalizes some previously reported techniques, such as discrete Fourier transform-based transceivers, but it also allows to extend it to other kind of transforms (e.g., discrete trigonometric transforms). As a particular case study, the use of discrete cosine transform Type-II even (DCT2e) is analyzed. In this case, a simple structure that recover the transmitted symbol at the receiver is also shown. Additionally, the expressions of the one-tap per subcarrier coefficients, also using the DCT2e, are derived.

***Index Terms*—** Multicarrier Modulation (MCM), Zero padding (ZP), Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Orthogonal Frequency-Division Multiplexing (OFDM).

1. INTRODUCTION

In digital multicarrier communications, Zero Padding (ZP) is one form of redundancy inserted in the transmitted sequence to avoid interblock interference. In DFT-Based OFDM systems, the effect of ZP has been previously studied in [1]. In a recent work [2], ZP has been also analyzed when using the Type-IV even Discrete Cosine Transform (DCT4e). In both approaches, some procedures have been proposed to handle the received vector, in order to recover the original signal.

In this work, we present a general formulation using matrices when ZP is inserted as redundancy in the transmitted sequence. We address the problem of obtaining a condition under which the original data can be reconstructed via one-tap per subcarrier frequency-domain equalizer (FEQ) at the receiver.

Furthermore, we apply it to a very interesting case, say, when Discrete Cosine Transform Type-II even (DCT2e) is used at both the transmiter and the receiver sides. DCT2e has been widely used in the context of multicarrier modulation (MCM), and it has been analyzed in many works as an alternative to DFT, due to its good properties (e.g., good performance under carrier frequency offset) [3–7]. So it is an interesting problem to be considered: How to handle ZP when using DCT2e in the transmission system. In [4], the effect of ZP is analyzed when using DCT2e over fast-fading channels, in order to suppress inter-symbol-interference (ISI); but there is a lack of a procedure which guarantees the reconstruction of the original signal by means of FEQ. In this work we provide that procedure, and show that it is as simple as theoretically sound. It is one of the main contributions of this paper.

This paper is organized as follows: In Section 2, we present the general formulation for the use of ZP jointly with any discrete transform in MCM. We also show that from this general result we derive some previously known techniques. In Section 3 we address the problem of using ZP with DCT2e, and obtain a new procedure for its implementation with ZP. We also present a simple expression for the 1-tap equalizers in this case. Section 4 contains the main conclusions.

Notation: Throughout the paper, superscript ^{T} stands for transposition, $\mathbf{I}_k$ and $\mathbf{J}_k$ denote respectively the identity matrix and the antidiagonal permutation matrix of order k, and $\mathbf{0}$ is the null matrix.

2. GENERAL FORMULATION OF THE ZP PROCEDURE IN MCM

We consider a channel whose impulse response is $\mathbf{h_{ch}} = (h_{-\nu}, \cdots, h_{-1}, h_0, h_1, \cdots, h_\nu)$, of length $2\nu+1$. Let $\mathbf{x}$ be the information symbol of size N and $\mathbf{x}_{zp}$ denote the zero-padded extension of the original symbol; if we append 4ν zeros to $\mathbf{x}$, then $\mathbf{x}_{zp}$ has length $N+4\nu$. If we transmit $\mathbf{x}_{zp}$, then the receiving symbol $\mathbf{y}_{zp}$ may be written in matricial form as $\mathbf{y}_{zp} = \mathbf{H} \cdot \mathbf{x}_{zp} + \mathbf{n}$ where $\mathbf{n}$ is a term related to the additive noise, and the transmission matrix $\mathbf{H}$ is the Toeplitz matrix of

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size $(N+2\nu)\times(N+4\nu)$ defined as

$$\mathbf{H}=\left[\begin{array}{cccccccc} h_\nu & \cdots & h_0 & \cdots & h_{-\nu} & 0 & \cdots & 0 \\ 0 & h_\nu & \ddots & h_0 & \ddots & h_{-\nu} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_\nu & \cdots & h_0 & \cdots & h_{-\nu} \end{array}\right]. \tag{1}$$

As 4ν coefficients of $\mathbf{x}_{zp}$ are null, we can rewrite this expression as $\mathbf{y}_{zp}=\mathbf{H}_m\cdot\mathbf{x}+\mathbf{n}$ where $\mathbf{H}_m$ is formed by the N columns of $\mathbf{H}$ which are not multiplied by the 4ν null components of $\mathbf{x}_{zp}$. Hence, $\mathbf{H}_m$ has size $(N+2\nu)\times N$. The received symbol $\mathbf{y}_{zp}$ has length $N+2\nu$ and notice that it equals the convolution of $\mathbf{h_{ch}}$ with the symbol $\mathbf{x}$. We would like to modify $\mathbf{y}_{zp}$ in order to recover $\mathbf{x}$.

With this in mind, we propose the general scheme in Fig. 1: Let us modify $\mathbf{y}_{zp}$ in order to get $\mathbf{y}=\mathbf{M}\cdot\mathbf{y}_{zp}$ of length N. This means that $\mathbf{M}$ is an $N\times(N+2\nu)$ matrix, and

$$\mathbf{y}=\mathbf{M}\cdot\mathbf{y}_{zp}=\mathbf{M}\cdot\mathbf{H}_m\cdot\mathbf{x}+\mathbf{M}\cdot\mathbf{n}. \tag{2}$$

Thus, the whole transformation can be rewritten as

$$\mathbf{y}=\mathbf{H}_{equiv}\cdot\mathbf{x}+\mathbf{M}\cdot\mathbf{n} \tag{3}$$

where $\mathbf{H}_{equiv}=\mathbf{M}\cdot\mathbf{H}_m$ is the equivalent channel matrix, which is a square matrix of order N.

Now, if we apply a discrete transform $\mathbf{T}^{-1}$ at the transmitter and $\mathbf{T}$ at the receiver (as in Fig.1), the aim is to obtain $\mathbf{M}$ such that $\mathbf{H}_{equiv}$ is diagonalized by $\mathbf{T}$. In that case, $\mathbf{H}_{equiv}=\mathbf{T}^{-1}\cdot\mathbf{D}\cdot\mathbf{T}$ with $\mathbf{D}$ diagonal, and

$$\begin{aligned}\mathbf{y}&=\mathbf{T}^{-1}\cdot\mathbf{D}\cdot\mathbf{T}\cdot\mathbf{x}+\mathbf{M}\cdot\mathbf{n}\\ &\Rightarrow\mathbf{T}\cdot\mathbf{y}=\mathbf{D}\cdot\mathbf{T}\cdot\mathbf{x}+\mathbf{T}\cdot\mathbf{M}\cdot\mathbf{n}.\end{aligned}$$

By denoting $\mathbf{Y}:=\mathbf{T}\cdot\mathbf{y}$, $\mathbf{X}:=\mathbf{T}\cdot\mathbf{x}$, and $\mathbf{N}:=\mathbf{T}\cdot\mathbf{M}\cdot\mathbf{n}$ we get the scheme

$$\mathbf{Y}=\mathbf{D}\cdot\mathbf{X}+\mathbf{N}$$

where $\mathbf{D}=diag\,(d_0,\ldots,d_{N-1})$ contains the eigenvalues of $\mathbf{H}_{equiv}$. If they are nonzero, we obtain an estimation of $\mathbf{X}$,

$$\hat{\mathbf{X}}_k=\mathbf{Y}_k/d_k$$

by means of the 1-tap filter d_k (see Fig. 1).

In summary, the question is: *How to design* $\mathbf{M}$ *such that the matrix* $\mathbf{H}_{equiv}=\mathbf{M}\cdot\mathbf{H}_m$ *can be diagonalized via a prescribed transformation* $\mathbf{T}$? Of course, it depends on the choice of the discrete transformation $\mathbf{T}$. Let us recall two cases studied in the literature in the next subsections.

2.1. ZP-OFDM-OLA

The use of ZP for DFT-based MCM is studied in [1]. The authors propose the following procedure when using DFT as the transform $\mathbf{T}$: First, the original vector symbol is zero–padded before transmission; then, at the receiver it suffices to make the modification $\mathbf{y}=\mathbf{M}\cdot\mathbf{y}_{zp}$ to the received vector, which consists on taking the last 2ν components of $\mathbf{y}_{zp}$ and summing them to the first 2ν components of $\mathbf{y}_{zp}$. In other words, the procedure is an *overlap-and-add* (OLA), and it is performed by the modification matrix

$$\mathbf{M}=\left[\begin{array}{ccc}\mathbf{I}_{2\nu} & \mathbf{0} & \mathbf{I}_{2\nu}\\ \mathbf{0} & \mathbf{I}_{N-2\nu} & \mathbf{0}\end{array}\right] \tag{4}$$

In this way, it is guaranteed that $\mathbf{H}_{equiv}=\mathbf{M}\cdot\mathbf{H}_m$ is a circulant matrix of order N, hence diagonalizable by the DFT. Moreover, its eigenvalues d_k are computed via the N-point DFT of the zero-padded filter $\mathbf{h}$. This is the summary of the so called ZP-OFDM-OLA algorithm, which is a particular case of the proposed general scheme in Fig. 1, when choosing $\mathbf{T}$ =DFT, and setting the block given by matrix $\mathbf{M}$ of Eq. (4) at the receiver.

2.2. ZP-DCT4-MIAS

In [2], the use of ZP for Discrete Cosine Transform Type-IV even (DCT4e)-based MCM is studied. As the filter $\mathbf{h}$ must be symmetric $(h_{-k}=h_k,\ k=1,\ldots,\nu)$, it is needed to include a prefilter $\mathbf{w}$ as first stage at the receiver (see Fig. 1) as explained in [3]. With this assumption, it is shown that it suffices to append 2ν zeros to the left and to the right of the original symbol $\mathbf{x}$ in order to get $\mathbf{x}_{zp}$. This means that $\mathbf{H}_m$ is the matrix whose columns are the N central columns of $\mathbf{H}$:

$$\mathbf{H}_m=\left[\begin{array}{cccc} h_\nu & 0 & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ h_0 & \ddots & \ddots & 0\\ \vdots & \ddots & \ddots & h_\nu\\ \vdots & \ddots & \ddots & \vdots\\ h_\nu & \ddots & \ddots & h_0\\ 0 & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & 0 & h_\nu \end{array}\right]. \tag{5}$$

Then, the received block is multiplied by a matrix defined as

$$\mathbf{M}=\left[\begin{array}{ccccc}\mathbf{J}_\nu & \mathbf{I}_\nu & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{N-2\nu} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_\nu & -\mathbf{J}_\nu\end{array}\right]. \tag{6}$$

This multiplication $\mathbf{y}=\mathbf{M}\cdot\mathbf{y}_{zp}$ can be explained as follows: The first ν components of $\mathbf{y}_{zp}$ are symmetrized as in a mirror, and added to their adjacent (next) ν components; in an analogous way, the last ν components of $\mathbf{y}_{zp}$ are also symmetrized as in a mirror, and substracted from their adjacent

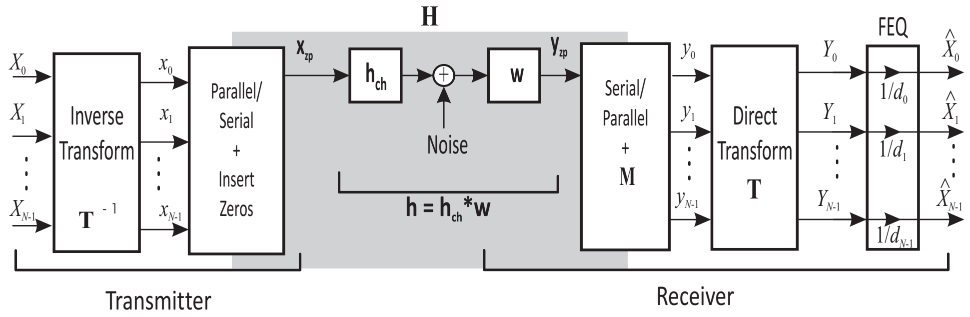

Fig. 1. Block diagram for a ZP-based MCM transceiver.

(previous) ν components. In other words, it is a procedure of *mirror and add/substract,* so it has been renamed as MIAS algorithm.

This procedure guarantees that the matrix $\mathbf{H}_{equiv} = \mathbf{M} \cdot \mathbf{H}_m$ is diagonalized by the DCT4e transform; moreover, the eigenvalues d_k can be computed as the Discrete Cosine Transform Type-III even (DCT3e) of the zero-padded vector of length N, $[h_0, h_1, ..., h_\nu, 0, ..., 0]$.

In summary, the MIAS procedure proposed in [2] corres–ponds to our general formulation, for the particular case of $\mathbf{T}$ =DCT4e, choosing the modification block given by matrix $\mathbf{M}$ of Eq. (6) at the receiver.

3. ZP AND DIAGONALIZATION VIA DCT2 EVEN

We will now focus on the study of the ZP when using the Discrete Cosine Transform Type-II even (DCT2e). Let us first recall the definition of the DCT2e matrix $\mathbf{C}_{2e}$ [8, 9]:

$$(\mathbf{C}_{2e})_{k,j} = 2\cos\left(\frac{k(2j+1)\pi}{2N}\right), \quad 0 \le k, j \le N-1.$$

Following the general formulation given in Section 2, the aim of this Section is to modify the channel matrix $\mathbf{H}$ of Eq. (1) into a new matrix $\mathbf{H}_{equiv}$ in order to render it diagonalizable via DCT2e. It is known that $\mathbf{C}_{2e}$ diagonalizes symmetric matrices that can be decomposed into the sum of a Toeplitz matrix and a Hankel one with related entries (see [8]).

With this in mind, let us explain how to achieve an equivalent channel matrix $\mathbf{H}_{equiv}$ diagonalizable by $\mathbf{C}_{2e}$. First of all, we must impose symmetry of the filter $\mathbf{h}$, $(h_{-k} = h_k)$ so as to get a symmetric matrix $\mathbf{H}_{equiv}$. If the channel filter $\mathbf{h}_{\mathbf{ch}}$ is not symmetric, we introduce a prefilter $\mathbf{w}$ as in [3], so we can assume $\mathbf{h} = \mathbf{h}_{\mathbf{ch}} * \mathbf{w}$ is symmetric (see Fig. 1).

The procedure is as follows: into each transmitted block, 2ν zeros are appended as prefix, and 2ν zeros as suffix. Hence,

$$\mathbf{x}_{zp} = [0, ...0, \mathbf{x}^T, 0, ...0]^T$$

is the $(N+4\nu) \times 1$ zero-padded version of the original symbol $\mathbf{x}$ of length N. This vector $\mathbf{x}_{zp}$ is transformed by the $(N+2\nu) \times (N+4\nu)$ matrix $\mathbf{H}$ of Eq.(1) to obtain the receiving data $\mathbf{y}_{zp}$ of length $N+2\nu$.

It is easy to see that $\mathbf{y}_{zp} = \mathbf{H} \cdot \mathbf{x}_{zp} + \mathbf{n} = \mathbf{H}_m \cdot \mathbf{x} + \mathbf{n}$, where $\mathbf{n}$ is a term related to the noise and $\mathbf{H}_m$ is the $(N+2\nu) \times N$ matrix of Eq.(5); recall that its columns are the N central columns of $\mathbf{H}$. Now, let us split $\mathbf{H}_m$ as

$$\mathbf{H}_m = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \mathbf{H}_c^T & \mathbf{H}_4^T & \mathbf{H}_5^T \end{bmatrix}^T,$$

where $\mathbf{H}_1$ contains the rows from 1 to ν, $\mathbf{H}_2$ from $\nu+1$ to 2ν, $\mathbf{H}_c$ from $2\nu+1$ to N, $\mathbf{H}_4$ from $N+1$ to $N+\nu$, and finally $\mathbf{H}_5$ the remaining ν last rows. Observe that all the above are $\nu \times N$ matrices, except for $\mathbf{H}_c$, which is an $(N-2\nu) \times N$ matrix. Next, $\mathbf{H}_{equiv}$ is built from $\mathbf{H}_m$ just folding its first (or last) ν rows (through the permutation $\mathbf{J}_\nu$) as in a mirror, and adding them to the adjacent (or previous) ν rows:

$$\mathbf{H}_{equiv} = \begin{bmatrix} \mathbf{J}_\nu \mathbf{H}_1 + \mathbf{H}_2 \\ \mathbf{H}_c \\ \mathbf{H}_4 + \mathbf{J}_\nu \mathbf{H}_5 \end{bmatrix}$$

We call this procedure "mirror and add" (MIA). This matrix $\mathbf{H}_{equiv}$ can be written as $\mathbf{H}_{equiv} = \mathbf{Z} + \mathbf{K}$, with $\mathbf{Z}$ the Toeplitz matrix

$$\mathbf{Z} = \begin{bmatrix} h_0 & \cdots & h_\nu & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \\ h_\nu & & & & \ddots & 0 \\ 0 & & & & & h_\nu \\ \vdots & \ddots & \ddots & & & \vdots \\ 0 & \cdots & 0 & h_\nu & \cdots & h_0 \end{bmatrix}$$

and $\mathbf{K}$ the Hankel matrix:

$$\mathbf{K} = \begin{bmatrix} h_1 & \cdots & h_\nu & 0 & \cdots & 0 \\ \vdots & & & & & \vdots \\ h_\nu & & & & & 0 \\ 0 & & & & & h_\nu \\ \vdots & & & & & \vdots \\ 0 & \cdots & 0 & h_\nu & \cdots & h_1 \end{bmatrix}$$

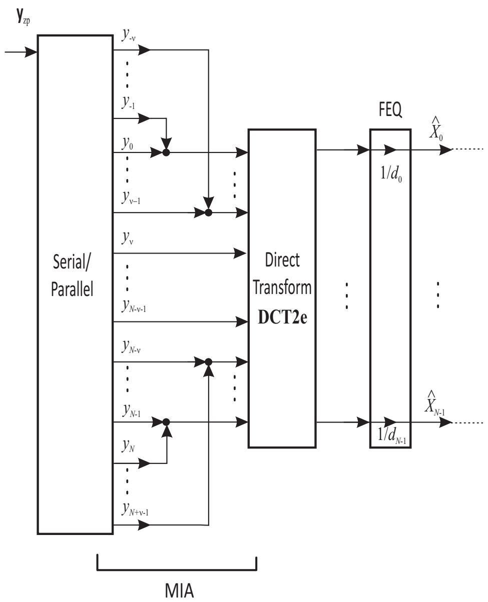

Fig. 2. Mirror and add (MIA) block processing at the receiver.

$\mathbf{Z}$ and $\mathbf{K}$ are symmetric as result of the symmetry of the filter and are related as indicated in [8]; hence, $\mathbf{H}_{equiv}$ can be diagonalized by means of the DCT2e.

To reconstruct the original symbol $\mathbf{x}$ from the receiving data $\mathbf{y}_{zp}$, we split $\mathbf{y}_{zp}$ and the noise $\mathbf{n}$ in the same way as $\mathbf{H}_m$:

$$\mathbf{y}_{zp} = \left[\begin{array}{ccccc} \mathbf{y}_1^T & \mathbf{y}_2^T & \mathbf{y}_c^T & \mathbf{y}_4^T & \mathbf{y}_5^T \end{array}\right]^T.$$

where

$$\mathbf{y}_1^T = \left[\begin{array}{ccc} y_{-\nu} & \cdots & y_{-1} \end{array}\right], \mathbf{y}_2^T = \left[\begin{array}{ccc} y_0 & \cdots & y_{\nu-1} \end{array}\right],$$

$$\mathbf{y}_c^T = \left[\begin{array}{ccc} y_\nu & \cdots & y_{N-\nu-1} \end{array}\right],$$

$$\mathbf{y}_4^T = \left[\begin{array}{ccc} y_{N-\nu} & \cdots & y_{N-1} \end{array}\right], \mathbf{y}_5^T = \left[\begin{array}{ccc} y_N & \cdots & y_{N+\nu-1} \end{array}\right].$$

Then, the same transformations on the rows of $\mathbf{y}_{zp}$ must be performed to get the $N \times 1$ receiving data:

$$\mathbf{y} = \left[\begin{array}{c} \mathbf{J}_\nu \mathbf{y}_1 + \mathbf{y}_2 \\ \mathbf{y}_c \\ \mathbf{y}_4 + \mathbf{J}_\nu \mathbf{y}_5 \end{array}\right] + \mathbf{M} \cdot \mathbf{n} = \left[\begin{array}{c} \mathbf{J}_\nu \mathbf{H}_1 + \mathbf{H}_2 \\ \mathbf{H}_c \\ \mathbf{H}_4 + \mathbf{J}_\nu \mathbf{H}_5 \end{array}\right] \mathbf{x} + \mathbf{M} \cdot \mathbf{n}$$
$$= \left[\begin{array}{ccccc} \mathbf{J}_\nu & \mathbf{I}_\nu & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{N-2\nu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_\nu & \mathbf{J}_\nu \end{array}\right] \mathbf{H}_m \cdot \mathbf{x} + \mathbf{M} \cdot \mathbf{n}.$$

Notice that this modification $\mathbf{y} = \mathbf{M} \cdot \mathbf{y}_{zp}$ performs in the following way: the first (and last) ν components of $\mathbf{y}_{zp}$ are symmetrized as in a mirror, and added to their adjacent ν components. In other words, it is a *mirror and add* procedure, so we will denote it as MIA.

To summarize, the whole transform is written as in Eq.(3):

$$\mathbf{y} = \mathbf{H}_{equiv} \cdot \mathbf{x} + \mathbf{M} \cdot \mathbf{n},$$

where the equivalent channel matrix can be expressed as $\mathbf{H}_{equiv} = \mathbf{M} \cdot \mathbf{H}_m$, being

$$\mathbf{M} = \left[\begin{array}{ccccc} \mathbf{J}_\nu & \mathbf{I}_\nu & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{N-2\nu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_\nu & \mathbf{J}_\nu \end{array}\right].$$

and $\mathbf{H}_{equiv}$ can be perfectly diagonalized via the DCT2e,

$$\mathbf{H}_{equiv} = \mathbf{C}_{2e}^{-1} \cdot \mathbf{D} \cdot \mathbf{C}_{2e}. \tag{7}$$

Notice that the MIA block processing, performed by $\mathbf{M}$, is required at the receiver before the DCT2e block transform (see Fig. 2).

3.1. FEQ coefficients

Now, we obtain the coefficients d_i $(i \leq 0 \leq N-1)$ to efficiently equalize the frequency-selective transmission channel under the zero-forcing criterion. Being d_i the entries of $\mathbf{D}$ in (7), it is shown (see [5, 8]) that these diagonal entries can be obtained as the first N components of

$$\mathbf{d} = \mathrm{DCT1e}(\mathbf{h}_{\mathrm{ZP}}^{\mathrm{r}}), \tag{8}$$

where DCT1e stands for Discrete Cosine Transform Type-I even, and $\mathbf{h}_{ZP}^r$ is the $(N+1)-$length vector defined as

$$\mathbf{h}_{ZP}^r = [h_0, \cdots, h_\nu, 0, \cdots, 0].$$

The result in (8) can also be expressed by means of the DCT2e of the first column of $\mathbf{H}_{equiv}$, which is

$$\mathbf{h}_{equiv} =$$
$$\left[\begin{array}{ccccccc} h_0 + h_1 & h_1 + h_2 & \cdots & h_{\nu-1} + h_\nu & h_\nu & 0 & \cdots & 0 \end{array}\right]^T.$$

One gets

$$d_i = \frac{(\mathbf{C}_{2e}\mathbf{h}_{equiv})_{i+1}}{(\mathbf{C}_{2e})_{i+1,1}},\ i = 0, \ldots, N-1. \tag{9}$$

Summarizing, the coefficients of the FEQ can be computed using the DCT1e as in (8) or using the DCT2e as in (9). In this last case, all the transforms are carried out by means of the DCT2e, which makes the transceiver implementation easier.

4. CONCLUSIONS

In this work, we have presented a general formulation for the use of ZP with any Discrete Transform in multicarrier communications. Moreover, we have derived conditions for the application of ZP with the Discrete Cosine Transform Type-II even. The channel matrix has been modified in order to be diagonalized by the DCT2e transform. Matrix formulation has been used to meet the conditions that allowed perfect reconstruction of the original symbol at the receiver. The technique consists in using zero padding and including a mirror and add (MIA) block processing at the receiver. Furthermore, the values of the coefficients for the one-tap per subcarrier equalizer have also been provided by means of a DCT transform.

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